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*Abstract-* **Heterogeneity is the key distinguishing characteristic of wireless sensor networks (WSN), compared to wireless cellular networks (WCN). It may appear in the form of communication ranges of sensor devices, synchronization and signal processing capabilities. In this article, we develop a hybrid location estimation scheme for heterogeneous WSNs with unsynchronized short-range simple relays and mobile sensors nodes, and synchronized stations. Our scheme employs combination of time difference of arrival (TDOA) and received signal strength (RSS) measurements, whereby the TDOA is used to filter out the clock offset that appears due to the lack of synchronization. We quantify the estimation accuracy of the scheme by deriving the Cramer-Rao bound (CRB), and discuss the performance trade-off between the number of synchronized and non-synchronized devices involved.**

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## I. INTRODUCTION

Accurate location information in wireless sensor networks is crucial for organizing both communication, and sensor-based measurement and computations. Locating techniques may be used for estimating sensor locations, tracking moving objects within a region and locating other physical entities such as sources of sounds. Applications requiring a certain degree of knowledge of sensor locations include environmental and structural monitoring (e.g., water quality monitoring and in-door air quality monitoring), indoor user tracking (e.g., locating of patients and medical personnel in hospitals and employee tracking in companies) and others. There is a variety of locating techniques developed for tracking mobile terminals in WCNs. However, they are not directly applicable to WSNs. Providing accurate location estimation in WSNs is a more difficult task than that in WCNs, due to multidimensional heterogeneity that may appear in the form of communication ranges of sensor devices, synchronization and signal processing, routing capabilities etc. For instance, the emerging ZigBee standards, which use IEEE 802.15.4 MAC/PHY, define two device types in the networking and routing layer: RN- and RN+. They are both full function devices (FFD), according to the IEEE 802.15.4 standard [1]. RN- nodes do not have any memory and they don't have routing tables, unlike RN+ nodes. On the other hand, both RN- and RN+ can measure the RSS of received signals, and they are not synchronized [2].

The lack of synchronization in WSNs is a major obstacle for the use of location estimation techniques designed for WCNs, particularly time of arrival (TOA), where transmitters and receivers need to have synchronized clocks and bi-directional communication [3], [4]. The TDOA approach takes the difference of arrival times of different TOA measurements, whereby sacrificing one TOA measurement to filter out the clock offset. On the other hand, the RSS-based location estimation is known to under-perform both TOA and TDOA, and is limited by the distance to the reference device [5], [6]. Synchronizing sensor nodes with the rest of the network requires a highly precise central clock inducing high design complexity. Furthermore, even though the synchronization is achieved, due to short range of sensor devices at least three such synchronized stations must be observed within the radio coverage of a sensor node for trilateration. This drastically increases the required number of synchronized devices in the network. In [5], the authors established the feasibility criteria for location estimation in wireless sensor networks, in terms of the relationship between the number of reference devices and the number of devices whose location is being determined (blindfolded devices). They also computed the CRB for TOA-based and RSS-based estimation for arbitrary number of reference and blindfolded devices satisfying the feasibility criteria. They did not consider the case of heterogeneous network and location estimation based on the combination on different location techniques.

In this article, we address the location estimation problem in a partially synchronized, heterogeneous WSN, comprised of sensor nodes (SN) that have very short transmission ranges (e.g., 30m) and no sense of timing, relay nodes (RN) that have simple routing capabilities and are not synchronized with the network, and mutually synchronized absolute position routers (APR). The SNs can be corresponded to the RN- devices in the ZigBee standards, and the RNs to the RN+ devices. We propose a hybrid location estimation scheme (H-LES), which is based on the combination of TDOA measurements between the SNs and the APRs that SNs can hear, and RSS measurements between the SNs and their neighboring RNs. The TDOA technique is used in order to mitigate the lack of synchronization between SNs and APs. The advantage of the H-LES is that it exploits the both TDOA and RSS. The TDOA measurements are used to provision improved location accuracy in an unsynchronized network, while the use of RSS measurements at RNs enhances the accuracy within the neighborhood of RNs. In the H-LES, it is easy to quantify the trade-off between the number of synchronized routers, the density of relay nodes and the achievable location estimation accuracy. This gives an answer to an important design consideration.

