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# Spectral Shape of UWB Signals – Influence of Modulation Format, Multiple Access Scheme and Pulse Shape

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**Abstract:** This paper studies how to design the spectrum of a UWB signal in accordance with the FCC regulations and IEEE 802.15.3a recommendations. We show that the “conventional” UWB system with pulse-position modulation and time-hopping multiple access gives rise to spectral lines that violate the regulations. The impact of different modulation and multiple access schemes on the spectrum shaping is derived from the power spectral density of a non-linear and memoryless modulation. Detailed theoretical and simulation results stress the difficulties raised by the use of dithered pulse trains. We thus propose several solutions to achieve compliance with the FCC spectral masks.

## I. INTRODUCTION

Ultra Wideband (UWB) systems have recently received considerable attention for future commercial and military wireless communication systems [5-7]. Especially, the decision of the US Federal Communications Commission (FCC) to allow preliminary UWB systems [2, 3] has created enormous interest. Following the FCC ruling, a standards committee has been established by the IEEE [4] that should develop wireless personal area networks (wireless PANs) with data rates in excess of 100 Mbits/s.

Commonly, UWB systems communicate with trains of short-duration pulses with a low duty-cycle, and thus spread the energy of the radio signal very thinly over a wide range of frequencies [5,6]. Almost all of the systems analyzed in the open literature use a combination of time-hopping spreading (for multiple access) and pulse position modulation (as modulation format). We will show in this paper that this combination results in spectral lines that either lead to a violation of the FCC requirements, or require a significant power back off, and thus loss of performance and/or range. We will also present alternative modulation and multiple-access schemes that remedy these problems.

In Section 2, we review the most relevant requirements of the FCC and the IEEE standards. Section 3 gives analytical formulations for the spectrum of “conventional” TH-PPM, showing the spectral lines created by this MA/modulation scheme. Subsequently, we discuss

remedies for this problem, namely the randomization of the polarity of the transmitted signals, either on a symbol-by-symbol or on a pulse-by-pulse basis. The impact of using long or short hopping sequences is also analyzed.

## II. STANDARDS REQUIREMENTS AND FCC REGULATIONS

The IEEE 802.15.3a group has defined the baselines for the use of UWB in short range indoor communication system [4]. Data rates of at least 110 Mbps at 10 meters are required at the PHY-SAP, which means after the FEC and any overhead. The transmission rate would be greater. Furthermore a bit rate of *at least* 200 Mb/s at 4 meters is required. Scalability to rates in excess of 480 Mb/s is desirable even if the rates can only be achieved at reduced ranges. These system requirements provide us with a range of values for the Pulse Repetition Frequency (PRF). In this paper, a data rate of 100Mbps will be used with the assumption of 1 bit per symbol to simplify the calculations.

In February 2002, the FCC released the First Order and Report [3] providing power limits. The average limits over all useable frequencies are different for indoor and outdoor systems. These limits are given in the form of a power spectral density mask. In the band from 3.1GHz to 10.6 GHz, the power spectral density (PSD) is limited to  $-41.25\text{dBm/MHz}$ . The limits on the PSD must be fulfilled for each possible 1MHz band, but not necessarily for smaller bandwidths.

For systems operating above 960 MHz, there is a limit on the peak emission level contained within a 50 MHz bandwidth centered on the frequency,  $f_M$ , at which the highest radiated emission occurs. The FCC has adopted a peak limit based on a sliding scale dependent on the actual resolution bandwidth (RBW) employed in the measurement. The peak EIRP limit is  $20 \log(\text{RBW}/50)$  dBm when measured with a resolution bandwidth ranging from 1 MHz to 50 MHz. Only one peak measurement, centered on  $f_M$ , is required. As a result, UWB emissions are average-limited for PRFs greater than 1 MHz and peak-limited for PRFs below 1 MHz.

These limits and the requirements from 802.15.3a and the FCC provide the foundation for our analysis. They result in constraints on the pulse shape, the level of the total power used, the PRF, and the positions and amplitudes of the spectral lines.

