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# Impulse Radio Pulse Shaping for Ultra-Wide Bandwidth (UWB) Systems

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***Abstract*— In this paper, we investigate the design of pulse shaping FIR filters for impulse radio ultrawideband (UWB) communications systems. The goal of the shaping is to meet an arbitrary spectrum mask, e.g., the mask mandated by the FCC for UWB emissions. Compared with classical FIR filter designs, the current problem introduces three new challenges: (1) it is minimax with quadratic constraints, (2) a single-sided distortion function is used, (3) delay positions are treated as tuning parameters. We first approach this problem by constructing a least-squares approximation to the minimax problem where the optimization over delays can be easily solved. With the LS solution serving as an initialization, nonlinear optimization techniques are employed to fine tune the solutions.**

## I. Introduction

With the release of the "First Report and Order," Feb. 14th, 2002, by the U.S. Federal Communications Commission (FCC), interest in ultra wide bandwidth (UWB) communication systems has increased [1]–[4]. The IEEE 802.15 standards organization (responsible for Personal Area Networks) has created a new task group, TG3a, that will standardize a high-data-rate Physical Layer based on UWB. One of the most important requirements is the fulfillment of the spectral masks mandated by the frequency administrations in different countries.

In addition, it is also required by IEEE 802.15 that UWB systems do not interfere with existing wireless systems like 802.15.1 (Bluetooth), 802.15.3 (Personal Area Networks), 802.15.4 (Zigbee) and 802.11a and 802.11b (wireless LANs). Furthermore, UWB systems should also be robust against interference from these devices, as well as from interference from microwave ovens and narrowband interferers.

All these requirements impose additional constraints on the spectral shaping. While some of those interferences are at fixed frequencies, others have variable center frequencies (like the different bands of 802.11a) or frequencies that cannot be predicted a priori. Finally, most devices will have to adapt to the frequency masks in different countries. It is thus both necessary and advantageous to be able to shape the spectrum adaptively.

While the FCC has not mandated a specific multiple-access and modulation scheme, time-hopping impulse radio (IR) is the most popular technique [1] [2]. In this, each symbol is represented by a series of time shifted pulses; the modulation can be pulse position modulation (PPM) or pulse amplitude modulation (PAM). In either case, the spectrum of the transmit signal is proportional to the spectrum of the transmitted pulses [4]. It is thus very important to design pulses that fulfill the spectral requirements.

In this paper, we undertake an optimization approach toward the problem of spectrum shaping for single-user communication systems. Due to the large bandwidth of the pulses, digital synthesization techniques can not be applied. However, it is straightforward to produce Gaussian pulses and their derivatives [3]. An analog FIR filter with adjustable real coefficients and time-shifts is used to shape the spectrum of those basis pulses. Compared with classical FIR filter designs, the present problem involves three distinct challenges:

1. the problem has a quadratic constrained min-max formulation.
2. the distortion measure between the desired and the actual response is single-sided, i.e., the actual magnitude response has to strictly lie below the desired response.
3. the delay positions are treated as variables. Equivalently, this can be regarded as a constraint on the number of nonzero coefficients. Hence the problem is inherently combinatorial.

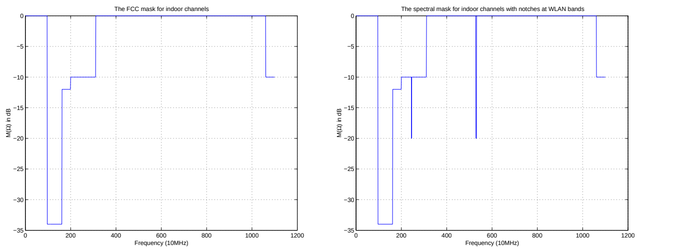
Our proposed approach is to first construct a bootstrapping solution by approximating the single-sided min-max problem with a least squares formulation. The particular form of least squares formulation also eliminates the need for combinatorial optimization over time-shifts. With the bootstrapping solution, nonlinear optimization techniques are then employed to fine tune the solution.

