

## **Antenna Selection with RF Pre-Processing: Robustness to RF and Selection Non-Idealities**

Pallav Sudarshan, Neelesh B. Mehta, Andreas F. Molisch and Jin Zhang

TR2004-103 October 2004

### **Abstract**

Multiple antenna transmitter and receiver architectures that combine antenna selection with RF pre-processing have been shown to significantly outperform conventional antenna selection with the same number of RF chains. Often, performance close to a full complexity architecture (with more RF chains) is also achieved. This paper studies the effect of hardware and signal processing non-idealities on such architectures. We show that they are robust to quantization, phase, and calibration errors introduced by RF phase-shifters, and also to the channel estimation errors. While insertion loss does lead to performance degradation, performance better than conventional antenna selection is observed for typical insertion loss values.

*RAWCON 2004*

This work may not be copied or reproduced in whole or in part for any commercial purpose. Permission to copy in whole or in part without payment of fee is granted for nonprofit educational and research purposes provided that all such whole or partial copies include the following: a notice that such copying is by permission of Mitsubishi Electric Research Laboratories, Inc.; an acknowledgment of the authors and individual contributions to the work; and all applicable portions of the copyright notice. Copying, reproduction, or republishing for any other purpose shall require a license with payment of fee to Mitsubishi Electric Research Laboratories, Inc. All rights reserved.



IEEE Radio-Wireless Conference (RAWCON)



# Antenna Selection with RF Pre-Processing: Robustness to RF and Selection Non-Idealities

Pallav Sudarshan, *Student Member, IEEE*,<sup>†</sup>  
Electrical and Computer Engineering Department,  
North Carolina State University,  
Raleigh, NC, USA.  
psudars@ncsu.edu

Neelesh B. Mehta, *Member, IEEE*,  
Andreas F. Molisch, *Senior Member, IEEE*,<sup>‡</sup>  
Jin Zhang, *Senior Member, IEEE*,  
Mitsubishi Electric Research Labs,  
201 Broadway, Cambridge, MA, USA.  
{mehta, molisch, jzhang}@merl.com

**Abstract**—Multiple antenna transmitter and receiver architectures that combine antenna selection with RF pre-processing have been shown to significantly outperform conventional antenna selection with the same number of RF chains. Often, performance close to a full complexity architecture (with more RF chains) is also achieved. This paper studies the effect of hardware and signal processing non-idealities on such architectures. We show that they are robust to quantization, phase, and calibration errors introduced by RF phase-shifters, and also to the channel estimation errors. While insertion loss does lead to performance degradation, performance better than conventional antenna selection is observed for typical insertion loss values.

**Index Terms**—MIMO systems, Spatial multiplexing, Diversity, Antenna arrays, Antenna selection, Information rates, Phase shifters, Quantization, Calibration, Estimation

## I. INTRODUCTION

Multiple input multiple output (MIMO) antenna systems promise dramatic improvements in link capacity [1] at the expense of increased hardware and signal processing complexity. Each antenna element at the receiver (transmitter) requires a low noise amplifier (power amplifier), a frequency converter, and an A/D (D/A) converter. Antenna selection, which adaptively chooses a subset  $L$  out of the  $N$  available antennas ( $L/N$ -selection), is a promising solution for reducing the RF chain count (see [2] and the references therein). While  $L/N$ -selection is better than a system with only  $L$  antenna elements, a penalty is paid in the form of reduced coding gain when compared to a full complexity (FC) system with  $N$  antenna elements and  $N$  RF chains. Selection schemes have been proposed for spatial multiplexing, in which multiple data streams are transmitted simultaneously from different antennas, and spatial diversity, in which the same data is transmitted from all antennas.

Recent approaches [3]–[5] have advocated linear RF pre-processing, which involves spatially filtering the received signal in RF, followed by selection. These have been shown to always outperform conventional antenna selection, and

achieve performance close to FC in many cases. The form of the RF pre-processing solution depends on whether spatial multiplexing or spatial diversity is used. However, in both cases, a phase-only restriction on the elements leads to practical implementations that use only variable phase-shifters and incurs a negligible loss in performance [4].

RF pre-processing circuits are familiar to the microwave community for applications such as analog beamforming [6], [7] that maximize the signal to noise ratio (SNR) and help in interference suppression [8]. Several designs for variable-phase shifters based on Silicon or GaAs PIN diodes, GaAs FETs, ferro-electric materials, piezo-electric transducers (PET), etc., have been investigated [9]–[12]. These designs differ in their insertion loss, chip area, operating voltage, phase error, time required to tune the elements, etc.