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### III. SPECTRA OF TH-PPM SIGNALS

#### III.1 M-PPM

Many UWB signals consider Pulse Position Modulation (PPM) for modulation. The spectrum can be obtained by considering this dithered signal as a M-PPM signal. Because PPM is a non-linear modulation we start the analysis with the equation for the PSD of signals with non-linear memoryless modulation. If the modulating sequence is composed of independent and equiprobable symbols the PSD is:

$$G_s(f) = \frac{1}{M^2 T_s^2} \cdot \sum_{n=-\infty}^{+\infty} \left( \left| \sum_{i=0}^{M-1} S_i \left( \frac{n}{T_s} \right) \right|^2 \delta \left( f - \frac{n}{T_s} \right) \right) \quad (1)$$

$$+ \frac{1}{T_s} \left( \sum_{i=0}^{M-1} \frac{1}{M} \cdot |S_i(f)|^2 - \left| \sum_{i=0}^{M-1} \frac{1}{M} \cdot S_i(f) \right|^2 \right)$$

where M denotes the number of symbols,  $T_s$  the symbol period, and  $S_i$  the PSD of the  $i^{\text{th}}$  symbol of the constellation.

As seen from the first term in (1), spectral lines are inherent in PPM. The spectrum of a signal with a 2-PPM usually contains spectral lines spaced by the PRF. Rewriting Eq. (1) as the sum of a discrete and a continuous part,

$$G_s(f) = \frac{1}{2T_s^2} \sum_{n=-\infty}^{+\infty} \left( \left| \sum_{i=0}^{M-1} F_p \left( \frac{n}{T_s} \right) \right|^2 (1 + \cos(2\pi n x)) \right) \cdot \delta \left( f - \frac{n}{T_s} \right)$$

$$+ \frac{1}{2T_s} |F_p(f)|^2 (1 - \cos(2\pi f x T_s)) \quad (2)$$

we find that the amplitude of these spectral lines can be  $10 \cdot \log_{10}(T_s^{-1})$  dB above the level of the continuous part of the spectrum. That corresponds to 80dB for 100Mbps data rate

The measurement procedures mandated by the FCC average the power of these lines over the resolution bandwidth; however even then their level remains higher than the continuum and they thus violate the FCC limits or require a reduction of the total power. Time hopping (TH) is generally proposed as a method that can help reduce the problem of spectral lines, reducing their number in a given frequency band. However, we will show below that TH does not necessarily attenuate the *amplitude* of the remaining lines.

#### III. Infinitely long TH sequence

In order to evaluate the impact of TH on the spectral lines we can simplify the analysis by exploiting the similarity between M-PPM and TH sequence. Both TH and PPM result in a dithered pulse train. In a TH scheme, the symbol duration  $T_s$  is split into N frames with 1 pulse per frame (Fig.1). Within each frame, the pulse can take M positions. Thus, a non-periodic unmodulated Time Hopped signal is a pulse train

where each individual pulse can be in one of M equiprobable positions within its frame

This signal has the same spectrum as a M-PPM signal with the same PRF and uncorrelated modulated data. The effective PRF (denoted henceforth as  $f_{PR}$ ) is the inverse of the frame duration  $T_f$  (instead of the inverse of the symbol duration).

Since the signal is formally equivalent to a M-PPM signal, Eqs. (1) and (2) remain valid. If the possible positions are equally spaced within the frame, the discrete part of the spectrum now becomes

$$G_d(f) = \frac{1}{T_s^2} \cdot \sum_{n=-\infty}^{+\infty} \left( \left| F_p \left( \frac{Mn}{T_f} \right) \right|^2 \delta \left( f - \frac{Mn}{T_f} \right) \right),$$

All the spectral lines are spaced  $MN/T_s$  apart. By increasing the number of pulse positions within the frame we remove some spectral lines.