The rest of the paper is organized the following way: in Section 2, we set up the mathematical formulation of the problem. Section 3 solves the approximate formulation that serves as initialization for the nonlinear optimization of Section 4. We then show simulation results. A summary and conclusions wrap up the paper.

## II. Problem Formulation

We deal with the problem of finding a pulse that fulfills a given spectrum. Figure 1(a) shows the FCC mask for UWB communications transmitters in indoor environments. Figure 1(b) shows the modified spectrum mask with notches at WLAN bands. Note that Figure 1 has been normalized without loss of generality.

Assume there is a single basic monocycle $p(t)$ which can be generated by the circuits easily. Typically, Gaussian pulses and their differentiations are good choices in this regard. A linear FIR filter can be used to shape the spectrum of $p(t)$ in




(a) (b)

Fig. 1. (a) FCC spectral mask. (b) Modified spectral mask with notches at WLAN bands.

order to fit better with the mask. The corresponding system diagram is shown in Figure 2.

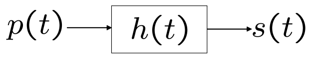



Fig. 2. Diagram for single user spectrum shaping.

The impulse response of $h(t)$ is assumed to be the sum of $\delta$-functions placed at different time and weighted differently. That is,

$$s(t) \equiv \sum_{i=0}^{M} s_i p(t-\tau_i) \tag{1}$$

$$S(j\Omega) \equiv \int_{-\infty}^{\infty} s(t)e^{-j\Omega t}dt = \sum_{i=0}^{M} s_i P(j\Omega)e^{-j\Omega\tau_i}. \tag{2}$$

The number of filter coefficients is restricted by complexity considerations. In contrast to tapped delay lines, where only certain discrete delays are feasible, we assume here that a continuum of delays can be chosen. This can be achieved by the use of programmable pulse generators. The range of allowed delays of the coefficients is determined by the pulse repetition frequency of the communication system.

Introduce the following notations:

$$\underline{s} \equiv [s_0 \; s_1 \; \ldots \; s_M]^T \tag{3}$$

$$\underline{\tau} \equiv [\tau_0 \; \tau_1 \; \ldots \; \tau_M]^T \tag{4}$$

$$r(\lambda) \equiv \int_{-\infty}^{\infty} p(t-\lambda)p(t)dt = r^*(-\lambda), \tag{5}$$

$$\mathbf{R}(\underline{\tau}) \equiv \begin{pmatrix} r(0) & r(\tau_0-\tau_1) & \cdots & r(\tau_0-\tau_M) \\ r(\tau_1-\tau_0) & r(0) & \ddots & r(\tau_1-\tau_M) \\ \vdots & \ddots & \ddots & \vdots \\ r(\tau_M-\tau_0) & r(\tau_M-\tau_1) & \cdots & r(0) \end{pmatrix}, \tag{6}$$

$$\langle s(t), s(t)\rangle \equiv \int_{-\infty}^{\infty} s(t)s^*(t)dt = \underline{s}^T\mathbf{R}(\underline{\tau})\underline{s} \tag{7}$$

The single user spectrum shaping problem can now be formulated as follows:

$$\max_{\underline{s},\underline{\tau}} \langle \mathbf{s}(t), \mathbf{s}(t)\rangle, \text{ subject to } |S(j\Omega)|^2 \le M(\Omega) \;, \forall\Omega \in [-\Omega_m, \Omega_m]. \tag{8}$$

where $M(\Omega)$ is the regulated upper-bound on the squared magnitude response and $\Omega_m$ is set to be 11GHz. This is equivalent to:

$$\min_{\underline{s},\underline{\tau}} \max_{\Omega\in[-\Omega_m,\Omega_m]} \frac{|S(j\Omega)|^2}{M(\Omega)}, \text{ subject to } \underline{s}^H\mathbf{R}(\underline{\tau})\underline{s} = 1. \tag{9}$$

## III. Approximate Solutions with Least-Squares FIR Filter Designs

The min-max formulation (9) (or the robust $\infty$-norm minimization) is well-known to be a difficult problem [5]–[7]. In this section, we propose to approximate (9) with a 2-norm minimization, in an effort to find a good bootstrapping solution.