Another advantage of the H-LES is that it takes into account the heterogeneity in communications ranges of network devices. The location estimation schemes available in the literature are based on the assumption that all the nodes have the same radio communication range. In other words, if node-A can hear node-B, node-B can also hear node-A. This is not realistic, because of the different transmission powers, battery capacities and overall complexity of the devices.

In Section II, the description of the H-LES scheme is given. The Cramer-Rao bound (CRB) for the estimation accuracy of this scheme is derived in Section III. The results are discussed in the light of the performance trade-off between the number and density of APRs (i.e. TDOA measurements) and RNs (i.e. RSS measurements) involved. In Section III, we present the conclusion and perspectives for future work. In the appendix, the details of the derivation of CRB are given.

## II. HYBRID LOCATION ESTIMATION TECHNIQUE (H-LES)

The hybrid location tracking system, as illustrated in Fig.1, consists of three device types: SNs, RNs and APRs. The SNs are simple sensor devices with a radio circuitry of very short communication range (e.g., 10-30 meters). They are only capable of listening to and forwarding signals. The RNs are low-cost devices with simple routing capabilities and ability to measure the RSS of received signals. They also have a very short communication range (e.g., 10-30meters). The APRs are stationary known-location devices with a precise central clock. The communication range of the APR is very long (e.g., 100 meters or longer). They are responsible for collecting messages from RNs and mobile SNs, and forwarding them to the Central Monitoring Unit (CMU) through an aggregation point. The APRs also distribute commands received from the central monitoring unit to the corresponding RNs and SNs.

### *A. Message Types and Messaging*

Figure 1 illustrates signaling and timing of messages among system components for tracking the location of SNs. The numbers inside the parentheses indicate the time order of the messages. The indexes assigned to the APRs are used to identify them; and 0 index is used for the SN to be tracked. The following routine explains how messages are initiated and forwarded in the two APR case. The extension to the cases involving three or more APRs is straightforward, in which one extra message would arrive to the SN from each additional APR. Note that the scheduling of the broadcast beacons from the APRs is not within the scope of the current work, and will be addressed in the future.

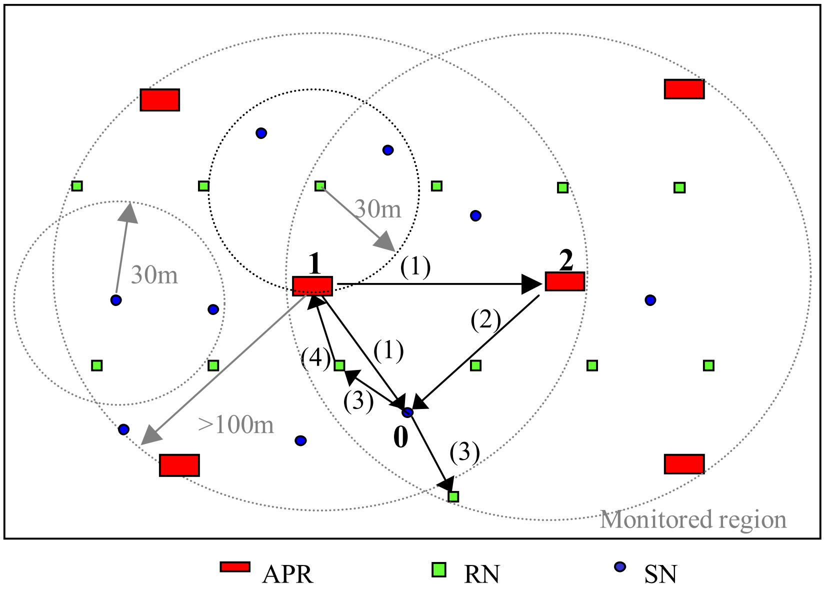

Figure 1: Messaging in the H-LES

**The two APR case:** Messaging

1: The APR-1 broadcasts a beacon with a unique sequence number, message (1). The sequence number is used to prevent duplicate processing of the same message. The broadcast may be event-driven and instructed by the CMU, or it may be periodic.