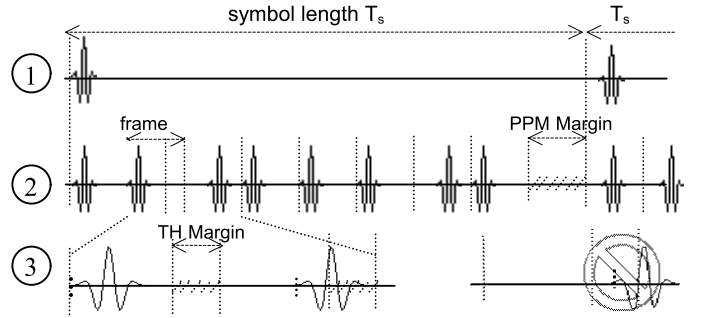


Fig.1 Symbol structure ①1pulse/symbol ②TH sequence of 8 pulses ③Frame

If M (or N) goes to infinity, which is equivalent to a uniform distribution of the pulse over the frame, all spectral lines occur at infinite spacing and thus effectively vanish. Unfortunately, practical considerations prevent the use of infinite M in UWB systems. The generation of TH positions via PN sequences, or other digital codes, or from stored tables, requires a rough quantization, and thus a finite number of possible positions M.

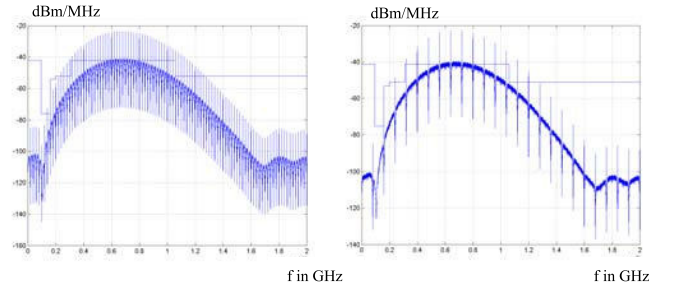


Fig. 2 Spectra of M-ary PPM with M=2 (left) and M=8 (right).

### III.3 PPM with finite length TH sequence

For realistic systems, we have to consider a finite duration of the time-hopping sequence. Almost all systems considered in the literature up to now assume that the duration of the TH sequence is identical to the symbol duration; in analogy to CDMA, we henceforth refer to this case as “short” hopping sequence. Furthermore, the number of possible positions within a frame, i.e.,  $M$ , is restricted to a the ratio of transmission bandwidth to data rate. In the case of IEEE 802.15.3a systems, this number cannot exceed 70 (for the base data rate) or even 20 (for the optional high data rate). Finally, it is necessary to introduce guard intervals to make sure that PPM-shifted pulses cannot overlap with pulses from the next symbol. All these effects lead to a modification of the spectrum (compared to the case of Sec. III.2). For space reasons, we omit here the equations, which derive from Eq.1 with the different symbols  $S_i$  combining PPM and short TH sequences, and just show a plot of a resulting signal spectrum in Fig. 3, with a 2-PPM and a short TH sequence (1001) of 4 pulses.

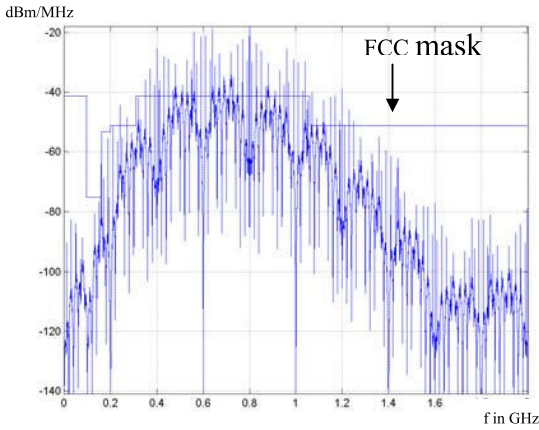


Fig.3 Spectrum of a 2.5% 2-PPM with TH sequence. 2.5% defines the shifted pulse position (the value of  $x$  in Eq.2)

## IV. POLARITY RANDOMIZATION

We have seen above that PPM-TH leads to the existence of spectral lines, which in turn require a power backoff of the total signal. In order to avoid this, we propose to randomize the polarity of the transmitted pulses. By randomizing, we mean changing the sign in a pseudorandom manner that is known to both the transmitter and receiver, and can thus be reversed by the receiver. We will also show that – under specific circumstances – it also allows to reduce the problem of designing an optimum transmit spectrum of the complete signal to the much simpler problem of designing optimum basis pulses.