Introduce

$$G(\Omega) \equiv \frac{\sqrt{M(\Omega)}}{|P(j\Omega)|}, \tag{10}$$

$$\phi(\Omega, \underline{s}, \underline{\tau}) \equiv s_0e^{-j\Omega\tau_0} + s_1e^{-j\Omega\tau_1} + \cdots + s_Me^{-j\Omega\tau_M}. \tag{11}$$

so that

$$\max_{\Omega\in[-\Omega_m,\Omega_m]} \frac{|S(j\Omega)|^2}{M(\Omega)} = \max_{\Omega\in[-\Omega_m,\Omega_m]} \frac{|\phi(\Omega,\underline{s},\underline{\tau})|^2}{G(\Omega)^2}. \tag{12}$$

Note that

$$\max_{\Omega\in[-\Omega_m,\Omega_m]} \frac{|\phi(\Omega,\underline{s},\underline{\tau})|}{G(\Omega)} - 1 = \max_{\Omega\in[-\Omega_m,\Omega_m]} \frac{|\phi(\Omega,\underline{s},\underline{\tau})| - G(\Omega)}{G(\Omega)}. \tag{13}$$

We can choose the energy constraint as being sufficiently big, i.e.,

$$\underline{s}^H\mathbf{R}(\underline{\tau})\underline{s} = b, \quad b \to \infty \tag{14}$$

such that (13) is equal to the $\infty$-norm. Although it does not make any difference in the original $\infty$-norm formulation, it affects the 2-norm approximation. A 2-norm approximation can be constructed by replacing the maximum with an integration weighted by $G(\Omega)$. Then the 2-norm approximation results in a FIR filter design problem with least-squares formulation:

$$\min_{\underline{s},\underline{\tau}} \int_{-\Omega_m}^{\Omega_m} \left||\phi(\Omega,\underline{s},\underline{\tau})| - G(\Omega)\right|^2 d\Omega. \tag{15}$$

One notable difference with traditional least-squares FIR filter design is that the delay vector $\tau$ is itself to be optimized here. In this step of our solution, we use the approximation of discrete spacing of the delays. Since the frequency range of interest is $[-\Omega_m, \Omega_m]$(Hz), if $|\phi(\Omega,\underline{s},\underline{\tau})|$ is required not to fold over in this frequency range, $\Delta\tau \le \frac{1}{2\Omega_m}$(sec).

Assume $\tau_i = d_i\Delta\tau, i = 0, \ldots, M$ and $d_i$'s are integers. Let $\tilde{G}(\omega) \equiv G(\omega\Delta\tau)$ and $g(n)$ be the impulse responses of $\tilde{G}(\omega)$. Consider

$$\min_{\underline{s},\underline{\tau},\lambda} \frac{1}{2\pi}\int_{-\Omega_m}^{\Omega_m} \left|\phi(\Omega,\underline{s},\underline{\tau}) - G(\Omega)e^{-j\Omega\lambda}\right|^2 d\Omega \tag{16}$$

$$\approx \min_{\underline{s},\underline{d},n} \frac{1}{2\pi}\int_{-\Omega_m}^{\Omega_m} |s_0e^{-j\Omega\Delta\tau d_0} + \cdots + s_Me^{-j\Omega\Delta\tau d_M} - G(\Omega)e^{-j\Omega n\Delta\tau}|^2 d\Omega \tag{17}$$

$$= \min_{\underline{s},\underline{d},n} \frac{1}{2\pi}\int_{-\pi}^{\pi} |s_0e^{-j\omega\tau d_0} + \cdots + s_Me^{-j\omega\tau d_M} - \tilde{G}(\omega)e^{-j\omega n}|^2 d\omega \tag{18}$$

$$= \min_{\underline{s},\underline{d},n} \sum_{i=0}^{M} |s_i - g_{d_i-n}|^2 + \sum_{\text{other } j} |g_j|^2 \quad (19)$$

$$= \min_{\underline{s},\underline{d},n} \sum_{i=0}^{M} (s_i^2 - 2s_i g_{d_i-n}) + \sum_{j=0}^{\infty} |g_j|^2 \quad (20)$$

$$\text{subject to } \underline{s}^H \mathbf{R}(\underline{\tau})\underline{s} = b \quad (21)$$

