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Abstract

A UWB positioning system which performs NLOS identification and TOA-based location estimation is described. For NLOS identification, we consider both a simple variance test, and a more complex non-parametric identification test. The ranging technique is based on TOA estimation, where the nodes can adopt a two-way ranging scheme in the absence of a common clock. From the NLOS identification and range estimation steps, an approximate MLE technique is employed for the node positioning.

Index Terms—Ultra-wideband (UWB), geolocation, time of arrival (TOA), maximum likelihood estimation.

I. NLOS Identification

The problem of NLOS identification can be seen as a detection problem, which compares the LOS hypothesis to the NLOS hypothesis. The probability distribution of the measurements under the LOS hypothesis is usually known except for its mean. If the distribution under NLOS hypothesis is also assumed to be known, then the problem can be solved by the conventional hypothesis testing method [1]. However, the probability distribution of NLOS errors, hence that of the measurements under the NLOS hypothesis, is usually unknown. Therefore, a technique which does not assume the knowledge of NLOS error statistics is needed.

The simplest of such a technique is the "variance test", which compares the sample variance of a set of measurements to the known variance of the measurement error [1]. Since the variance becomes larger in a NLOS situation, the simple test

$$\hat{\sigma}^2 > \sigma_m^2 \longrightarrow \text{NLOS} \tag{1}$$

$$\hat{\sigma}^2 \leq \sigma_m^2 \longrightarrow \text{LOS} \tag{2}$$

can be used for identification, where $\hat{\sigma}^2$ is the sample variance, and σ_m^2 is the known measurement error in a LOS situation.

Instead of comparing the variances, we can also take a non-parametric approach for the NLOS identification [2]. Since the statistics of TOA delays due to NLOS are not known exactly, a non-parametric approach is adopted to approximate the probability density function of the measurements. Then, a suitable distance metric between a known measurement error distribution and a non-parametrically estimated distance measurement distribution is defined to determine whether a given BS is within LOS or NLOS of the MS. The distance between these two distributions can also be used as a reliability measure for the measurements from the given BS.

Consider a situation in which m independent identically distributed (iid) range measurements (obtained from TOA measurements multiplied by the speed of light) between an MS and a BS are taken. Assume that the change in the location of the MS during these measurements can be ignored. Hence the distance between the MS and the BS can be considered approximately constant for the geolocation purpose. Then, for the ith measurement, the hypotheses can be expressed as:

$$\begin{aligned} H_0 &: \quad r_i = d + n_i \\ H_1 &: \quad r_i = d + n_i + e_i, \end{aligned} \tag{3}$$

for $i = 1, ..., m$, where H_0 is the LOS hypothesis and H_1 is the NLOS hypothesis. In the former case, the measurement is modelled as the summation of the true distance d and a measurement noise, n_i, while in the latter case, the NLOS error e_i is also present, which is modelled by a positive random variable.

We assume that the measurement noise statistics are completely known and is modelled by a zero mean Gaussian random variable. However, neither d nor the probability density function of the NLOS error are known. Therefore, it is not possible to invoke conventional hypothesis testing techniques like generalized likelihood ratios.

Let the probability density function (pdf) of the measurement noise be $p_n(x)$, which is completely known. Then, the pdf of the measurements in the LOS hypothesis case is given by $p_n(x - d)$. Note that this distribution is completely known except for one parameter, d, which affects only the mean of the distribution. The main idea in the non-parametric NLOS identification test is to compare the closeness of this pdf to the pdf of range measurements. Thus we first approximate the pdf of the range measurements non-parametrically, compare the closeness of this pdf to the LOS pdf by defining a distance metric, and then decide LOS/NLOS after a threshold test. This test can be summarized as follows:

1) Estimate the pdf of the distance from m iid range measurements. Let this estimate be denoted by $\hat{p}_r(x)$.

2) Calculate the distance between $p_n(x - d)$ and $\hat{p}_r(x)$ for all possible d values and find the minimum distance.

3) Compare this minimum distance to a threshold: Decide H_0 if the minimum distance is smaller than the threshold, and decide H_1 otherwise.