### IV.1 Symbol-based randomization

The first case we consider randomizes the polarity (sign) of the transmitted pulses on a symbol-by-symbol basis.

This has a formal similarity to a combination of joint PPM and PAM as modulation format (combined with TH for MA). The difference is that in our case, the sign of the transmit pulses does not bear any information, and can thus be easily discarded by simplified receiver structures (envelope detectors). The introduction of pseudorandom polarity serves only to improve the spectral properties of the transmit signal. As we change randomly the polarity of the  $M$  symbols  $S_i$ ; the signal can be viewed as a pulse train composed of  $2M$  antipodal symbols. Thus the Equation (1) can be rewritten as the spectral density of a signal with antipodal modulation:

$$G_s(f) = \frac{1}{M^2 T_s^2} \cdot \sum_{n=-\infty}^{+\infty} \left( \left| \frac{1}{2} \sum_{i=0}^{M-1} S_i \left( \frac{n}{T_s} \right) + \frac{1}{2} \sum_{j=0}^{M-1} S_j \left( \frac{n}{T_s} \right) \right|^2 \delta \left( f - \frac{n}{T_s} \right) \right) + \frac{1}{T_s} \left( \sum_{i=0}^{M-1} \frac{1}{M} \cdot |S_i(f)|^2 - \left| \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{2} S_i(f) + \frac{1}{M} \sum_{j=0}^{M-1} \frac{1}{2} S_j(f) \right|^2 \right)$$

where  $S_i$  is the Fourier Transform of the  $i^{\text{th}}$  shifted version of the short TH sequence (not a single pulse!). The symbols  $S_{i+M}$  are the symbols  $s_i$  with an opposite polarity. Hence  $S_{i+M} = -S_i$  for  $i$  from 0 to  $M-1$ . The continuous and discrete part of the spectrum then becomes

$$G_s(f) = \frac{1}{M^2 T_s^2} \cdot \frac{1}{2} \sum_{n=-\infty}^{+\infty} \left( \left| \sum_{i=0}^{M-1} \left( S_i \left( \frac{n}{T_s} \right) - S_i \left( \frac{n}{T_s} \right) \right) \right|^2 \delta \left( f - \frac{n}{T_s} \right) \right) + \frac{1}{T_s} \left( \sum_{i=0}^{M-1} \frac{1}{M} \cdot |S_i(f)|^2 - \left| \frac{1}{2M} \sum_{i=0}^{M-1} (S_i(f) - S_i(f)) \right|^2 \right)$$

$$G_s(f) = \frac{1}{M T_s} \sum_{i=0}^{M-1} |S_i(f)|^2. \quad (3)$$

Thus, the spectral lines disappear. Furthermore, from Equation (3), it appears that the spectrum of the signal is defined by the summation of the spectrum of the symbols. If the symbols have the same waveform, the spectral properties of the signal are identical to the spectral properties of this waveform.

From Eq. (3), it is also clear that the same statement holds for different modulation formats, like PPM, BPSK, and on-off keying. Furthermore it is unnecessary to have zero mean *information stream* to control the spectral characteristics of the modulated signal. That solves the problem of spectral lines caused by non-equiprobable symbols and non-antipodal modulation schemes at the same time.

Thus changing randomly the polarity from symbol to symbol (symbol based polarity hopping) provides an efficient way to shape the spectrum. The task of spectrum shaping of the signal is determined by the design of the symbol waveform (TH sequence). The spectral density of the symbol is the multiplication between the spectral density of the pulse and the spectral density of the TH sequence. In the following example we show that the efficient design of the continuous part of the spectrum is also a daunting task. The TH sequence leads to variations of the spectrum that also lead to a reduction of the total transmit power. Figure 4 shows the spectrum of a

signal with the TH seq 1001; the peak values are at 6dB above envelope.

We note that the design of TH sequences with low peak-to-average ratio has a formal similarity with the design of OFDM signals with low crest factor [10].

