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PSD of UWB signals

Yves-Paul Nakache, *Member, IEEE,* and Andreas F. Molisch, *Fellow, IEEE*

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***Index Terms*— polarity randomization, Power Spectral Density, Pulse Position Modulation (PPM), spectral lines, time hopping impulse radio (TH-IR).**

Introduction

This document is organized in the following manner: Section 1 introduces the general Power Spectral Density equation that will be used in the remainder of this document. Section 2 gives analytical formulations for the spectrum of "conventional" TH-PPM, showing the spectral lines created by this multiple-access (MA)/modulation scheme, and also presents the spectral characteristics for OOK and PAM modulations. Section 3 introduces the symbol-based polarity randomization which eliminates the spectral lines and the pulse-based polarity randomization, which solves the problem of the spectral crest factor due to the structure of the TH sequence. Remarks on finite sequences, a summary and a conclusion end this paper.

The results presented in this paper were verified by theoretical analysis and Matlab simulations. In order to present clear illustrations of the results, a bandwidth of 4 GHz (frequency range 3-7 GHz) was used to display the PSDs and their spectral crest factor. However, the spectrum of our waveforms also fits the FCC mask at frequencies >7 GHz as shown in [1]. The parameters for the examples were chosen such that they were suitable for a high-data-rate system (100 Mbit/s). However, we stress that the basic approach is equally valid for other data rates. In particular, it can be applied to low-data rate systems as envisioned, e.g., for the IEEE 802.15.4a standard.

I. - Power Spectral Density (PSD)

The following equation of the PSD of M-ary modulation when the digital input is Markovian will be used for the calculation of the PSDs of UWB signals, [2]

$$G_s(f)=\frac{1}{T_s^{\,2}}.\sum_{n=-\infty}^{+\infty}\left(\left|\sum_{i=0}^{M-1}P_iS_i\left(\frac{n}{T_s}\right)\right|^2\delta\left(f-\frac{n}{T_s}\right)\right)+\frac{1}{T_s}\left(\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}P_i.S_i(f)S_j^*(f)\left[\sum_{n=-\infty}^{+\infty}\left(a_{ij}^m-P_j\right).e^{j2\pi mfT_s}\right]\right)\quad(1)$$

Here, M denotes the number of symbols, T_s the symbol period, $S_i(f)$ the Fourier transform of the i^{th} symbol of the constellation, P_i the marginal probability of the i^{th} symbol, and a_{ij}^m the probability that the symbol $s_j(t)$ is transmitted m time units following transmission of $s_i(t)$.

Since we consider independent input sequences:

$$a_{ij}^{(n)}=\begin{cases}\delta_{ij} & n=0\\ P_j & n\neq 0\end{cases}$$

and we obtain:

$$G_s(f)=\frac{1}{T_s^{\,2}}.\sum_{n=-\infty}^{+\infty}\left(\left|\sum_{i=0}^{M-1}P_i.S_i\left(\frac{n}{T_s}\right)\right|^2\delta\left(f-\frac{n}{T_s}\right)\right)+\frac{1}{T_s}\left(\sum_{i=0}^{M-1}P_i.\left|S_i(f)\right|^2-\left|\sum_{i=0}^{M-1}P_i.S_i(f)\right|^2\right)\quad(2)$$

We will use Eq. (2) in the remainder of this document.

II. - Spectral characteristics of classical TH-IR

A Transmit waveform of TH-IR

In this section, we review and re-interpret the spectral characteristics of classical TH-IR. The structure of the transmit signal is shown in Fig. 1. The transmit signal is given as [3], [4]

$$s_{tr}(t)=\sum_{j=-\infty}^{\infty}x(t-jT_f-c_jT_c-b_{\lfloor j/N_f\rfloor}\Delta)\quad(3)$$

Where $x(t)$ is the transmitted pulse, T_f is the average pulse repetition time, N_f is the number of frames (and therefore also the number of pulses N_s) representing one

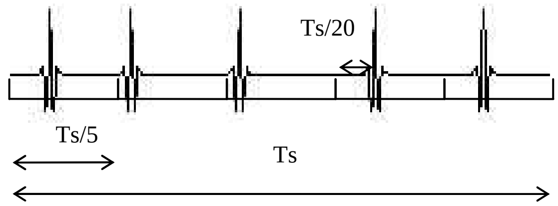


Fig. 1 TH structure of sequence [1 0 0 1 1]

information symbol of length T_s, and b is the information symbol transmitted. Δ is the time shift of the Pulse Position Modulation.

The TH sequence provides an additional time shift of $c_j T_c$ seconds to the j^{th} pulse of the signal, where T_c is the chip interval, and c_j are the elements of a pseudorandom sequence, taking on integer values between 0 and $N_c - 1$. To prevent pulses from overlapping, the chip interval is selected to satisfy $T_c \le T_f / N_c$. In the remainder of this paper, we assume $T_f / T_c = N_c$ so that N_c is the number of chips per frame.