In this paper, we study the robustness of antenna selection with RF pre-processing to the non-idealities of the hardware implementations as well as the signal processing non-idealities, both of which can potentially erode the predicted gains. For such systems, we study the effect of phase and calibration errors and insertion loss on the fundamental Shannon capacity for spatial multiplexing and the output SINR for spatial diversity. Imperfect channel estimates that occur due to noise during channel estimation can also degrade the performance of RF pre-processing with selection. Therefore, we study its impact as well. We thus obtain the requirements on the RF elements for such a design to succeed. While only receiver side selection is illustrated in this paper, analogous arguments hold for the transmitter as well.

The rest of the paper is organized as follows. Section II describes the system model. Various receiver architectures are described in Section III and the RF and channel estimate imperfections are modeled in Section IV. Results are presented in Section V, followed by conclusions in Section VI.

## II. SYSTEM MODEL

Let  $N_t$  denote the number of transmit antennas and  $N_r$  denote the number of receive antennas. The received vector,

<sup>†</sup> This work was done when the author was at Mitsubishi Electric Research Labs.

<sup>‡</sup> A. F. Molisch is also at the Department of Electrosience, Lund University, Lund, Sweden.

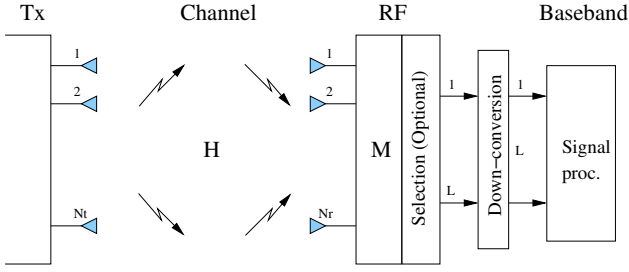


Fig. 1: Block diagram for spatial multiplexing transmission with RF pre-processing.

$\mathbf{y}$ , in baseband representation, is given by

$$\mathbf{y} = \sqrt{\frac{\rho}{N_t}} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{x}$  is the transmitted data vector,  $\mathbf{H}$  is  $N_r \times N_t$  channel matrix and  $\rho$  is the received SNR input to a receiver's antenna.  $\mathbf{n}$  is the additive white Gaussian noise and follows the distribution  $\mathcal{N}_c(\mathbf{0}, \mathbf{I}_{N_r})$ , where  $\mathcal{N}_c$  denotes the complex Gaussian distribution,  $\mathbf{0}$  is the all zeros mean vector and  $\mathbf{I}_{N_r}$  is the  $N_r \times N_r$  identity covariance matrix. The received vector,  $\mathbf{y}$ , is sent through a RF pre-processing matrix  $\mathbf{M}$ , followed by selection, down-conversion, and baseband processing.

In spatial multiplexing, multiple data streams are simultaneously transmitted, as shown in Fig. 1. In spatial diversity, the transmitted vector takes the form  $\mathbf{x} = \mathbf{v}b$ , where  $b$  is the data symbol and  $\mathbf{v}$  is the transmit weight vector.

We adopt the widely used Kronecker correlation model [16] that accurately represents many practical channels of interest. The instantaneous channel matrix can be represented as

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}} \mathbf{H}_w \mathbf{T}^{\frac{1}{2}}, \quad (2)$$

where the entries of  $\mathbf{H}_w$  are i. i. d. complex Gaussian  $\sim \mathcal{N}_c(0, 1)$ , and  $\mathbf{R}$  and  $\mathbf{T}$  are the receiver and transmitter correlation matrices, respectively.

### III. PERFORMANCE OF RECEIVER ARCHITECTURES

In this section, we list the performance metrics for various receiver architectures considered in the literature. For spatial multiplexing, the metric is the capacity in bits/s/Hz, and for spatial diversity, it is the output SNR. The receiver has instantaneous channel state information (CSI). Each receiver has  $L \leq N_r$  demodulator (demod) chains, except for the FC receiver that has  $N_r$  demod chains. Spatial multiplexing receivers are described below. Descriptions of spatial diversity receivers are omitted due to space constraints and are given in [3].