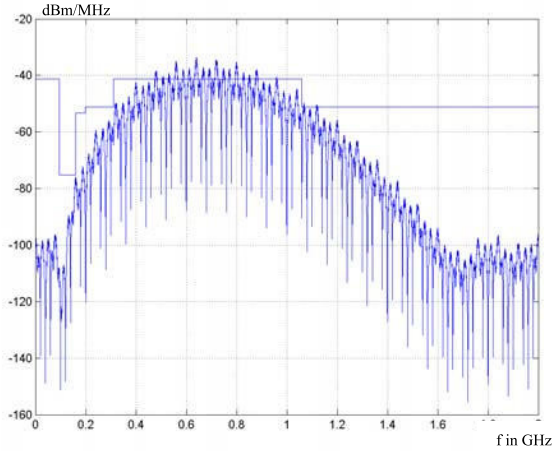


Fig. 4 Spectrum of a PPM signal with TH sequence

#### IV.2 Pulse-based polarity randomization with long randomization code

Using a symbol-based randomization of the polarity, we are essentially using a whole *sequence of pulses* as the “basis function” of our modulation. Consequently, it is the spectral characteristic of this sequence that determines the spectrum of the transmit signal. If we randomize, however, the polarity of each frame separately, then the *pulse* becomes the “basis function”. In the limit of a very long randomization sequence, designing the spectrum of the *pulse* becomes identical to designing the spectrum of the *transmit signal*.

To demonstrate that fact, we consider at first an a short TH code, i.e., which has a length equal to the symbol length, but with a long pulse based polarity randomization code. The signal can be decomposed into N data streams, each of which has a symbol period  $T_s$ . Thus, each of these signals can be interpreted as a PPM-only signal with random polarity for each symbol. Because the signals are completely uncorrelated, we can add their spectra. Consequently, the spectrum of this signal is the spectrum of the pulse.

This result also holds true with a TH sequence over several symbol durations. Once we split the signal into N streams, each of them can be rewritten as a signal with an antipodal modulation scheme because of the pulse based polarity hopping sequence. Finally we can generalize this result to TH sequences without a frame structure. We stress again that these considerations are valid for different modulation schemes, including PPM, BPSK, and OOK.

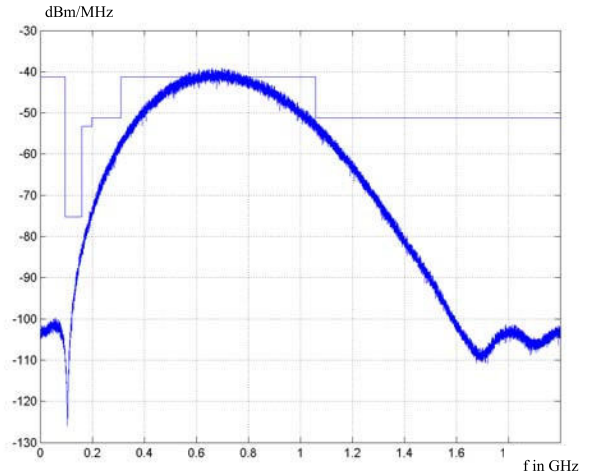


Fig. 5 2-PPM Signal Spectrum using this TH sequence and the pulse based polarity hopping sequence.

## V. PULSE DESIGN

Since the spectral characteristics of our signal are determined by the pulse shape, we can transfer directly some required properties of our UWB signal to the spectrum of the pulse waveform itself. Shaping this waveform in order to get a better power efficiency or to reduce interference and get better coexistence with other systems such as 802.11a can be achieved without any consideration for the modulation scheme.

Reference [7] considers shaping a waveform to avoid interference. The use of a waveform formed by two closely spaced pulses with opposite polarity, allows to create nulls in the waveform spectrum. If this waveform is used in conjunction with PPM modulation only, we have spectral lines. Following the results of the previous sections, we remove spectral lines and can create a TH sequence that keeps the spectral characteristics of the waveform by using a long waveform based polarity hopping sequence. The spectrum becomes:

$$\begin{aligned} |S_{dualp}(f)|^2 &= \left| \frac{F_p(f)}{\sqrt{2}} - \frac{F_p(f)}{\sqrt{2}} e^{-j2\pi f x T_s} \right|^2 \\ &= 2 |F_p(f)|^2 \cdot \sin^2(\pi f x T_s) \end{aligned}$$

where  $x$  gives the distance between the 2 pulses. Figure 6 shows such a spectrum that is designed to suppress radiation at 5.0GHz, and thus minimize interference to IEEE 802.11a systems.