2: The APR-2 and the sensor node SN-0, which are within the coverage area of APR-1, receive message (1).

3: The APR-2 modifies message (1) by appending its own ID, but maintaining the original sequence number. Afterwards, it broadcasts the modified message, (2).

4: Upon receiving a message from an APR, the SN-0 inserts its ID in the header, and forwards the message to its neighboring RNs, because it may not reach the APR directly due to its shorter communication range. The destination address for this message, (3), is the CMU.

5: The one-hop RN (could be one RN, like in the figure, or more) that receives message (3) inserts its ID to the message, and the RSS level of the signal. It then forwards the message, (4) for the CMU, through other intermediate RNs. These intermediate RNs do not need to modify the packet, but simply forward it until it reaches the destination. Note that by moving the RSS computation to the relay devices, mobile sensors are spared from measuring the RSS of signals from routers.

As described before, there are four different messages. The content of each message is given below.

**The two APR case:** Message Contents

1: Message (1) contains the time stamp of departure from APR-1, the ID of APR-1 and a unique sequence number to identify the broadcast beacon.

2: Message (2) contains the time stamp of departure from APR-1, the time stamp of departure from APR-2, the ID of APR-1, the ID of APR-2 and the same sequence number in message (1).

3: Message (3) contains the time stamp of departure from APR-1, the time stamp of departure from APR-2, the ID of APR-1, the ID of APR-2, the same sequence number in message (1) and the ID of mobile sensor SN-0.

4: Message (4) contains the time stamp of departure from APR-1, the time stamp of departure from APR-2, the ID of APR-1, the ID of APR-2, the same sequence number in message (1), the ID of mobile sensor SN-0 and the RSS of message (3).

### *B. Timing Diagram*

Table-I presents the notations that are used in illustration of the timing of the messages in Fig.2 and in Section III. Taking the difference of the arrival times of messages (1) and (2) at SN-0 yields $t_{1,2}+t_{P2}+t_{2,0}-t_{1,0}$, which is an observation that is free from the unknown time offset $t_{0\_OFF}$. The CMU knows a-priori what $t_{1,2}$ is, and can derive the processing delay $t_{P2}$ from the difference of timestamps $t_1$ and $t_2$. So, the CMU can filter out $t_{1,2}$ and $t_{P2}$, and retrieve the observation of the *difference of the propagation times* $t_{i,0}$ and $t_{j,0}$, i.e. $T_{i,j} = t_{i,0} - t_{j,0}$. Note that with $m$ APRs, $m$-1 independent TDOA observations $T_{i,j}$ are available.

TABLE-I

TIMING NOTATIONS OF THE MESSAGES

| | |
|---|---|
| $t_i$ | Timestamp of departure at APR-*i* |
| $t_{i,j}$ | Time for the signal to traverse from APR-*i* to APR-*j* |
| $t_{Pi}$ | Processing delay at APR-*i* |
| $t_{i,0}$ | Time for the signal to traverse from APR-*i* to SN-0 |
| $t_{0\_OFF}$ | Time offset between APRs and the SN-0 |
| $d_{i,j}$ | Distance between APR-*i* and APR-*j* |
| $d_{i,0}$ | Distance between APR-*i* and SN-0 |

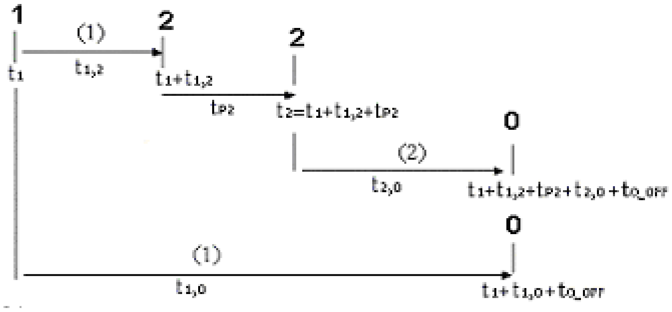

Figure 2: Timing diagram of messages (1) and (2)

The actual derivation of the location estimate is omitted here due to space limitations. Essentially, any estimator based on the above described set of TDOA observations and RSS observations can be used. The Maximum Likelihood Estimator (MLE) can easily be obtained along the lines of the derivation in [5]. In the following section, we derive the minimum achievable variance of any given estimator for H-LES.