We will discuss these steps in more details in the following subsections.

A. Parzen Window Density Estimation

In order to estimate the pdf of the distance, a non-parametric density estimation technique, called Parzen window density estimation, is employed, which approximates the pdf using some window functions around the samples. The reason for employing this technique is its flexibility in choosing density estimation parameters depending on the sample size.

Given iid distance measurements $r_1, ..., r_m$, the distance pdf can be estimated by the following formula [3]

$$\hat{p}_r(x) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{h_m} \phi(\frac{x - r_i}{h_m}), \tag{4}$$

where $\phi(.)$ is the window function and h_m is a scaling parameter. The window function must be a probability density function in order for $\hat{p}_r(x)$ to be a valid pdf. In other words, it is always non-negative and integrates to one. Commonly used window functions include Gaussian and rectangular windows [3].

B. Distance Function

After obtaining the approximate pdf of the distance, our next step is to determine whether these distance measurements are coming from $p_n(x-d)$ or the pdf under the NLOS hypothesis. Since the pdf under the NLOS hypothesis is unknown, it is reasonable to compare the distance between $p_n(x-d)$ and $\hat{p}_r(x)$ and accept the LOS hypothesis if the distance is smaller than a threshold, that is, if the two distributions are sufficiently close. Since the true distance, d, is unknown, the minimum distance between $p_n(x-d)$ and $\hat{p}_r(x)$ must be calculated among all possible d's.

The Kullback-Leibler (KL) distance [4] can be used to calculate the distance between two probability distributions. For given pdf's p_1 and p_2, the KL distance between them is given by

$$D(p_1 \| p_2) = \int p_1(x) \, \log \frac{p_1(x)}{p_2(x)} \, dx. \tag{5}$$

C. Decision Criterion

The decision criterion to determine LOS or NLOS hypothesis becomes the following test:

$$\inf_{d} \; \{D(\hat{p}_r(x) \| p_n(x-d))\} \; \underset{H_1}{\overset{H_0}{\lesseqqgtr}} \; \delta, \tag{6}$$

where δ is the threshold.

If the value of the d minimizing the decision variable can be found, the test can be expressed simply as

$$D(\hat{p}_r(x) \| p_n(x-\hat{d})) \; \underset{H_1}{\overset{H_0}{\lesseqqgtr}} \; \delta. \tag{7}$$

We assume that the measurement noise is a zero mean Gaussian random variable, which is a valid approximation when the TOA's are acquired with a matched filter approach at high signal-to-noise ratio (SNR) [5]. Then, $p_n(x-d)$ is expressed as

$$p_n(x-d) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-d)^2/(2\sigma^2)}. \tag{8}$$

In this case, the following result indicates the simplification of the decision test.

Proposition 1 *For a zero mean Gaussian measurement error and a symmetric window function, i.e.,* $\phi(x) = \phi(-x)$ *for all* x*, the value of* d *minimizing the distance function of (6) is the sample mean of the measurements, that is,* $\hat{d} = \frac{1}{m}\sum_{i=1}^{m} r_i$.

Proof *See [2].*

Proposition 2.1 states that for a symmetric window function, the minimum distance to be used in the decision criterion can be computed by simply shifting the Gaussian measurement error pdf by the sample mean of the measurements and calculating the KL distance between this shifted pdf and the estimated pdf, $\hat{p}_r(x)$.

Another important issue is the appropriate choice of the threshold value, δ. Since the pdf's are not known exactly under either hypothesis, it does not seem possible to set the "false alarm" (i.e. misinterpret a LOS situation as NLOS) and "miss detection" (i.e. misinterpret an NLOS situation as LOS) probabilities. However, the following result states that in some situations the false alarm probability can be set even though the true distance d is not known, that is, without any information about the mean of the random variable under H_0.

Proposition 2 *For a zero mean Gaussian measurement error and a symmetric window function, the false alarm probability can be set independently of the true distance between the mobile and the base station.*

Proof *See [2].*

Proposition 2.2 states that under suitable conditions the distance function is independent of the true distance under the LOS hypothesis and it is therefore theoretically possible to set the false alarm rate.

