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Abstract

In this letter, a threshold selection technique for time of arrival estimation of ultra-wideband signals is proposed. It exploits the Kurtosis of the received signal samples. The dependency between the Kurtosis and optimal normalized threshold are investigated via simulations. The proposed technique yields efficient threshold selections, has low complexity and sampling rate requirements, and accounts both the signal to noise ratio and the statistics of individual channel realization.

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Threshold Selection for UWB TOA Estimation Based on Kurtosis Analysis

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***Abstract*— In this letter, a threshold selection technique for time of arrival estimation of ultra-wideband signals is proposed, which uses the Kurtosis of the received signal samples. The dependency between the Kurtosis and optimal normalized threshold are investigated via simulations. The proposed technique yields efficient threshold selection, has low complexity and sampling rate requirements, and accounts both the signal to noise ratio and the statistics of individual channel realizations.**

I. Introduction

Impulse radio ultra-wideband (IR-UWB) enables precise ranging and location estimation due to extremely short duration pulses employed. Accurate time of arrival (TOA) calculation based on the received signal samples is the key aspect for precise ranging, and is a challenging issue due to hundreds of multipath components observed. If a coarse timing estimate is available, comparing the received samples with a threshold and choosing the first threshold-exceeding sample is a convenient technique that directly yields the leading edge estimate of the received signal. However, the major challenge is the selection of an appropriate threshold based on the received signal statistics (i.e. the signal to noise ratio (SNR), channel realization etc.). Even though threshold based TOA estimation was discussed in [1], it was not addressed how to select the thresholds. In [2], a normalized threshold technique was proposed that accounts the minimum and maximum sample values, and its dependency on the SNR was investigated. However, calculation of the SNR without the knowledge of the TOA is an extremely challenging task, making it impractical to adapt the normalized threshold based on it. Moreover, using only the SNR of the received signal does not account the individual channel realizations, and therefore in essence yields suboptimal threshold selection.

The contribution of this letter is to use the Kurtosis of the signal samples as a metric for threshold selection. Unlike the SNR of the received signal, Kurtosis captures both the statistics of individual channel realizations, and the relative energy of the signal to noise.

II. System Model

Let the received UWB multipath signal be represented as

$$r(t) = \sum_{j=-\infty}^{\infty} d_j \omega_{mp}\big(t - jT_f - c_j T_c - \tau_{toa}\big) + n(t) \qquad (1)$$

where frame index and frame duration are denoted by j and T_f, N_s represents the number of pulses per symbol, T_c is the chip duration, T_s is the symbol duration, τ_{toa} is the TOA of the received signal, and N_h is the possible number of chip positions per frame, given by $N_h = T_f/T_c$. Effective pulse after the channel impulse response is given by $\omega_{mp}(t) = \sqrt{E}\sum_{l=1}^{L}\alpha_l \omega(t-\tau_l)$, where $\omega(t)$ is the received UWB pulse with unit energy, E is the pulse energy, α_l and τ_l are the fading coefficients and delays of the multipath components, respectively. Additive white Gaussian noise (AWGN) with zero-mean and double-sided power spectral density $\mathcal{N}_0/2$ and variance σ^2 is denoted by $n(t)$. No modulation is considered for the ranging process. Time-hopping codes $c_j^{(k)} \in \{0,\ 1, ..., N_h-1\}$, and random-polarity codes $d_j \in \{\pm 1\}$ are used to introduce additional processing gain. Assume that a coarse acquisition on the order of frame-length is acquired [3], such that $\tau_{toa} \sim \mathcal{U}(0, T_f)$, where $\mathcal{U}(.)$ denotes the uniform distribution. For the search region, the signal within time frame T_f plus half of the next frame is considered to factor-in inter-frame leakage due to multipath, and the signal is input to a square-law device with an integration interval of T_b[1]. The number of samples (or blocks) is denoted by $N_b = \frac{3}{2}\frac{T_f}{T_b}$, and $n \in \{1,\ 2, ..., N_b\}$ denotes the sample index with respect to the starting point of the uncertainty region. With a sampling interval of t_s (which is equal to block length T_b), the sample values at the output of the square-law device are given by

$$z[n] = \sum_{j=1}^{N_s} \int_{(j-1)T_f+(c_j+n-1)T_b}^{(j-1)T_f+(c_j+n)T_b} |r(t)|^2 dt\ , \qquad (2)$$

