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Abstract

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DFT-based Hybrid Antenna Selection Schemes for spatially correlated MIMO channels

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***Abstract*—We address the antenna subset selection problem in spatially correlated MIMO channels. To reduce the severe performance degradation of the traditional antenna selection scheme in correlated channels, we propose to embed DFT operations in the RF chains. The resulting system shows a significant advantage both for diversity schemes and for the capacity of spatial multiplexing, while requiring only a minor hardware overhead.**

I. Introduction

Wireless communications systems show large performance improvements when using multiple antenna elements at both link ends. Analytical as well as simulation studies have verified the advantages of these so-called MIMO (Multiple-Input-Multiple-Output) systems [16] [17] [12]. Specifically, those systems can increase the data rate by transmitting different data streams from different antenna elements (spatial multiplexing, [14], [13]), or improve the quality of a single data stream by exploitation of transmit and receive diversity.

In either case, a major drawback is the requirement for multiple RF chains (one for each antenna element), which leads to high implementation costs. For this reason, recent papers [6], [8], [9], [7], have suggested antenna selection schemes that optimally choose a subset of the available transmit and/or receive antennas, and process the signals associated with those antennas. This allows to combine a large number of low-cost antenna elements (e.g., patch or dipole antennas) with a small number of (high-cost) RF chains, allowing to maximally benefit from the multiple antenna diversities within the RF cost constraint.

These antenna selection schemes work well for the uncorrelated MIMO channels (e.g., i.i.d. Rayleigh fading at each antenna element). Hybrid selection / maximum ratio combining (HS-MRC) schemes perform almost as well as maximum ratio combining with the same number of antenna elements [9]. Similarly, spatial-multiplexing MIMO systems with antenna selection (HS-MIMO) show high capacity in uncorrelated channels as long as the number of RF chains is as least as large as the number of available data streams [6],[8]. Also space-time codes combined with antenna selection perform well [5].

However, most practically occurring cellular channels exhibit fading correlation due to a nonuniform power azimuth spectrum (APS) at the base station (BS) [3], [18]. In such channels, HS schemes performs considerably worse than full-complexity schemes [10], because the signals at the different antenna elements exhibit correlation, which in turn decreases the gain of the antenna selection. In the current paper, we present a novel, simple but highly effective, hybrid antenna selection scheme that performs as well as full-complexity schemes in fully correlated channels, and as well as HS in uncorrelated channels. The new scheme uses a Discrete Fourier Transformation (DFT) to the (spatial) received signal vector in the RF domain. This can be realized in a simple, low-cost way by placing a Butler matrix (a butterfly structure consisting of phase shifters, adders and power splitters) between the antenna elements and the receiver switch. This system shows significant performance improvements for HS-MIMO as well.

The rest of the paper is organized the following way: Section 2 describes the system model, and the assumptions about the propagation channel. Next, we describe the performance of the traditional HS-MRC scheme as well as of our new scheme for transmit/receive diversity schemes. Section 4 then describes the performance with spatial multiplexing. A summary and conclusions wrap up this paper.

II. Channel Model

We consider a multiple antenna system with t transmit and r receive antenna elements. The channel is described

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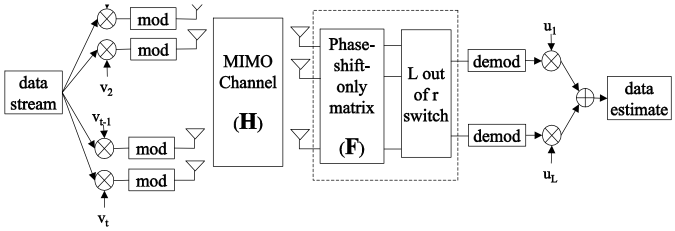

Fig. 1. MIMO channel model and system diagram.

by $\mathbf{H}$, the $r \times t$ transfer function of the MIMO channel. We adopt the widely used model [11] [2][7] :

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{W}\mathbf{T}^{1/2}, \tag{1}$$

where $\mathbf{W}$ is a matrix with i.i.d. complex Gaussian entries $\sim \mathcal{N}_C(0,1)$, and $\mathbf{R}$, $\mathbf{T}$ are $r \times r$, $t \times t$ matrices denoting receive and transmit correlations respectively. Such a model is usually valid when assuming independent transmit and receive correlations. We furthermore assume that the antenna arrays at both sides are uniformly spaced linear arrays, and that the angles of arrival at the transmitter (and receiver) are Gaussian distributed around the mean values: $\theta = \theta_t + \epsilon;\ \epsilon \sim \mathcal{N}(0, \sigma_t^2)$, and the angle spread ϵ is small