Now let $b \to \infty$, if $\mathbf{R}(\underline{\tau}) = \mathbf{I}$, the optimal set of $d_i - n$ has to match the taps of $g(n)$ with the largest magnitudes. Even for general $\mathbf{R}(\underline{\tau})$, the above choices of $d_i$'s are still reasonable. Once $d_i$ has been fixed, we are dealing with:

$$\min_{\underline{s}} \|\underline{s} - \underline{g}\|^2, \text{subject to } \underline{s}^H \mathbf{R}(\underline{\tau})\underline{s} = b, \quad (22)$$

where $\underline{g}$ is constructed by stacking the chosen $g_{d_i-n}$'s with the largest magnitudes. Let $b \to \infty$. It can be shown that the solution is given by:

$$\underline{s} \propto \underline{u}_1 \underline{u}_1^H \underline{g} \quad (23)$$

where $\underline{u}_1$ is the principal component of $\mathbf{R}$.

The above quadratic approximation is only one out of the many possible quadratic approximations. In essence, without constraints on the degree of freedom, the optimal solution for (9) is an IIR filter with frequency response $\tilde{G}(\omega)$, if $\langle \mathbf{s}(t), \mathbf{s}(t) \rangle$ in (8) is changed to the energy within the band of interest $[-\Omega_m, \Omega_m]$. The various quadratic approximations can be viewed as minimizing some distance measure between the IIR solution and the pursued FIR solution. The above quadratic approximation is pursued mainly because the joint optimizations over the weights and delays are easily solvable with its special structure of $\min_{\underline{s},\underline{\tau}} \|\underline{s} - \underline{g}\|^2$. A general quadratic formulation of the form,

$$\min_{\underline{s},\underline{\tau}} \text{trace}(\underline{s} - \underline{g})^H \mathbf{W} (\underline{s} - \underline{g}),$$

would require combinatorial optimizations to search for the best placement of the delays out of the allowed pulse positions on the quantization grid, which is itself a difficult problem.

## IV. Nonlinear Optimization with Neural Networks

Our general strategy for the solution of (9) is to initialize with the bootstrapping solution provided above and further exploit nonlinear optimization techniques to gradually refine the solutions. Here, we discuss an implementation with multi-layer perceptron (MLP) neural networks.

### *A. Putting into the MLP Framework*

In order to put the current problem into the general framework of MLP, we first simplify the maximization over frequency range $[-\Omega_m, \Omega_m]$ with that over a uniformly quantized frequency points, i.e.,

$$\max_{\Omega \in [-\Omega_m, \Omega_m]} \frac{|\phi(\Omega, \underline{s}, \underline{\tau})|^2}{G(\Omega)^2} \approx \max_{i \in \{0,\ldots,L-1\}} \frac{|\phi(\Omega_i, \underline{s}, \underline{\tau})|^2}{G(\Omega_i)^2} \quad (24)$$

where $\Omega_i \equiv i\Delta\Omega$. In this way, we essentially introduced $L$ hidden units in the MLP.

A second simplification is to replace the max-function with a differentiable soft-max. Given a set of continuous functions $f_i(\underline{x}), i = 0, \ldots, L-1$, introduce

$$\varphi(\underline{x}) \equiv \max_{i=0,\ldots,L-1} f_i(\underline{x}). \quad (25)$$

At any $\underline{x}$ where there is a unique $i$ such that $f_i(\underline{x}) = \varphi(\underline{x})$, for a sufficiently large positive number $\alpha$, we have the following soft-max approximation:

$$\varphi(\underline{x}) = \max_{i=0,\ldots,L-1} f_i(\underline{x}) \approx \frac{1}{\alpha} \log \sum_{i=0,\ldots,L-1} e^{\alpha f_i(\underline{x})}. \quad (26)$$