## *III. THE CRB OF THE H-LES*

In this section, we derive the CRB, i.e. minimum variance of the location estimation error of any estimator, achievable by the H-LES. We consider a general case with *m* synchronized APRs, with indexes 1,…,*m*, and *n* RNs, indexed as *m*+1,…,*m*+*n*. We assume that the locations of the APRs and RNs are known. We denote the SN, of which the location is being estimated, with index 0. The results presented here can be straightforwardly extended to the more general case of location estimation of arbitrary number of SNs. For the sake of brevity, only the case of a single SN is considered in this paper.

Assume that the actual coordinate of the SN to be tracked is $\theta_0 = [x_0, y_0]$. The location estimation problem is the estimation of the coordinate vector, $\hat{\theta}_0$, given the known coordinates of the APRs and the RNs. The estimation is based on *m*-1 TDOA observations and $n$ RSS observations, organized in the observation vector $X = [X_T , X_R]= [[T_{1,2}, T_{2,3}\ldots,T_{m-1,m}], [P_{0,m+1},\ldots, P_{0,m+n}]]$, in which:

- $T_{i,i+1} = t_{i,0} - t_{i+1,0}$ , $i < m$, is the observed difference of the propagation times between APR-$i$ and SN-0, APR-$i$+1 and SN-0 respectively, as described in Section II. It is worth pointing out again that with $m$ APRs, $m$-1 observations are available, while the $m^{\text{th}}$ observation is sacrificed to eliminate the unknown clock offset of the SN-0 with respect to the clock of the APRs.
- $P_{0,i}$ , $i>m$, represents an RSS observation of the received power of message (3), transmitted by SN-0 and received by RN $i$.

Propagation times $t_{i,0}$ and $t_{i+1,0}$ are commonly assumed to have a normal distribution with means $d_{i,0}/c$ and $d_{i+1,0}/c$ , respectively, where $d_{i,j} =[(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$. We denote the variance of the distribution of the multipath delay profile of the propagation environment as $\sigma_T^2$. The pdf of $T_{i,j}$ is then $T_{i,j} \sim \mathrm{N}((d_{0,i} - d_{0,1+1})/c, 2\sigma_T^2)$. Hence, the cost of eliminating the unknown clock offset of the SN includes the drawback of doubling the variance of the error.

The pdf of $P_{0,i}$ is $P_{0,i}(dB) \sim \mathrm{N}(P_0(dB), \sigma_{dB}^2)$, where $P_0(dB) = P_t(dB) - 10n_p \log_{10}(d_{0,i})$ is the mean received power, $P_t(dB)$ the mean transmitted power of SN-0, $n_p$ the propagation exponent, and $\sigma_{dB}^2$ is the variance of lognormal shadowing. The CRB of an unbiased estimator $\hat{\theta}_0$ is $\mathrm{cov}(\hat{\theta}_0) \geq I(\theta_0)^{-1}$, where $I(\theta_0)$ is the Fisher information matrix (FIM), defined as

$$I(\theta_0) = -E\nabla_{\theta_0}(\nabla_{\theta_0} l(X|\theta_0)) \tag{1}$$

where $l(X|\theta_0)$ is the logarithm of the joint conditional probability density function.

$$l(X|\theta_0) = \sum_{\substack{i=1 \\ i\neq m}}^{m+n} l_{0,i} = \sum_{i=1}^{m-1} \log f_{X_T|\theta_0}(T_{i,i+1}|\theta_0) + \sum_{i=m+1}^{m+n} \log f_{X_R|\theta_0}(P_{0,i}|\theta_0) \tag{2}$$