Denoting $d_t(d_r)$ as the relative antenna spacing of transmitter (receiver) with respect to the carrier wavelength, the k,l-th element of the correlation matrix $\mathbf{R}$ can be expressed as [1]

$$\begin{aligned}[\mathbf{R}]_{m,n} \approx \exp[-j2\pi(m-n)d\cos(\theta_r)]\times \\ \exp\{-\frac{1}{2}[2\pi(m-n)d\sin(\theta_r)\sigma_r]^2\}.\end{aligned} \tag{2}$$

To simplify the analysis we assume only receive correlation here, while the fading at the transmitter is uncorrelated $\mathbf{T} = \mathbf{I}_t$.This is realistic for the uplink of a cellular system, where the mobile station sees a uniform APS.

We furthermore assume the directions-of-arrival at the receiver are Gaussian-distributed around the mean values: $\theta = \theta_t + \epsilon;\ \epsilon \sim \mathcal{N}(0, \sigma_t^2)$. This allows a closed-form computation of the entries of $\mathbf{R}$, see [1]. The directions-of-arrival at the transmitter are uniformly distributed, so that $\mathbf{T}$ is a $t \times t$ identity matrix $\mathbf{I}_t$. This is a reasonable model for the uplink of a cellular system.

III. Transmit/receive diversity

We first consider a system with a single data stream 1. For simplicity only receive antenna selection is discussed in this paper, while the transmitter fully exploits all available antennas. However, the transmit selection can be handled in duality. To maximize the diversity gain, an information stream is multiplied by a t-dimensional complex weighting vector before it is modulated to the passband and applied to each of the t transmitting antennas. In a conventional HS-MRC receiver, L out of the r observation streams are selected, downconverted, and linearly combined. In our new scheme, the observation streams are passed through a $r \times r$ Fourier transformation before the selection; the purpose of the Fourier transform will be explained below. The (conventional) system can be mathematically expressed by

$$\vec{x}(k) = \mathbf{H}\vec{v}s(k) + \vec{n}(k), \tag{3}$$

where $s(k) \in \mathcal{C}$ is the transmitting stream, $\vec{x}(k) \in \mathcal{C}^r$ is the sample stacks of the complex-valued receiver data sequence, and $\mathbf{H}$ is the $r \times t$ channel transfer function. The total transmission power is constrained to P. The thermal noises $\vec{n}(k) \in \mathcal{C}^r$ are white i.i.d Gaussian random processes with independent real and imaginary parts and variance $N\mathbf{I}_r$, and $\vec{v}$ is the t-dimensional transmitter weighting vector satisfying $\|\vec{v}\| = 1$.

For the determination of the optimum weights we introduce the singular value decomposition (SVD) of $\mathbf{H}$: $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices representing the left and right singular vector spaces of $\mathbf{H}$, respectively; and $\mathbf{\Sigma}$ is the diagonal matrices consisting of all the singular values of $\mathbf{H}$. For convenience we will denote $\lambda_i(A)$ as the i-th largest singular value of a matrix A.

1) **No Antenna Selection** When there is no antenna selection, to estimate the information stream $s(k)$, a linear combination of all the r observation streams with coefficient vector $\vec{u}$ is performed at the receiver: $\hat{s}(k) = \vec{u}^*\mathbf{H}\vec{v} + \vec{u}^*\vec{n}(k)$.To maximize the estimate SNR (Signal-to-Noise Ratio), it is obvious that MRT and MRC should be adopted in this case, i.e. $\vec{u}$ ($\vec{v}$) should be the singular vector in $\mathbf{U}$ ($\mathbf{V}$) corresponding to the largest singular value $\lambda_1(\mathbf{H})$. The resulting SNR is then $\rho\lambda_1^2()$ where $\rho = \frac{P}{N}$ is the nominal SNR.
2) **Antenna Selection** Now we assume that L out of the r antenna elements are selected at the receiver. Mathematically, each selection option corresponds to a reduced-size transfer function matrix, which is formed by extracting the L rows of $\mathbf{H}$ that are associated with the selected antennas. We denote the set of all such submatrices as $\mathcal{S}_L(\mathbf{H})$. Therefore for the pure L/r antenna selection, the optimal SNR

achieved is

$$\max_{\tilde{\mathbf{H}} \in \mathcal{S}_L(\mathbf{H})} \rho \lambda_1^2(\tilde{\mathbf{H}}).$$