With these two simplifications, the problem becomes:

$$\min_{\underline{s},\underline{\tau}} \psi(\underline{s}, \underline{\tau}), \qquad \text{subject to } \underline{s}^H \mathbf{R}(\underline{\tau})\underline{s} = 1. \quad (27)$$

where

$$\psi(\underline{s}, \underline{\tau}) \equiv \sum_{i=0}^{L-1} e^{\frac{\alpha \phi_i(\underline{s},\underline{\tau})}{G(\Omega_i)^2}}, \quad (28)$$

$$\phi_i(\underline{s}, \underline{\tau}) \equiv |\phi(\Omega_i, \underline{s}, \underline{\tau})|^2. \quad (29)$$

The right part of Figure 3 depicts the the feedforward evaluations of the function and the left part shows the back propagation update of the function.

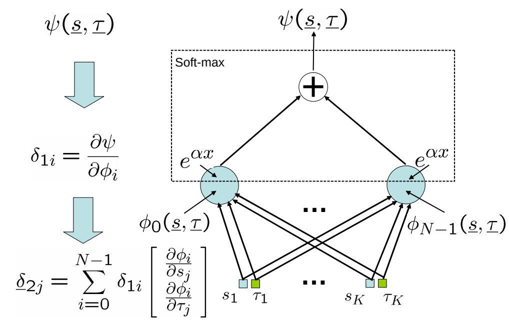



Fig. 3. MLP with Back-Propagation Learning

The computation terminates when the soft-max cost function cannot be improved further. Since the soft-max is not exactly the maximum, with approximation errors especially noticeable at places around the cross-points of the maximizer functions, the minimax solution among all the $(\underline{s}, \underline{\tau})$'s ever visited is output as the solution.

*1) Conditional Maximization:* While it is theoretically possible to simultaneously adjust the two set of parameters $\underline{s}$ and $\underline{\tau}$, practically we prefer to decouple their tuning by adopting a conditional maximization approach, i.e., optimizing one with the other fixed. In addition, this decoupling may be justified by the different nature of the two parameter sets. To better appreciate this, suppose a small change $\mu\Delta\underline{\tau}$ is to be applied to the current $\underline{\tau}$. If $\mu$ is too big, then the algorithm may not converge; otherwise, such fine changes in delays may not be supportable by the timing accuracy in circuit implementation. Hence in our current implementation, with $\underline{\tau}$ quantized, we scale $\mu$ such that it "hops" to the nearest valid quantization point on the multi-dimensional grid.

*2) Escaping from Local Minima:* Typically, numerical nonlinear optimizations can only be assured to arrive at a local optima rather than a global one. A popular approach to escape from possible local minima is to initialize the solution with several distributed (randomized) initializations and choose the best solution among the different trial pathes.

### *B. Implications about Differentiability*

In essence, the above procedure is a gradient descent approach to iteratively optimize the function. Generically, $\varphi(\underline{x})$ is not differentiable everywhere. The above procedure treats the problem by simply taking the gradient of the soft-max. We should point out this is only an *ad hoc* approach. At places where $\varphi(\underline{x})$ is not differentiable because more than one function achieve the maximum, the computed gradient is essentially an average of the gradients of the maximizers. Thus the computed gradient does not carry a physical meaning as a direction used for local first-order approximation.

It has been established in the literature [6] [5] that under general conditions, any directional derivative (e.g., left derivative or right derivative in one-dimensional case) of $\varphi(\underline{x})$ exists and is continuous. Iterative methods to identify beneficial directions to move and implement gradient descent based on the directional derivatives have been proposed [5]–[7] and supported theoretically [5], [6]. For example, the procedure in [7] involves iterations of three subroutines, linear programming, quadratic programming, and one-dimensional search. Therefore, if further enhancement to the performance and robustness is critically demanded, it is recommended that these procedures based on directional derivatives be used. A performance evaluation of them is left as a future work.