It can be shown that Eq.1 simplifies to

$$\begin{aligned} I(\theta_0) &= -\sum_{\substack{i=1 \\ i\neq m}}^{m+n} E\left[\partial^2 l_{0,i} / \partial\theta_0^2\right] = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix} = \begin{bmatrix} I_{Txx} + I_{Rxx} & I_{Txy} + I_{Rxy} \\ I_{Txy} + I_{Rxy} & I_{Tyy} + I_{Ryy} \end{bmatrix} \\ &= \begin{bmatrix} I_{Txx} & I_{Txy} \\ I_{Txy} & I_{Tyy} \end{bmatrix} + \begin{bmatrix} I_{Rxx} & I_{Rxy} \\ I_{Rxy} & I_{Ryy} \end{bmatrix} = I_T(\theta_0) + I_T(\theta_0) \end{aligned} \tag{3}$$

where

$$I_{xx} = -\sum_{\substack{i=1 \\ i\neq m}}^{m+n} E\left[\partial^2 l_{0,i} / \partial x_0^2\right] = -\sum_{i=1}^{m-1} E\left[\partial^2 l_{0,i} / \partial x_0^2\right] - \sum_{i=m+1}^{m+n} E\left[\partial^2 l_{0,i} / \partial x_0^2\right] = I_{T_{xx}} + I_{R_{xx}} \tag{4}$$

Equivalent definitions are valid for $I_{xy}$ and $I_{yy}$. Finally, the CRB becomes

$$\sigma_{CRB}^2 = \min_{\hat{\theta}_0} E[(\hat{x}_0 - x_0)^2 + (\hat{y}_0 - y_0)^2] = \min tr\{\mathrm{cov}(\hat{\theta}_0)\} = tr\{I(\theta_0)^{-1}\} = \frac{I_{xx} + I_{yy}}{I_{xx} I_{yy} - I_{xy}^2} \tag{5}$$

The derivatives $\partial^2 l_{0,i}/\partial x_0^2$, $\partial^2 l_{0,i}/\partial y_0^2$, $\partial^2 l_{0,i}/\partial x_0 \partial y_0$ for RSS observations were computed in [5]. As for $T_{i,j}$ observations, we derive these terms along with $I_{xx}$, $I_{xy}$, $I_{yy}$, and compute the FIM from Eq.3 and Eq.4. The details are given in the appendix. The resulting CRB can then be obtained from (5). General expression for the CRB being rather cumbersome, here we examine the CRB of the special case of $m = 3$, $n = 2$.

$$\sigma_{CRB\ (m=3,n=2)}^2 = \frac{3\left(c^2 2\sigma_T^2\right)^{-1} + b\sum\limits_{i=m+1}^{m+n} d_{0,i}^{-2}}{\left(c^2 2\sigma_T^2\right)^{-2}(A_{1,2}^2 + A_{2,3}^2 - A_{1,3}^2) + b\left(c^2 2\sigma_T^2\right)^{-1}\sum\limits_{i=1}^{m-1}\sum\limits_{j=m+1}^{m+n}\left(\frac{A_{i,j} - A_{i+1,j}}{d_{o,j}}\right)^2 + b^2\sum\limits_{i=m+1}^{m+n}\sum\limits_{\substack{j=m+1\\ j<i}}^{m+n}\left(\frac{A_{i,j}}{d_{i,j}^2}\right)^2} \tag{6}$$

where $A_{i,j} = \dfrac{d_{0xi,j} d_{i,j}}{d_{0,i} d_{0,j}}$ is a unit-less parameter, called *geometric conditioning* of SN-0 with respect to nodes *i* and *j* [7], and $d_{0xi,j}$ is the length of the shortest line segment connecting SN-0 with the line going through nodes *i* and *j*.

The first term in the denominator represents the contribution of TDOA measurements to the CRB, and it depends only on the geometric conditioning of the APRs with respect to the SN, but not on their distances. It is important to note that the distance between the APR and the SN impacts $\sigma_T^2$. However, given the order of magnitude of the distances involved, it is reasonable to neglect this effect in the first approximation. Therefore, the accuracy of the estimation depends only on the number of APRs that the SN-0 can hear and their geometry, regardless of their separation. The third term in the denominator represents the contribution of RSS observations of RNs. Due to $d_{i,j}^2$ in the denominator, the distance from the RNs to the SN plays a major role in the estimation accuracy. Reducing RN-SN separation by increasing the density of RNs improves the estimation accuracy. It may also lead to less transmission power requirement and hence extended battery life of SNs. The second term represents joint contribution of TDOA and RSS measurements. Its analysis is beyond the scope of this paper and will be the topic of future research.