As mentioned above, this scheme shows good performance when the fading at the antenna elements is independent. However, for strongly correlated fading, performance is bad. In the limit of a single incident fading wave, maximum ratio combining reduces to pure beamforming, resulting in an increase of the average SNR, but no change of the slope of the SNR distribution. The beamforming gain, i.e., the gain in the average SNR is proportional to the number of combined signals r. A selection diversity scheme performs badly because it has a low beamforming gain L, while (due to the strong correlation of the fading) it cannot provide diversity gain.

3) **DFT-based selection:** For our new scheme we send all received observation streams through a (spatial) Fourier transform before selection and downconversion. This can be implemented easily by means of a Butler matrix, which performs a DFT in the RF domain. The r-point DFT matrix is illustrated by the $r \times r$ matrix $\mathbf{F}$ in Figure 1, which is of the form

$$\mathbf{F} = \frac{1}{\sqrt{r}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j\omega_r} & \dots & e^{-j(r-1)\omega_r} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j(r-1)\omega_r} & \dots & e^{-j(r-1)^2\omega_r} \end{bmatrix}$$

where $\omega_r = \frac{2\pi}{r}$. When $\mathbf{F}$ is inserted before selection, the antenna selection is performed on the virtual channel $\mathbf{FH}$. In the meanwhile, the thermal noises are also multiplied by $\mathbf{F}$, resulting an a (different) vector of i.i.d. Gaussian noise variables. Following the same argument as in part 2, the optimal SNR after maximal ratio combining is now

$$\begin{aligned} SNR_{opt} &= \max_{\tilde{\mathbf{F}} \in_L(\mathbf{F})} \max_{\vec{u}, \|\vec{v}\|=1} \frac{\rho |\vec{u}^* \tilde{\mathbf{F}} \mathbf{H} \vec{v}|^2}{|\vec{u}^* \tilde{\mathbf{F}}|^2} \\ &= \max_{\tilde{\mathbf{F}} \in_L(\mathbf{F})} \rho \lambda_1^2(\tilde{\mathbf{F}} \mathbf{H}). \end{aligned} \tag{4}$$

Let us next give an intuitive argument for the use of the DFT. The output of the DFT can be regarded as "beams" oriented into different directions in space. Each beam implicitly has a beamforming gain proportional to the dimension of the DFT, which is r. In a strongly correlated channel, the scheme just picks the strongest beam, and is thus as good as MRC. When the PAS is uniform, the DFT has no effect on the performance: selecting the

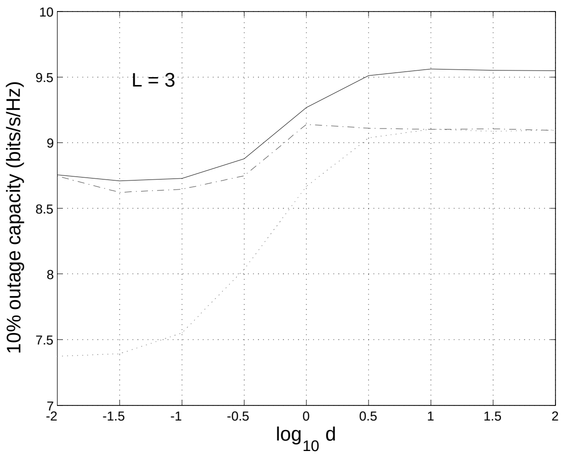

Fig. 2. 10% outage capacity with respect to the relative antenna spacing d: original channel (solid surve), pure antenna selection (dotted curve) and DFT-antenna selection (dash-dotted curve) with $t = 2$, $r = 8$, $\rho = 20db$, $\theta = \frac{\pi}{4}$ and $\sigma_r = \frac{\pi}{12}$.

best L beams and combining them with maximum ratio combining gives the same performance as selecting the best L antenna signals. This interpretation is supported by Fig. 2 which shows the 10% outage capacity (as defined by [14]) as a function of the fading correlation for a system with 2 transmit antennas, $r = 8$ receive antenna elements, and $L = 3$ RF chains. We see that for large correlation (meaning a small ratio of antenna spacing to correlation length of the channel, antenna selection performs considerably worse than the DFT-based selection or the full-complexity scheme. At low correlation, DFT-based selection shows the same performance as antenna selection. Fig. 3 shows the capacity distribution functions for all three schemes with different numbers of receiver chains L. We see that our DFT-based selection outperforms antenna selection especially for small L.