#### A. Full Complexity (FC)

The FC receiver optimally combines signals from all the antennas, and, by definition, achieves the largest achievable capacity among all receivers with  $N_r$  receive antennas. Its channel capacity is given by

$$C_{\text{FC}} = \log_2 \left| \mathbf{I}_{N_t} + \frac{\rho}{N_t} \mathbf{H}^\dagger \mathbf{H} \right|,$$

where  $|\cdot|$  denotes the determinant and  $(\cdot)^\dagger$  denotes the Hermitian of a matrix.

#### B. Pure Antenna Selection

A pure selection receiver selects the best  $L$  out of  $N_r$  rows of  $\mathbf{H}$ . Its capacity is

$$C_{\text{sel}} = \max_{\tilde{\mathbf{H}} \in S_L(\mathbf{H})} \log_2 \left| \mathbf{I}_{N_t} + \frac{\rho}{N_t} \tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} \right|.$$

$S_L(\mathbf{H})$  denotes the set of all  $L \times N_r$  sub-matrices of  $\mathbf{H}$ .

#### C. Instantaneous Time-Variant Pre-Processing (TV)

In this receiver, the entries of the  $L \times N_r$  pre-processing matrix,  $\mathbf{M}_{\text{TV}}$ , are allowed to vary as fast as the instantaneous channel state  $\mathbf{H}$ . Thus, TV tracks the small-scale fading in the channel. The optimal  $\mathbf{M}_{\text{TV}}$  is the conjugate transpose of the  $L$  left singular vectors of  $\mathbf{H}$  corresponding to its  $L$  largest singular values [3]. The capacity is given by

$$C_{\text{TV}} = \sum_{i=1}^L \log_2 \left( 1 + \frac{\rho}{N_t} \lambda_i^2 \right),$$

where  $\lambda_i$  is the  $i^{\text{th}}$  largest singular value of  $\mathbf{H}$ . This  $L \times N_r$  pre-processing matrix outputs  $L$  streams, thereby eliminating the need for subsequent selection.

#### D. FFT Pre-Processing Followed by Selection

This is an alternate approach that uses the FFT Butler matrix,  $\mathbf{F}$ , for pre-processing [5]. Note that  $\mathbf{F}$  is completely independent of the channel state. Selection is performed on the virtual channel  $\mathbf{F}\mathbf{H}$ , and the capacity is given by

$$C_{\text{FFT}} = \max_{\tilde{\mathbf{H}} \in S_L(\mathbf{F}\mathbf{H})} \log_2 \left| \mathbf{I}_{N_t} + \frac{\rho}{N_t} \tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} \right|.$$

#### E. Channel Statistics-Based Pre-Processing (TI)

In TI, the pre-processing matrix, denoted by  $\mathbf{M}_{\text{TI}}$ , depends only on the large-scale slowly-varying parameters of the channel such as the mean angle of arrival (AoA), angular spread, etc., [4]. It is for this reason that we refer to it as the time-invariant solution (TI).

When  $\mathbf{M}_{\text{TI}}$  is of size  $L \times N_r$ , no subsequent selection is required. The optimal  $\mathbf{M}_{\text{TI}}$  that maximizes the ergodic capacity takes the form  $\mathbf{M}_{\text{TI}} = [\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_L]^\dagger$ , where  $\boldsymbol{\mu}_l$  is the eigenvector of the receiver correlation matrix  $\mathbf{R}$  corresponding to its  $l^{\text{th}}$  largest eigenvalue. The capacity is given by

$$C_{\text{TI}} = \log_2 \left| \mathbf{I}_L + \frac{\rho}{N_t} \mathbf{M}_{\text{TI}} \mathbf{H} \mathbf{H}^\dagger \mathbf{M}_{\text{TI}}^\dagger \right|.$$

When  $\mathbf{M}_{\text{TI}}$  is of size  $N_r \times N_r$ , it is followed by  $L/N$ -selection.  $\mathbf{M}_{\text{TI}}$  then consists of all the eigenvectors of  $\mathbf{R}$ .

Phase-only approximations to TV and TI that require only variable-phase shifters to implement them shall be referred to as TV-Ph and TI-Ph, respectively. These have been shown to incur a negligible performance loss [4].

#### IV. MODEL FOR NON-IDEALITIES

Implementing the phase-shifters using finite bit-resolution digital phase-shifters introduces quantization errors. In addition, the phase-shifters suffer from phase and calibration errors that cause an offset in the desired phase. The phase errors are typically  $1^\circ$ – $3^\circ$  [13]–[15]. The calibration error is taken to be uniformly distributed with mean  $0^\circ$  and a range  $20^\circ$ .