Under the conditions stated in the above two propositions, the decision test can be expressed as follows [2]:

$$\int \hat{p}_r(x)\log(\sqrt{2\pi}\sigma\hat{p}_r(x))dx + \frac{\hat{\sigma}^2}{2\sigma^2} \underset{H_1}{\overset{H_0}{\lesseqgtr}} \delta', \tag{9}$$

where $\hat{\sigma}^2$ is the sample variance[2] of the range measurements, that is $\hat{\sigma}^2 = \frac{1}{m}\sum_{i=1}^{m}(r_i - \hat{d})^2$.

Depending on the technique to locate the mobile user, the classification of BS's may not be necessary. Instead some reliability information about the measurements from each BS might be required. In this case, the distance value between the LOS and NLOS pdf's can be used as a reliability information, which can help us to locate the mobile more accurately.

II. Time-of-Arrival Estimation

Due to the fine resolution of UWB signals, time-based geolocation techniques are more appropriate for UWB systems. Here we consider a TOA estimation approach, which tries to estimate the first arriving signal path in order to reduce the effects of NLOS propagation. The estimation technique can be considered as a two-step algorithm:

(1) Estimate the delay of a signal path of the incoming signal (acquisition step),

(2) Starting from the detected path, estimate the delay of the first arriving path.

The first step can be very time-consuming for a UWB system due to its high time resolution. Therefore, fast acquisition algorithms have been considered in the literature [6]-[8].

For the second step, we consider the first path detection algorithm in [9]. Given the location of the path detected in the previous step, the suboptimal algorithm searches backwards from the the given paths by estimating the delays and the amplitudes of the previous paths.

[2]The sample variance is often defined as $s^2 = \frac{1}{m-1}\sum_{i=1}^{m}(r_i - \bar{r})^2$, where $\bar{r}$ is the sample mean. This definition makes s^2 an unbiased estimate of the population variance.

III. Location Estimation

After obtaining the TOA estimates from the previous step, the final job is to estimate the location of the mobile node. The conventional technique uses the least-squares technique to estimate the location [10]:

$$\hat{\boldsymbol{p}} = \arg\min_{\boldsymbol{p}} \left\{ \sum_{i=1}^{B} \alpha_i (r_i - \|\boldsymbol{p} - \boldsymbol{p}_i\|_2)^2 \right\}, \tag{10}$$

where α_i is the reliability of the ith measurement, $\boldsymbol{p}_i$ and $\boldsymbol{p}$ are the locations of the ith node and the node in question, respectively, r_i is the ith distance measurement (obtained from TOA measurement) from the ith node, and B is the number of reference nodes. Assuming a two-dimensional positioning problem, that is, $\boldsymbol{p} = [x\ y]$ and $\boldsymbol{p}_i = [x_i\ y_i]$, $\|\boldsymbol{p} - \boldsymbol{p}_i\|_2$ is given by

$$\|\boldsymbol{p} - \boldsymbol{p}_i\|_2 = d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}. \tag{11}$$

The LS technique in (10) is optimal if the ith measurement error is modelled by zero mean Gaussian random variable with variance $1/\alpha_i$. This model is approximately true for most LOS scenarios. However, in an NLOS situation, the error is usually biased since the first arriving signal has travelled an extra distance.

Considering the IEEE channel model [11], we see that multipath arrival times follow a Poisson distribution. In other words, the time difference between any two paths is an exponentially distributed random variable. The IEEE channel measurements provide the mean of this exponential random variable in different scenarios. Assuming that we are able to detect the first arriving path in the NLOS situation by the algorithm in Section II, the absent LOS path can be considered as the preceding path of that first arriving NLOS path. Hence, the NLOS error can be modelled as an exponentially distributed random variable as specified by the channel measurements. Therefore, we can now model the measurements from B reference nodes as

$$r_i = d_i + \begin{cases} n_i + e_i, & i = 1, \ldots, M \\ n_i, & i = M+1, \ldots, B \end{cases}, \tag{12}$$

where $n_i \sim \mathcal{N}(0\,,\,\sigma_i^2)$ and $e_i \sim \mathcal{E}(\lambda_i)$. We assume, without loss of generality, that the first M nodes are NLOS and the remaining ones are LOS.