## V. SIMULATIONS

In this section, design examples for the proposed method are given, with $M = 3$ and $p(t)$ being the 5th derivative of Gaussian pulse. The basis pulses, delays, and number of coefficients are typical for an TH-IR system that fulfills the IEEE 802.15.3a requirements. The efficiency is measured in terms of the maximum (normalized) transmission power $P_m \equiv \langle \mathrm{s}(t), \mathrm{s}(t) \rangle$ as in (8). The two spectrum masks shown in Figure 1(a) and Figure 1(b) are tested. In the results reported below, the nonlinear optimization uses only one initialization, which is provided by the LS approximation.

### *A. Fulfilling the FCC Mask*

For the FCC mask in Figure 1(a), the impulse reponses of $G$ is shown in Figure 4.

When restricted to a FIR with only $M+1 = 4$ coefficients, the discussions in Section III on the LS approximations would suggest setting $\underline{\tau}$ to correspond to the first four pulse positions. The results with LS approximations are:

$$\underline{s} = [0.4373 \;\; -0.2677 \;\; -0.2677 \; 0.4373]^T \tag{30}$$
$$\underline{\tau} = [0 \; 1 \; 2 \; 3]^T / 22ns \tag{31}$$
$$P_m = 97.7306dB. \tag{32}$$

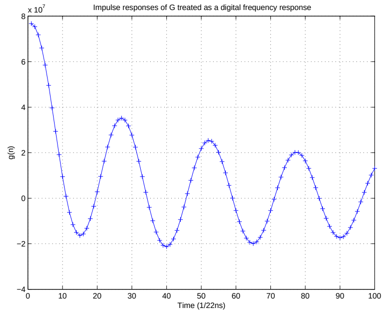



Fig. 4. Impulse response of $G$, which is truncated to obtain the initial solution.

The results with nonlinear optimizations initiated with the LS approximation solution are:

$$\underline{s} = [-0.2524 \;\; -0.9988 \;\; -0.9988 \;\; -0.2524]^T \tag{33}$$
$$\underline{\tau} = [0 \; 10 \; 17 \; 27]^T / 200ns \tag{34}$$
$$P_m = 98.8112dB. \tag{35}$$

The initial monocycle already fits the FCC mask very well with $P_m = 98.1263$. Compared with the initial monocycle, the LS approximate solution is worse by about $0.4$dB; the fine tuning result with nonlinear optimizations is better than the initial monocycle by about $0.7$dB.

Figure 5 shows the spectrum shaping results after normalization, compared with the FCC mask. In the time domain, the shaped pulse obtained from the nonlinear optimizations are shown in Figure 6, together with the original monocycle. Since in this case the shaped pulse still has a very short support, the potentially achievable data rate is high.

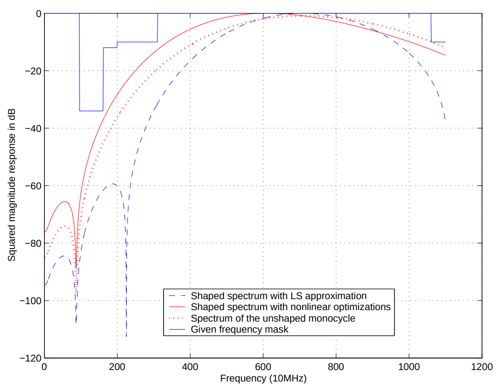



Fig. 5. The shaped spectrum for the FCC mask

### *B. Fulfiling the Modified Mask with Notches at WLAN Bands*

For the modified mask with notches in WLAN bands in Figure 1(b), the impulse response of $G$ is shown in Figure 7. The two notches do not result in a significant change to the impulse response. The results with LS approximations are:

$$\underline{s} = [0.4373 \;\; -0.2677 \;\; -0.2677 \; 0.4373]^T \tag{36}$$
$$\underline{\tau} = [0 \; 1 \; 2 \; 3]^T / 22ns \tag{37}$$
$$P_m = 82.6555dB. \tag{38}$$