Insertion loss is defined as the measured power loss through the phase-shifter. The RF phase-shifters are associated with insertion losses of the order of 0.9–6.0 dB [12]–[14].<sup>1</sup> If the phase-shifters are placed before the low noise amplifiers (LNA) (as shown in Fig. 1), insertion loss and losses in the switch lead to a reduction in the SNR.

Due to noise during estimation and the inherent time-varying nature of the wireless channel, the channel estimates used in RF pre-processing and selection are imperfect. The imperfect channel estimate,  $\mathbf{H}_{\text{est}}$ , is modeled as

$$\mathbf{H}_{\text{est}} = \mathbf{H} + \sigma_H \mathbf{H}_{\text{err}}, \quad (3)$$

where  $\mathbf{H}$  is the actual channel matrix between the transmitter and the receiver and has Gaussian entries  $\sim \mathcal{N}_c(0, 1)$ .  $\sigma_H$  is the RMS error and  $\mathbf{H}_{\text{err}}$  is the channel error matrix, with i. i. d. Gaussian entries  $\sim \mathcal{N}_c(0, 1)$ .

#### V. RESULTS AND SIMULATIONS

We compare the different receiver architectures, described earlier, in the presence of the various non-idealities. A uniform linear array is considered with 4 transmit and receive antennas ( $N_t = N_r = 4$ ) and only one demod chain ( $L = 1$ ). Spatial multiplexing systems will typically have more RF chains;  $L = 1$  is the worst case. The capacities (or average output SNR) of the receivers are compared. The cumulative distribution function (CDF), which describes the entire distribution, of the capacity is also plotted. The spacing between the antenna elements is taken to be  $d = 0.5\lambda$ , where  $\lambda$  is the wavelength, and the mean AoA is  $\theta = 45^\circ$ . The RMS angular spread shall be denoted by  $\sigma_\theta$ . For simplicity, we assume that there is no transmit correlation, *i.e.*,  $\mathbf{T} = \mathbf{I}_{N_t}$ .

##### A. Impact of Channel Estimation Error

Table I compares the ergodic capacity for various receiver architectures when the pre-processing matrix and the selection are based on imperfect CSI.<sup>2</sup> We see that TI-Ph receiver's capacity is approximately 1 bits/s/Hz greater than that of pure antenna selection. Also, the statistics-based solution, TI-Ph, is more robust to channel estimation errors than the instantaneous solutions. When the channel estimates at the receiver have an RMS error  $\sigma_H = 0.6$ , both TV and pure antenna selection incur an approximately 0.2 bits/s/Hz loss. However, TI-Ph's

<sup>1</sup>Note that the losses in the citations are not at the same frequency. However, they do provide an estimate of the range of insertion losses to be expected from such devices.

<sup>2</sup>Not only does imperfect channel estimates lead to incorrect selection, it also reduces the MIMO capacity. However, the latter topic is beyond the scope of this paper, and  $\mathbf{H}$  is taken to be perfect.

TABLE I: Ergodic capacity with imperfect CSI for  $\theta = 45^\circ$  and  $\sigma_\theta = 15^\circ$ .

$\sigma_H$	FC	TV-Ph	TI-Ph	Ant. Sel.
0	5.78	3.70	3.47	2.61
0.6	5.78	3.52	3.46	2.45

TABLE II: Ergodic capacity with phase quantization and calibration error for  $\theta = 45^\circ$  and  $\sigma_\theta = 6^\circ$ .

Resolution	FC	TV-Ph (0°)	TV-Ph ( $\pm 10^\circ$ )	TI-Ph (0°)	TI-Ph ( $\pm 10^\circ$ )	Ant. Sel
Ideal	4.65	3.86	3.85	3.86	3.85	2.43
3 bit	4.65	3.80	3.80	3.78	3.77	2.43
2 bit	4.65	3.61	3.60	3.56	3.55	2.43

performance does not degrade. Here Figure 2 plots the corresponding capacity CDFs and studies the effect of channel estimation errors for spatial multiplexing. For reference, the CDF of the capacity achieved by a system with  $N_t = 4$  transmit antennas and  $N_r = 1$  receive antenna (and  $L = 1$ ) is also shown.

Spatial diversity behaves similarly – TI-Ph is insensitive to estimation non-idealities, while TV-Ph shows more sensitivity.