In most cases, the NLOS error is much more significant than the Gaussian measurement error. Therefore, we can model the measurements more simply as

$$r_i = d_i + \begin{cases} e_i, & i = 1, \ldots, M \\ n_i, & i = M+1, \ldots, B \end{cases}, \tag{13}$$

which will result in a much simpler estimator at the end.

The MLE for the node location is given by

$$\hat{\boldsymbol{p}} = \arg\max_{\boldsymbol{p}} p\,(\boldsymbol{r}|\boldsymbol{p}), \tag{14}$$

where $\boldsymbol{r} = [r_1 \cdots r_B]$.

Using the approximate model in (13), the MLE for the node location can be obtained, after some manipulation, as

$$\hat{\boldsymbol{p}} = \arg\min_{\boldsymbol{p}} \left\{ \sum_{i=1}^{M} \lambda_i (r_i - d_i) + \sum_{i=M+1}^{B} \frac{1}{2\sigma_i^2} (r_i - d_i)^2 \right\}. \tag{15}$$

In order to obtain the exact decision rule, consider the summation of exponential and Gaussian r.v.'s. For an exponential r.v. with parameter λ and for a zero mean Gaussian r.v. with variance σ^2, the p.d.f. for the sum can be obtained as

$$p(x) = \lambda e^{-\lambda(x - \lambda\sigma^2/2)} Q\left(\lambda\sigma - \frac{x}{\sigma}\right). \tag{16}$$

Then the MLE for the node position becomes

$$\hat{\boldsymbol{p}} = \arg\min_{\boldsymbol{p}} \left\{ \sum_{i=1}^{M} \lambda_i (r_i - d_i - \lambda_i \sigma_i^2/2) - \sum_{i=1}^{M} \log \left[Q\left(\lambda_i \sigma_i - \frac{r_i - d_i}{\sigma_i}\right)\right] + \sum_{i=M+1}^{B} \frac{1}{2\sigma_i^2} (r_i - d_i)^2 \right\}. \tag{17}$$

Note that for $\sigma_i^2 = 0$ for $i = 1, \ldots, M$, (17) reduces to (15).

IV. Location Tracking [13]

When the aim is to track a specific node, some smoothing operation on the location estimates, obtained as described in the previous section, is needed. This smoothing operation is achieved by a Kalman-Bucy filter.

Assume that each reference node takes measurements from the mobile node every $\triangle t$ seconds. The location estimate obtained from the location estimation algorithm in Section III method at time t is denoted by $\mathbf{Y}(t)$ where

$$\mathbf{Y}(t) = [Y_1(t) \quad Y_2(t)]^T. \tag{18}$$

Let the state vector be defined as

$$\mathbf{X}(t) = [X_1(t) \quad X_2(t) \quad X_3(t) \quad X_4(t)]^T \tag{19}$$

where $X_1(t)$ and $X_2(t)$ denote the x and y coordinates, respectively, of the mobile node, whereas $X_3(t)$ and $X_4(t)$ denote the x and y coordinates, respectively, of the velocity vector at time t. Then, the state and measurement equations can be expressed as follows:

$$\mathbf{X}_{i+1} = \mathbf{F}_i \mathbf{X}_i + \mathbf{G}_i \mathbf{U}_i \tag{20}$$

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{X}_i + \mathbf{V}_i \tag{21}$$

for $i = 0, 1, ...$ where i is the time at which the i^{th} location estimation is performed, $\mathbf{F}_i$, $\mathbf{G}_i$ and $\mathbf{H}_i$ are the following matrices:

$$\mathbf{F}_i = \begin{bmatrix} 1 & 0 & \triangle t & 0 \\ 0 & 1 & 0 & \triangle t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{G}_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \triangle t & 0 \\ 0 & \triangle t \end{bmatrix}, \mathbf{H}_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T.$$

$\mathbf{U}_i$ is the two dimensional random acceleration component which is modelled by a zero mean Gaussian process, and $\mathbf{V}_i$ is the two-dimensional zero mean Gaussian measurement error.