IV. Spatial multiplexing

Contrary to the transmission of a single information stream over multiple antennas in Section III, different streams can be applied on different antenna elements to provide a maximal data rate, which is illustrated in Figure 4 In this case, the system model is

$$\vec{x}(k) = \mathbf{H}\vec{s}(k) + \vec{n}(k), \tag{5}$$

where $\vec{s}(k)$ is now a $t \times 1$ vector denoting the transmit sequences. As the channel realization is unavailable at the transmitter, we assume even power distribution, i.e $\mathcal{E}[\vec{s}(k)\vec{s}^*(k)] = \frac{1}{t}\mathbf{I}_t$. With spatial multiplexing, capacity is the vital parameter to evaluate the system performance.

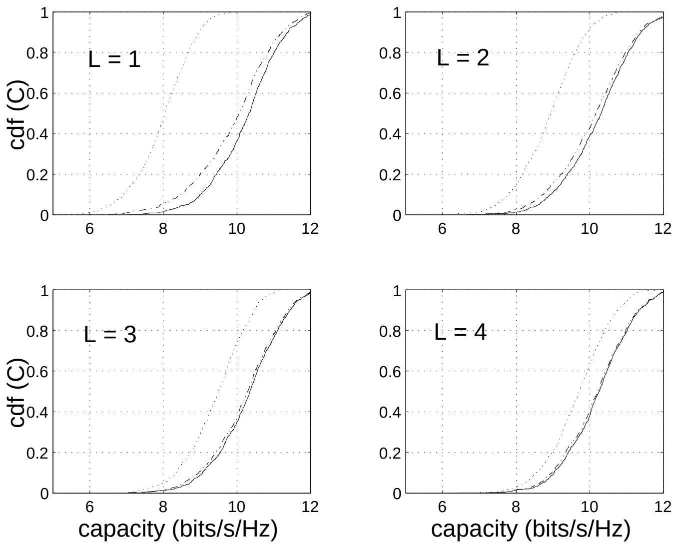

Fig. 3. Capacity cdf : full-complexity system (solid surve), pure antenna selection (dotted curve) and DFT-antenna selection (dash-dotted curve) with $t = 2$, $r = 8$, $\rho = 20db$, $d = 0.5$, $\theta = \frac{\pi}{12}$ and $\sigma_r = \frac{\pi}{12}$.

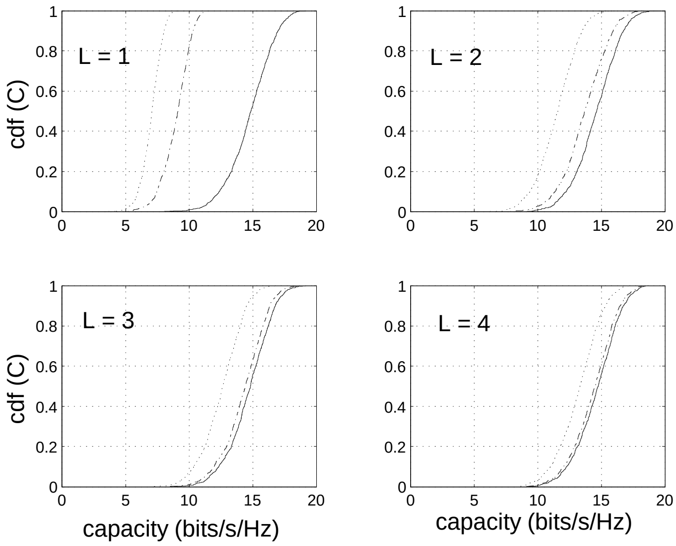

Fig. 5. Capacity cdf with spatial multiplexing: full-complexity system (solid curve), pure antenna selection (dotted curve) and DFT-antenna selection (dash-dotted curve) with $t = 2$, $r = 8$, $\rho = 20db$, $d = 0.5$, $\theta = \frac{\pi}{12}$ and $\sigma_r = \frac{\pi}{12}$.