##### B. Impact of Phase Quantization and Phase error

The ergodic capacities of TV-Ph and TI-Ph with different bit-resolutions and calibration errors are tabulated in Table II. The capacity of TI-Ph using a 2-bit phase-shifter (steps of  $90^\circ$ ) and 3-bit phase-shifter (steps of  $45^\circ$ ) is within 0.3 bits/s/Hz and 0.1 bits/s/Hz, respectively, of an ideal TI-Ph receiver with infinite phase resolution. Figure 3 shows the effect of phase quantization and phase errors on the capacity of TI-Ph. Also measured is the effect of calibration error that is uniformly distributed between  $\pm 10^\circ$ . We see that RF pre-processing is robust to calibration error. TV-Ph behaved in a similar manner.

For spatial diversity, the effect of phase quantization is slightly worse. For example, the average output SNR for TI-Ph for  $\rho = 10$  dB with infinite phase resolution is 15.78 dB, while that with 2-bit quantization is 14.78 dB.

##### C. Impact of Insertion Loss

Table III tabulates the ergodic capacity of TV-Ph and TI-Ph for different insertion losses and compares them with the ideal (no insertion loss) FFT, FC, and conventional selection. Figure 4 plots the corresponding CDFs of capacity. It can be seen that a 2 dB insertion loss reduces the TI-Ph capacity to that of ideal FFT-selection. An insertion loss of about 5 dB degrades the performance of TI-Ph to conventional selection. Similar trends were also observed in spatial diversity.

#### VI. CONCLUSIONS

We investigated the robustness of various receiver architectures, including RF pre-processing and antenna selection, to

TABLE III: Ergodic capacity with insertion loss for  $\theta = 45^\circ$  and  $\sigma_\theta = 6^\circ$ .

Ins. loss	FC	TV-Ph	TI-Ph	FFT	Ant. Sel.
0 dB	4.58	3.82	3.80	3.17	2.43
2 dB	-	3.22	3.20	-	-

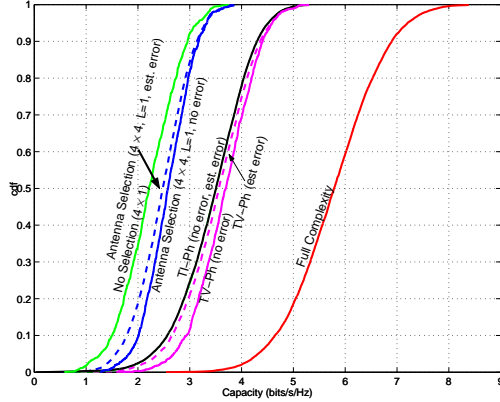


Fig. 2: Impact of channel estimation error: Capacity CDF for  $\rho = 6$  dB,  $\theta = 45^\circ$  and  $\sigma_\theta = 15^\circ$ .

RF non-idealities and channel estimation errors. Both spatial multiplexing and spatial diversity were evaluated. We showed that RF pre-processing based on only large-scale statistics, such as covariance [4], is very robust to the channel estimation errors, while the performance of instantaneous CSI-based pre-processing [3] degrades slightly. This result is notable because, under ideal conditions, the latter performs better. This makes the statistics-based solution more suitable for rapidly varying channels. Furthermore, RF pre-processing solutions suffer a negligible performance loss due to calibration errors and finite-bit quantization. A 3-bit quantization gives performance close to that of infinite resolution phase-shifters. The insertion loss introduced by the RF elements is the main reason for performance degradation. If it is high enough, the performance of RF pre-processing is worse than that of conventional antenna selection. In this case, the LNAs must be placed before RF pre-processing.

## REFERENCES

- [1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, pp. 585–595, 1999.
- [2] A. F. Molisch, M. Z. Win "MIMO systems with Antenna Selection," *IEEE Microwave Mag.*, Vol. 5, No. 1, pp. 46–56, Mar. 2004.
- [3] X. Zhang, A. F. Molisch, and S. Y. Kung, "Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection," *Submitted to IEEE Trans. Sig. Proc.*
- [4] P. Sudarshan, N. B. Mehta, A. F. Molisch, and J. Zhang, "Channel statistics-based joint RF-baseband design for antenna selection for spatial multiplexing," in *Globecom 2004*. Also see P. Sudarshan, N. B. Mehta, A. F. Molisch, J. Zhang "Channel Statistics-Based RF Pre-Processing with Antenna Selection," *submitted to IEEE Trans. Wireless Comm. 2004*.
- [5] A. F. Molisch, X. Zhang, S. Y. Kung, J. Zhang, "FFT-Based Hybrid Antenna Selection Schemes for Spatially Correlated MIMO Channels," *IEEE PIMRC*, Beijing, China, Sep. 2003.