For the linear stochastic system of (20) and (21), the discrete-time Kalman-Bucy filter equations can be used assuming that $\mathbf{U}_i$ and $\mathbf{V}_i$ are independent sequences that are also independent from the initial state. The estimated state at time i given i measurements is the conditional expectation of the state given those previous measurements $(\mathbf{Y_0}, \mathbf{Y_1}, ..., \mathbf{Y_i})$, which is simply denoted as follows:

$$\hat{\mathbf{X}}_{i|i} = \mathrm{E}\{\mathbf{X}_i | \mathbf{Y}_0^i\}. \tag{22}$$

The estimates $\hat{\mathbf{X}}_{i|i} = \mathrm{E}\{\mathbf{X}_i|\mathbf{Y}_0^i\}$ and $\hat{\mathbf{X}}_{i+1|i} = \mathrm{E}\{\mathbf{X}_{i+1}|\mathbf{Y}_0^i\}$ are given recursively by the following equations [12]:

$$\hat{\mathbf{X}}_{i|i} = \hat{\mathbf{X}}_{i|i-1} + \mathbf{K}_i(\mathbf{Y}_i - \mathbf{H}_i\hat{\mathbf{X}}_{i|i-1}) \quad i = 0, 1, ..., \tag{23}$$

and

$$\hat{\mathbf{X}}_{i+1|i} = \mathbf{F}_i\hat{\mathbf{X}}_{i|i} \quad i = 0, 1, ..., \tag{24}$$

where the Kalman gain matrix $\mathbf{K}_i$ is given by

$$\mathbf{K}_i = \mathbf{\Sigma}_{i|i-1}\mathbf{H}_i^T(\mathbf{H}_i\mathbf{\Sigma}_{i|i-1}\mathbf{H}_i^T + \mathbf{R}_i)^{-1}, \tag{25}$$

with $\mathbf{\Sigma}_{i|i-1} = \mathrm{Cov}\{\mathbf{X}_i|\mathbf{Y}_0^{i-1}\}$ and $\mathbf{R}_i = \mathrm{Cov}\{\mathbf{V}_i\}$. Here, $\mathbf{\Sigma}_{i|i-1}$ represents the covariance matrix of the prediction error $\mathbf{X}_i - \hat{\mathbf{X}}_{i|i-1}$, conditioned on $\mathbf{Y}_0^{i-1}$. This matrix and the filtering error covariance, $\mathbf{\Sigma}_{i|i} = \mathrm{Cov}\{\mathbf{X}_i|\mathbf{Y}_0^i\}$, can be computed using the following recursion:

$$\mathbf{\Sigma}_{i|i} = \mathbf{\Sigma}_{i|i-1} - \mathbf{K}_i\mathbf{H}_i\mathbf{\Sigma}_{i|i-1} \quad i = 0, 1, ..., \tag{26}$$

$$\mathbf{\Sigma}_{i+1|i} = \mathbf{F}_i\mathbf{\Sigma}_{i|i}\mathbf{F}_i^T + \mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^T \quad i = 0, 1, ..., \tag{27}$$

where $\mathbf{Q}_i = \mathrm{Cov}\{\mathbf{U}_i\}$.

With proper initial values, equations (23) through (27) can be used to evaluate the state of the mobile node at each time. The initial value for the state variable estimate can be set as follows:

$$\hat{\mathbf{X}}_{0|0} = [\hat{x}_1(i) \quad \hat{x}_2(i) \quad 0 \quad 0]^T, \tag{28}$$

where $\hat{\mathbf{x}} = [\hat{x}_1(i) \quad \hat{x}_2(i)]^T$ is the first location estimate obtained by the algorithm in Section III, and the initial velocity is assumed to be zero.

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