where means and variances of noise-only and energy bearing blocks are given by $\mu_0 = M\sigma^2$, $\sigma_0^2 = 2M\sigma^4$, $\mu_e = M\sigma^2 + E_n$, $\sigma_e^2 = 2M\sigma^4 + 4\sigma^2 E_n$, respectively, where M is the degree of freedom given by $M = 2BT_b + 1$, E_n is the total signal energy within the nth block, and B is the signal bandwidth. Received bit energy (which is not available to the receiver) is given by $\sum_{n=n_{toa}}^{n_{toa}+n_{eb}-1} E_n$, where n_{eb} is the number of blocks that sweeps the signal samples. Since τ_{toa} is continuous, first multipath component may arrive anywhere within the first energy block, which is accounted in the sequel.

Received samples can be compared to an appropriate threshold, and the first threshold-exceeding sample index can be corresponded as the TOA estimate, i.e.

$$\hat{t}_{TC} = \Big[\min\big\{n | z[n] > \xi\big\} - 0.5\Big]T_b\ , \qquad (3)$$

[1]Note that extension of the approach to transmitted or stored reference systems is trivial with a similar methodology as in energy detection [4].

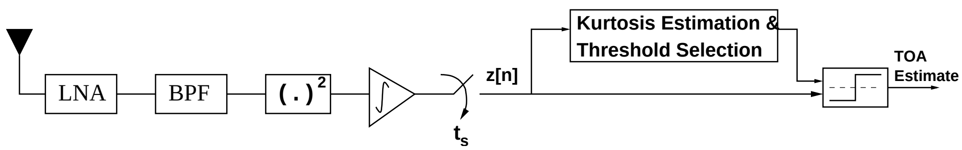

Fig. 1. Block diagram for threshold selection based on Kurtosis analysis.

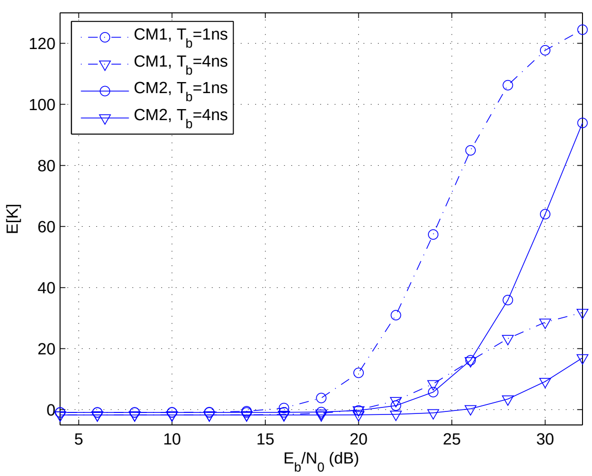

Fig. 2. Dependence of the expected value of the Kurtosis to E_b/N_0 for different channel models and block sizes for energy detection.

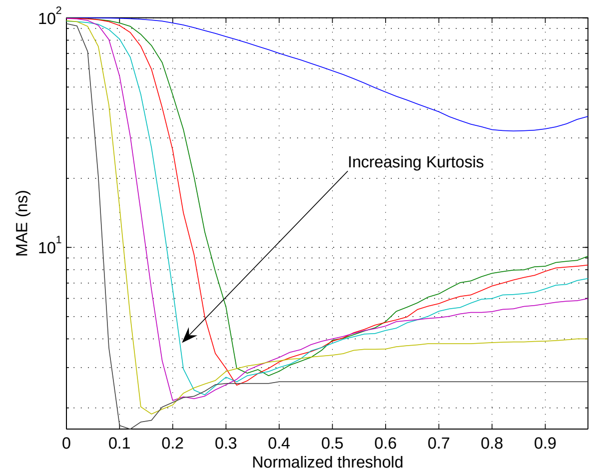

Fig. 3. MAE with respect to normalized threshold for $\log_2 K\big(z[n]\big) \in \{0, 1, 2, 3, 4, 5, 6\}$ (CM1, $T_b = 4$ns).

where ξ is a threshold that must be set based on the received signal statistics. Given the minimum and maximum energy sample values, the following normalized adaptive threshold can be used

$$\xi_{norm} = \frac{\xi - \min\{z[n]\}}{\max\{z[n]\} - \min\{z[n]\}} \ . \tag{4}$$

In [2], ξ_{norm} that minimizes the mean absolute error (MAE) defined by $E[|\hat{t}_{TC} - \tau_{toa}|]$ for a particular E_b/N_0 value was analyzed via simulations. However, estimation of E_b/N_0 is very challenging task. Moreover, optimal normalized threshold ξ_{opt} may change for different channel realizations at the same E_b/N_0, which motivates other metrics for threshold selection.