## V. CONCLUSIONS

This paper discussed the influence of the modulation format, multiple access format, and pulse shape on the spectral characteristics of impulse radio. We showed that PPM and time hopping lead to similar spectral characteristics, namely spectral lines. We showed that the use of polarity randomization eliminates the spectral lines. Furthermore, the use of long polarity randomization codes makes sure that the spectrum of the composite signal is equal to the spectrum of the basic waveform.

This paper has concentrated on the *spectral properties* of the transmitted signal; these are, however, not the only consideration in the signal design. While the use of long spreading sequences (either time-hopping or polarity randomization) simplifies the spectrum design, it complicates receiver design. This situation is completely analogue to CDMA systems, where the use of long spreading codes complicates the design of equalizers, multiuser detectors, etc.

Optimizing the spectral shape is very important in IEEE 802.15.3a systems, since they have a high data rate, and have to comply to the FCC emission limits; thus their signal-to-noise ratio is rather low at the boundaries of the required operating range. Our results allow a better exploitation of the limited resource “signal power”.

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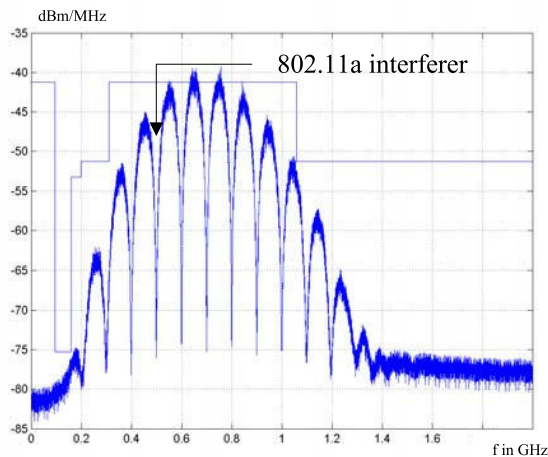


Fig. 6a Spectrum of TH-PPM signal with long frame-based polarity hopping sequence and basis waveform consisting of two opposite-polarity pulse.

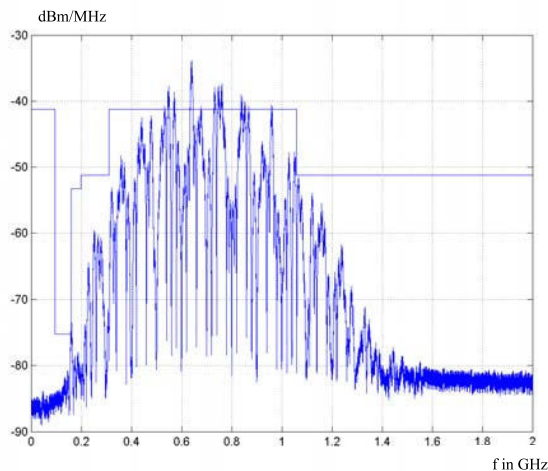


Fig. 6b Same as Fig. 6a, but with a symbol-based polarity hopping.

Fig. 6b uses a symbol-based polarity hopping sequence. The envelope carries the spectral characteristics of the TH sequence. Attenuations every 1GHz are preserved but the TH sequence has to be designed in order to limit the power back of. That can be avoid by using a frame based polarity hopping sequence, as shown in Fig. 6a.

The only modification of the envelope of the spectrum – compared to that of a single pulse – comes from the basic waveform structure, i.e., the fact that two pulses are always used in conjunction. This fact leads to a crest factor of 3dB.

The example given above used a very simple method for shaping the spectrum of the basic waveform, which resulted in a limited possibility of shaping the spectrum (for example, the width of the frequency notch could not be influenced). More general methods of influencing the waveform spectrum are discussed in [9].