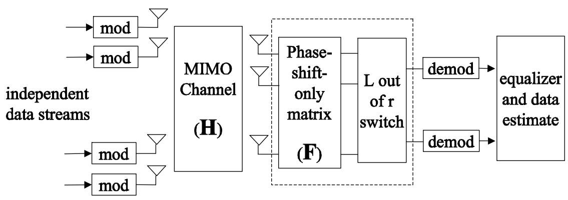

Fig. 4. System and channel model for spatial multiplexing.

1) **Capacity using all antennas**. The channel capacity of the original MIMO system in (5) at equal power distribution is well known to be

$$C = \sum_{i=1}^{t} \log_2[1 + \frac{\rho}{t}|\lambda_i(\mathbf{H})|^2]. \quad (6)$$

2) **Antenna Selection.** With antenna selection in the receiver end, the optimal choice that maximizes the resulting capacity is

$$C = \max_{\tilde{\mathbf{H}} \in S_L(\mathbf{H})} \sum_{i=1}^{L} \log_2[1 + \frac{\rho}{t}|\lambda_i(\tilde{\mathbf{H}})|^2]. \quad (7)$$

3) **DFT-based selection.** Similarly, with DFT involved, the optimal achievable capacity after antenna selection is given by (7) with $\mathbf{H}$ replaced by $\mathbf{FH}$.

Again, we see that the DFT-based selection shows considerably better performance than antenna selection (Fig. 5). For $L = 1$, the performance of both antenna selection and DFT-based selection is much worse than for a full-complexity scheme, since the number of receive RF chains is smaller than the number of transmit antennas, so that the separation of data streams becomes difficult. For $L \geq t$, both selection schemes can support the t data streams, but the DFT-based scheme outperforms antenna selection scheme by about 2bit/s/Hz, because it has better SNR. Figure 6 shows the 10% outage capacity as a function of the ratio of antenna spacing to channel correlation length.

V. Summary and Conclusions

We presented a new antenna selection scheme that shows excellent performance for arbitrary fading correlation of the received signals. The received signals are first spatially Fourier-transformed, and then the best L out of the total r received signals are downconverted and processed. We show that this scheme performs as well as L/r HS-MRC in uncorrelated-fading channels, and much better, namely as well as r-signal MRC in strongly correlated channels. It has (apart from a Butler matrix) the same hardware effort as L/r HS-MRC, which means the saving of $r - L$ RF chains compared to $r-$signal MRC. Computer experiments confirm our conclusions. We note that while the formulation here was given for selection at the receiver, the scheme can be implemented in a com-

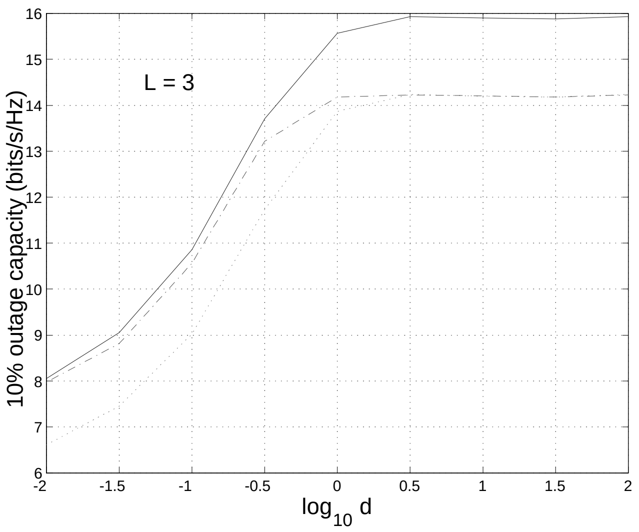

Fig. 6. 10% outage capacity with respect to the relative antenna spacing d: original channel (solid surve), pure antenna selection (dotted curve) and DFT-antenna selection (dash-dotted curve) with $t = 2$, $r = 8$, $\rho = 20db$, $\theta = \frac{\pi}{4}$ and $\sigma_r = \frac{\pi}{12}$.

pletely analogous fashion at the transmitter, or at both link ends.

This scheme has the advantage that it requires only a fixed RF circuit, namely a Butler matrix. An alternative approach would be the use of an adaptive scheme, where an adaptive phase transformation matrix is used instead of the Butler matrix [15]. This approach shows better performance in uncorrelated channels, but requires higher implementation complexity. Furthermore, the current approach can also be used when only partial channel information (e.g., average power azimuth spectrum) is available, as is often the case in FDD (frequency division duplexing).

Summarizing, we have presented a simple yet effective scheme for performance improvement of reduced-complexity multiple-antenna schemes.

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