Figures 3-7 show $\sqrt{\sigma_{CRB}^2}$ for the H-LES, computed inside the 50m by 50m region. The figures are produced by taking the channel measurements in [5] as a reference, and setting $\sigma_{dB}/n_P = 1.7$ and $\sigma_T = 6.1ns$. The RNs are located inside this square, while the APRs are located on the outer 100m by 100m region. It is assumed that at each point inside the inner square, the SN is able to hear the signals of all APRs and the RNs. The figures clearly show the difference between TDOA and RSS observations, and the benefits from their combination. TDOA observations are virtually independent on the distance to the

APRs (except in the close proximity of APRs), and hence provide an almost uniform contribution to the estimation accuracy within their coverage region. The number of APRs involved in the estimation has major effect on the performance of H-LES. By comparing Figures 3, 6 and 7, one can infer that replacing two RNs by two APRs reduces the minimum achievable CRB in the region by roughly three times. RSS observations, on the other hand, have significant effect on the accuracy only in the proximity of RNs. Therefore, the number of APRs and the density of RNs are the network design parameters that can be fine-tuned to achieve the desired trade-off between the network installation cost and the achievable location estimation accuracy.

Allocating one APR to mitigate the lack of synchronization, the case of two APRs is the most inefficient, since 50% of APRs are not exploited for TOA measurements. However, since only one APR needs to be sacrificed regardless of the total number of APRs used, the efficiency of the H-LES will get higher as the number of APRs in the network increases. It will asymptotically reach the CRB of a location estimation scheme that is based on convenional TOA and RSS measurements in a fully synchronized network in which the noise of the TOA observations is doubled. Note that for a direct use of the TOA in a fully synchronized network, the SNs must be able to directly communicate with the APRs. On the other hand, the H-LES removes this constraint.

It is evident from the comparison of Figures 5 and 6 that the impact of geometric conditioning of RNs with respect to APRs is measurable to the impact of the number of RNs involved. Hence, the mutual placement of APRs and RNs is an additional tool that can be used to provide the desired location estimation accuracy in the network, even if the number of RNs and APRs is fixed. Further analysis of the impact of geometric conditioning of APRs and RNS onto the location estimation accuracy in WSNs is the topic for future research.

## IV. CONCLUSION

We developed a hybrid location estimation scheme for partially synchronized sensor networks. The contribution with respect to existing location estimation techniques consists in taking into account the heterogeneity of sensor networks, in terms of communication range, time synchronization and routing capabilities of network devices. It provides the mean to take advantage of the benefits of the accuracy of TDOA-based location estimation of wireless sensor nodes that are unsynchronized with the rest of the network, hence greatly improving the accuracy with respect to the RSS-based estimation. At the same time, RSS measurements, supplied by low-cost unsynchronized simple relay nodes, are used to fine-tune the estimation. The scheme requires full synchronization only among the long-range fixed routers (APRs). The CRB of the proposed scheme is computed. The future work will focus on the investigation of the effect of mutual geometric conditioning of fixed routers and simple relay nodes on the achievable location estimation accuracy, and the ways to its optimization.