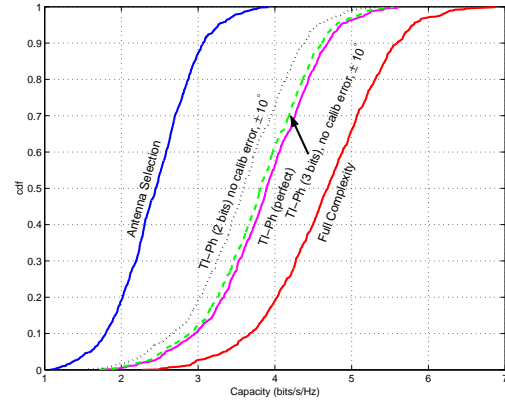


Fig. 3: Impact of phase quantization and calibration error: Capacity CDF for  $\rho = 6$  dB,  $\theta = 45^\circ$ , and  $\sigma_\theta = 6^\circ$ .

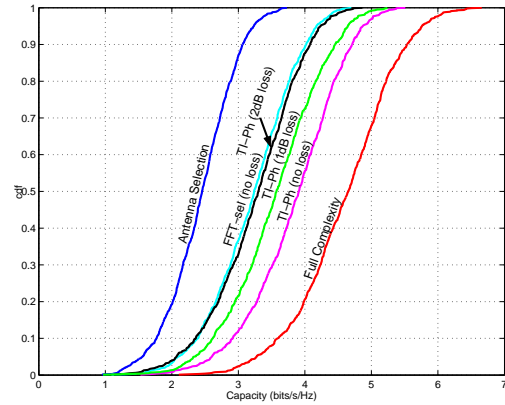


Fig. 4: Impact of insertion loss: Capacity CDF for  $\rho = 6$  dB,  $\theta = 45^\circ$  and  $\sigma_\theta = 6^\circ$ .

- [6] T. Ohira, "Analog smart antennas: An overview," in *Proc. PIMRC*, pp. 1502–1506, 2002.
- [7] T. Ohira, "Microwave signal processing and devices for analog beam-forming," in *IEEE AP-S Intl. Symp. USNC/URSI National Radio Science Meeting*, pp. 583–586, 2000.
- [8] S. T. Smith, "Phase-only adaptive nulling," *IEEE Trans. Sig. Proc.*, vol. 47, pp. 1835–1843, 1999.
- [9] G. M. Rebeiz, G.-L. Tan, and J. S. Hayden, "RF MEMS phase shifters: Design and applications," *IEEE Microwave Mag.*, pp. 72–81, 2002.
- [10] D. Kim *et al.*, "2.4 GHz continuously variable ferroelectric phase shifters using all-pass networks," *IEEE Microwave and Wireless Components Letter*, vol. 13, pp. 434–436, 2003.
- [11] T.-Y. Yun *et al.*, "A 10- to 21-GHz, low-cost, multifrequency, and full-duplex phased-array antenna system," *IEEE Trans. Antennas and Propagation*, vol. 50, pp. 641–650, 2002.
- [12] F. Ellinger, H. Jacket, and W. Bachtold, "Varactor-loaded transmission-line phase shifter at C-band using lumped elements," *IEEE Trans. Microwave Theory Tech.*, vol. 51, pp. 1135–1140, 2003.
- [13] G.-L. Tan *et al.*, "Low-loss 2- and 4-bit TTD MEMS phase shifters based on SP4T switches," *IEEE Trans. Microwave Theory Tech.*, vol. 51, pp. 297–303, 2003.
- [14] D. Yongsheng *et al.*, "A novel microminiature 2-22 GHz high performance 11.25° monolithic phase shifter," in *Proc. 2<sup>nd</sup> Intl. Conf. on Microwave and Millimeter Wave Technology*, pp. 138–141, 2000.
- [15] C. Campbell and S. Brown, "Compact 5-bit phase-shifter MMIC for K-band satellite communication systems," *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 2652–2656, 2000.
- [16] J. P. Kermaol *et al.*, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Select. Areas Commun.*, pp. 1211–1226, 2002.