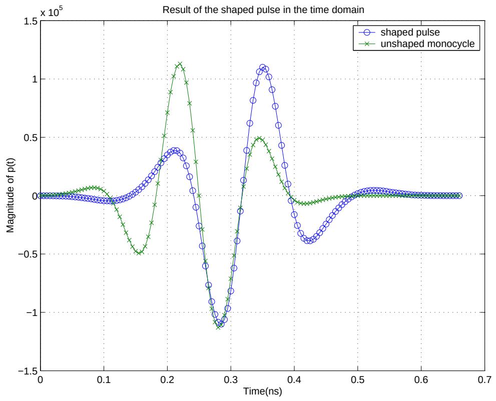



Fig. 6. The shaped pulse for the FCC mask

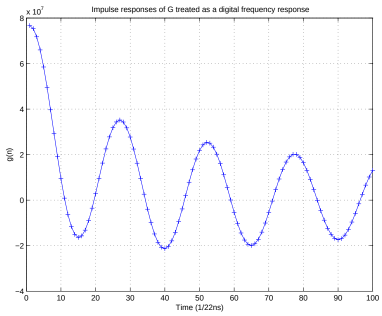



Fig. 7. Impulse response of $G$, which is truncated to obtain the initial solution.

The results with nonlinear optimizations initiated with the LS approximation solution are:

$$\underline{s} = [0.9832\ 0.7258\ 0.7258\ 0.9832]^T \quad (39)$$

$$\underline{\tau} = [0\ 9\ 18\ 27]^T/200ns \quad (40)$$

$$P_m = 97.1118dB. \quad (41)$$

The presence of the notch around 5.3GHz imposes a strict power limitation if the 5th derivative of Gaussian is used without shaping ($P_m = 82.1172$). The LS approximate solution improves by about $0.5$dB; the fine tuning result with nonlinear optimizations is better than the initial monocycle by about $15$dB.

As before, Figure 8 and Figure 9 show the spectrum shaping results in frequency domain and time domain, respectively.

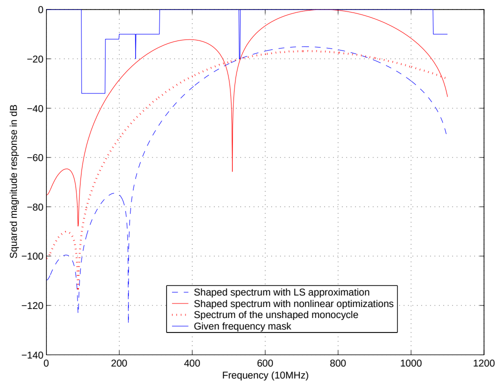



Fig. 8. The shaped spectrum for the modified mask

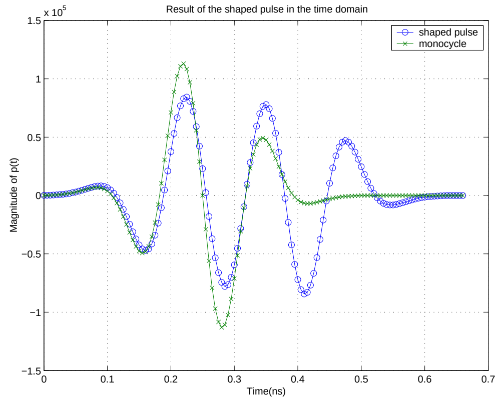



Fig. 9. The shaped pulse for the modified mask

## VI. Conclusions

In this paper, the design of pulse shaping filters to fit an arbitrary spectrum mask is studied. The problem is formulated as a quadratically constrained minimax with single-sided distortion function. In addition, delay positions are treated as tuning parameters. A least-squares approximation to the minimax problem is constructed for which the optimization over delay can be easily solved. With the approximate solution as an initialization, nonlinear optimization techniques are employed to fine tune the solutions. Simulation results demonstrate satisfactory performance of the proposed procedure, especially in the example of modified spectrum mask with notches at wireless LAN bands. Although the exact optimization over the delays is a combinatorial problem, it is empirically observed that the proposed approaches yield solutions with clustered delay patterns.

The work presented here is applicable, among others, to the pulseshaping in UWB communications systems. It is a central component of one of the standards proposals submitted to IEEE 802.15.

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