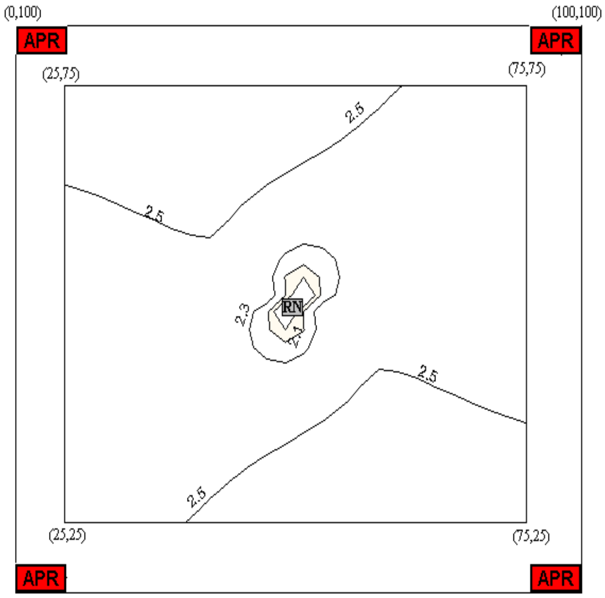

Fig. 3 – The CRB for 4 APRs and 1

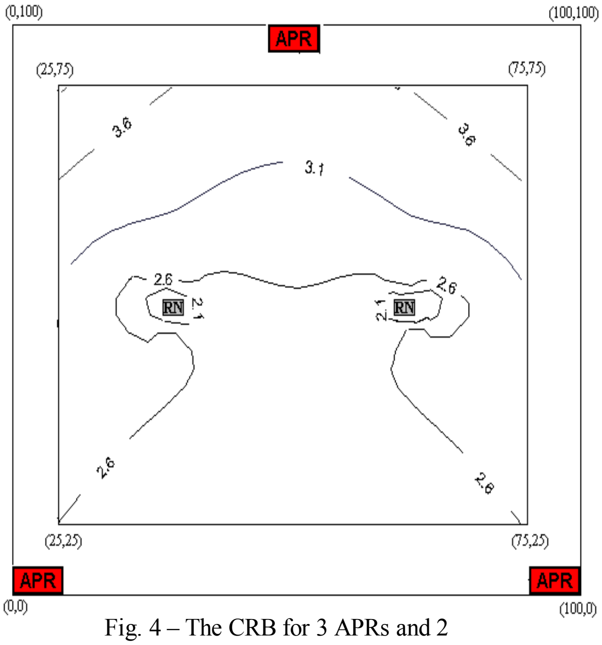

Fig. 4 – The CRB for 3 APRs and 2

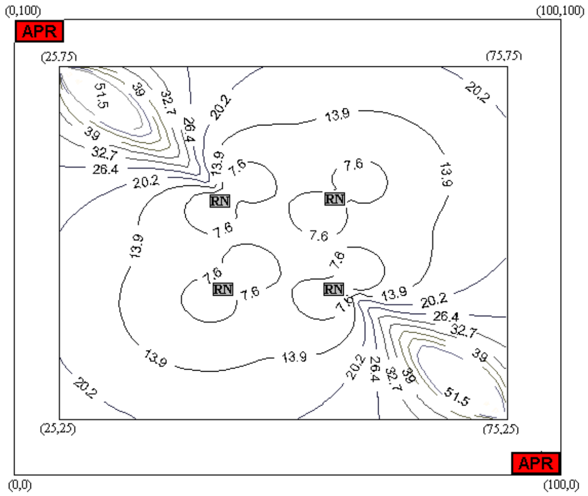

Fig. 5 – The CRB for 2 APRs and 3

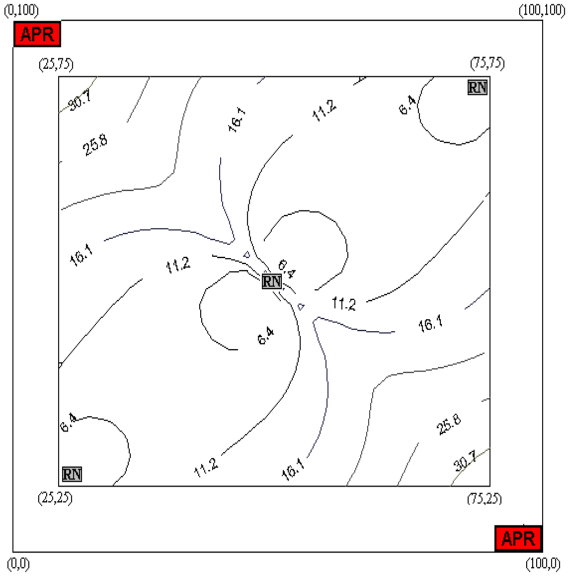

Fig. 6 – The CRB for 2 APRs and 3

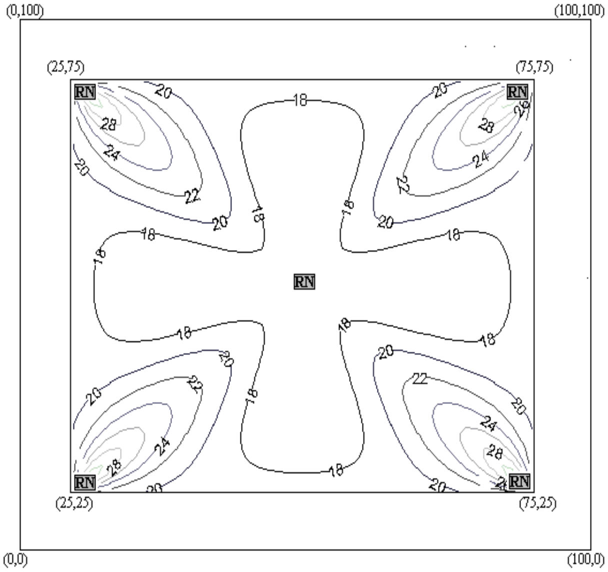

Fig. 7 – The CRB for 0 APRs and 5

## *Appendix*

In this appendix, we derive $\partial^2 l_{0,i}/\partial x_0^2$ , $\partial^2 l_{0,i}/\partial y_0^2$ , $\partial^2 l_{0,i}/\partial x_0 \partial y_0$ , $I_{xx}$, $I_{xy}$, $I_{yy}$.

For TDOA observations $T_{i,j}$, we derive them as follows:

$$l_{0,i} = \log f_{X_T|\theta_0}(T_{i,i+1}|\theta_0) = -\log(2\sqrt{\pi}\sigma_T) - [T_{i,j} - (d_{0,i} - d_{0,j})/c]/4\sigma_T^2 \quad i < m \tag{7}$$

$$E[\partial^2 l_{0,i}/\partial x_0^2] = -\frac{1}{2c^2\sigma_T^2}\left(\frac{x_0 - x_i}{d_{0,i}} - \frac{x_0 - x_{i+1}}{d_{0,i+1}}\right)^2 \qquad E[\partial^2 l_{0,i}/\partial y_0^2] = -\frac{1}{2c^2\sigma_T^2}\left(\frac{y_0 - y_i}{d_{0,i}} - \frac{y_0 - y_{i+1}}{d_{0,i+1}}\right)^2 \tag{8}$$