III. Threshold Selection Based on Kurtosis Analysis

The Kurtosis of the received signal samples is calculated using the second and fourth order moments of the received signal, and is expressed as

$$\kappa\big(z[n]\big) = \frac{\mathcal{E}(z^4[n])}{\mathcal{E}^2(z^2[n])} \ , \qquad n = 1, 2, ..., N_b \ , \tag{5}$$

where $\mathcal{E}(.)$ denotes the expectation operation. The Kurtosis relative to a Gaussian can be defined as $K\big(z[n]\big) = \kappa\big(z[n]\big) - 3$, which is zero for the Gaussian distribution. The Kurtosis is also commonly referred as Gaussian *unlikeness*, since a larger value of K implies a stronger non-Gaussianity[2].

In the absence of signal (or for low SNR), and for sufficiently large M, $z[n]$ will be Gaussian distributed, yielding $K = 0$. On the other hand, as SNR increases, K will tend to increase, and it may as well take different values for the same SNR value. In Fig. 2, expected value of K with respect to E_b/N_0 are plotted for different block sizes, and CM1 and CM2 channel models of IEEE802.15.4a [5] (averaged over 1000 channel realizations). The parameters employed are $T_f = 200$ns, $T_c = 1$ns, $B = 4$GHz, and $N_s = 1$. Note that the relationship in Fig. 2 is an average relationship, and Kurtosis values for individual channel realizations may show deviations depending on the clustering of the multipath components, which also affects the optimality of the threshold for the same E_b/N_0 value.

In Fig. 3, MAEs of the TOA estimates are observed with respect to normalized threshold and the Kurtosis values rounded to logarithmic integers[3]. The Kurtosis values are obtained for 1000 CM1 channel realizations with $E_b/N_0 = \{10, 12, 14, 16, 18, 20, 22, 24, 26\}$dB, i.e. for 9000 test cases. The optimal achievable MAE improves with increasing Kurtosis value. The optimal normalized threshold value with respect to the logarithm of Kurtosis, and the corresponding MAE are plotted in Fig. 4. While the channel model does not much affect the relation between ξ_{opt} and $\log_2 K$, the dependency changes for different block sizes. In order to model the relationship, a double exponential function fit is used for $T_b = 4$ns, while a linear function fit is used for

[2]Dropped the Kurtosis index term in the sequel for brevity.

[3]In order to account the clustering of the Kurtosis values at low E_b/N_0.

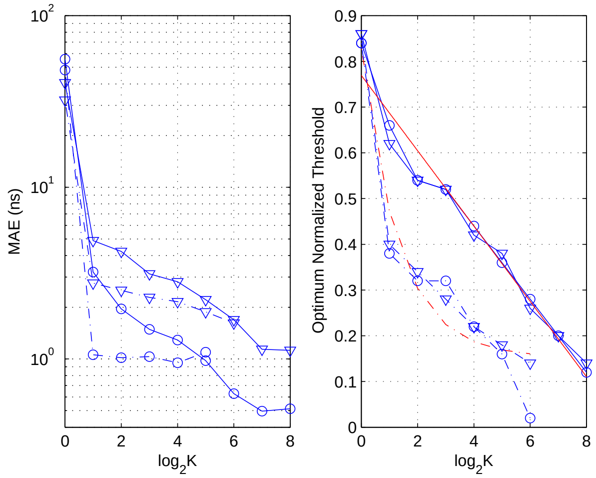

Fig. 4. Dependence of the optimal normalized threshold to Kurtosis value, and corresponding MAE for CM1 (triangle) and CM2 (circle), and for $T_b =$ 1ns (solid) and $T_b =$ 4ns (dashed). Exponential and linear curve fits are also shown.

$T_b = 1$ns

$$\xi_{opt}^{(4ns)} = 0.673e^{-0.75 \log_2 K} + 0.154e^{-0.001 \log_2 K} \ , \quad (6)$$

$$\xi_{opt}^{(1ns)} = -0.082 \log_2 K + 0.77 \ , \quad (7)$$

where the model coefficients are obtained using both CM1 and CM2 results.