$$E[\partial^2 l_{0,i}/\partial x_0^2] = -\frac{1}{2c^2\sigma_T^2}\left(\frac{x_0 - x_i}{d_{0,i}} - \frac{x_0 - x_{i+1}}{d_{0,i+1}}\right)\left(\frac{y_0 - y_i}{d_{0,i}} - \frac{y_0 - y_{i+1}}{d_{0,i+1}}\right)$$

Based on the above derivatives for TDOA observations, and the derivatives for RSS observations given in [5], we have

$$I_{xx} = I_{Txx} + I_{Rxx} = \frac{1}{2c^2\sigma_T^2}\sum_{i=1}^{m-1}\left(\frac{x_0 - x_i}{d_{0,i}} - \frac{x_0 - x_{i+1}}{d_{0,i+1}}\right)^2 \qquad I_{Rxx} = b\sum_{i=m+1}^{m+n}\frac{(x_0 - x_i)^2}{d_{0,i}^4} \tag{9}$$

$$I_{xy} = I_{Txy} + I_{Rxy} = \frac{1}{2c^2\sigma_T^2}\sum_{i=1}^{m-1}\left(\frac{x_0 - x_i}{d_{0,i}} - \frac{x_0 - x_{i+1}}{d_{0,i+1}}\right)\left(\frac{y_0 - y_i}{d_{0,i}} - \frac{y_0 - y_{i+1}}{d_{0,i+1}}\right) \qquad I_{Rxy} = \sum_{i=m+1}^{m+n}\frac{(x_0 - x_i)(y_0 - y_i)}{d_{0,i}^4} \tag{10}$$

$b = \left(\frac{10 n_p}{\sigma_{dB}\ \log 10}\right)^2$. $I_{Tyy}$ and $I_{Ryy}$ is analogous to $I_{Txx}$ and $I_{Rxx}$. Finally, the CRB and the FIM can be computed from Eq.3 and Eq.5 respectively.

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