IV. Simulation Results and Discussion

Computer simulations are performed to compare the threshold comparison and maximum energy selection (MES) based TOA estimation algorithms [2]. Results in Fig. 5 and Fig. 6 show that using the Kurtosis metric, estimation error can be significantly decreased compared to fixed $\xi_{norm} = 0.4$ and MES techniques, and also yields better results than ξ_{opt} solely based on SNR values. Confidence level of 3ns estimation error (i.e. 1 meter ranging accuracy) are plotted in Fig 7, which shows that %70 confidence level is achievable with E_b/N_0 larger than 22dB. Better results can be obtained via coherent ranging at lower E_b/N_0 values [4].

The proposed threshold selection approach can be easily implemented by calibrating the system for a particular block size and frame duration, and is fairly independent of the channel model.

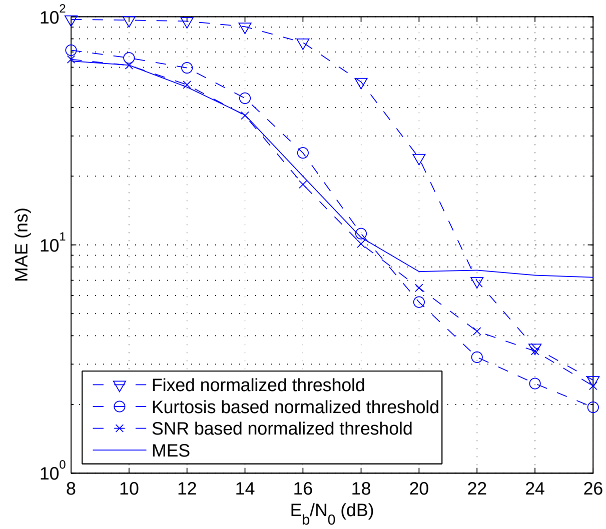

Fig. 5. MAE with respect to E_b/N_0 using different algorithms (CM1, $T_b = 1$ns).

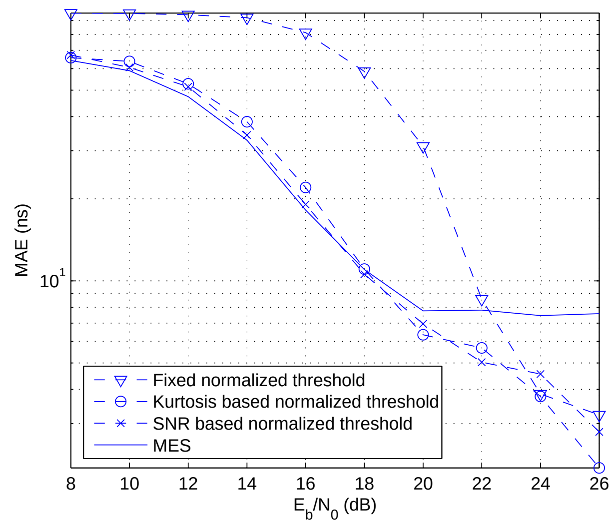

Fig. 6. MAE with respect to E_b/N_0 using different algorithms (CM1, $T_b = 4$ns).

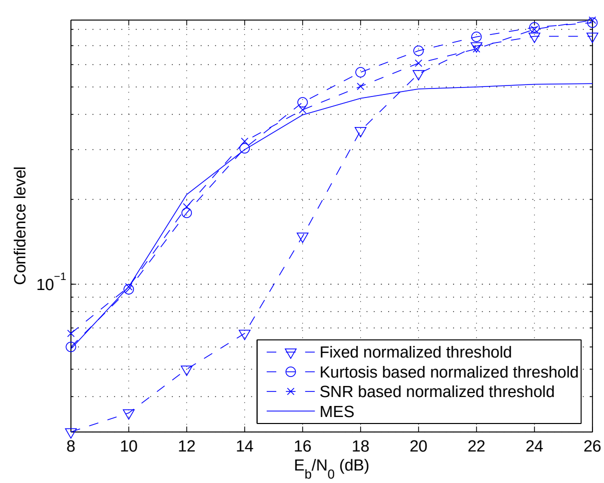

Fig. 7. Confidence level of 3ns error with respect to E_b/N_0 using different algorithms (CM1, $T_b = 4$ns).

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