B. PSD of TH-IR with short TH sequence

The PSD of the transmit signal is determined both by the pulse shape $\mathrm{s}(\mathrm{t})$, and the time-hopping sequence c_j . A majority of systems considered in the literature and standardization working groups up to now assume that the duration of the TH sequence is identical to the symbol duration; in analogy to CDMA, we henceforth refer to this case as “short” hopping sequence. Therefore, this TH sequence has a duration of $\mathrm{T_s}$ and it contains $\mathrm{N_s}$ pulses. In that case, the Fourier transform of the TH sequence can be written as

$$FT\{TH(t)\} = X(f).\sum_{k=0}^{N_s-1} e^{j2\pi\left(T_f.k+c_kT_c\right).f} = S(f) \quad (4)$$

The underlying pulse shape for the simulations of this paper is the fifth derivative of a Gaussian pulse, which was shown to be a good choice in [5]. $X(f)$ represents its Fourier Transform.

For most of the examples in the remainder of the text, we use a TH sequence with $\mathrm{N_s} = 5$ pulses following the sequence c_k [1,0,0,1,1], and 4 chips per frame (see Fig. 1), [6].

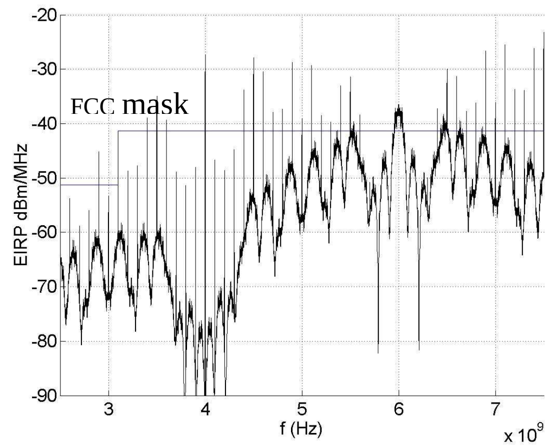


Fig. 2: PSD of short TH-IR signal with 2-PPM

1. 2-PPM

Rewriting Eq. (2) in the case of a 2-PPM with equiprobable input sequence, it becomes:

$$\begin{aligned}
G_s(f) &= \frac{1}{M^2 T_s^{\,2}}.\sum_{n=-\infty}^{+\infty}\left(\left|\sum_{i=0}^{M-1} S_i\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f-\frac{n}{T_s}\right)\right) \\
&+\frac{1}{M.T_s}.\left(\sum_{i=0}^{M-1}\left|S_i(f)\right|^2 - \frac{1}{M}.\left|\sum_{i=0}^{M-1} S_i(f)\right|^2\right) \qquad (5)\\
&=\frac{1}{2^2 T_s^{\,2}}.\sum_{n=-\infty}^{+\infty}\left(\left|S_0\left(\frac{n}{T_s}\right)+S_1\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f-\frac{n}{T_s}\right)\right) \\
&+\frac{1}{2.T_s}.\left(\left|S_0(f)\right|^2+\left|S_1(f)\right|^2-\frac{1}{2}.\left|S_0(f)+S_1(f)\right|^2\right)
\end{aligned}$$

and

$$\begin{aligned}
S_0(f) &= S(f).e^{-j0} = S(f) \\
S_1(f) &= S(f).e^{-j2\pi\Delta f}
\end{aligned} \qquad (6)$$

where Δ is the delay due to the Pulse Position Modulation, $\mathrm{S(f)}$ is the Fourier transform of the basic pulse waveform and $\mathrm{S_0(f)}$ and $\mathrm{S_1(f)}$ are the Fourier transforms of symbols 0 and 1.

Then:

$$G_s(f)=\frac{1}{2^2T_s^2}.\sum_{n=-\infty}^{+\infty}\left(\left|S\left(\frac{n}{T_s}\right)+S\left(\frac{n}{T_s}\right).e^{-j2\pi\frac{\Delta}{T_s}n}\right|^2\delta\left(f-\frac{n}{T_s}\right)\right)$$

$$+\frac{1}{2.T_s}.\left(\left|S(f)\right|^2+\left|S(f).e^{-j2\pi\Delta f}\right|^2-\frac{1}{2}.\left|S(f)+S(f).e^{-j2\pi\Delta f}\right|^2\right)$$

$$= \frac{1}{2^2 T_s^2}\sum_{n=-\infty}^{+\infty}\left|S\left(\frac{n}{T_s}\right)\right|^2 \left(\begin{array}{c}\left(1+\cos\left(2\pi n\frac{\Delta}{T_s}\right)\right)^2 \\ +\sin^2\left(2\pi n \frac{\Delta}{T_s}\right)\end{array}\right).\delta\left(f-\frac{n}{T_s}\right)$$

$$+\frac{1}{2T_s}\left|S(f)\right|^2\left(2-\frac{1}{2}.2.\left(1+\cos\left(2\pi f\Delta\right)\right)\right)$$

$$= \frac{1}{2T_s^2}\sum_{n=-\infty}^{+\infty}\left|S\left(\frac{n}{T_s}\right)\right|^2\left(1+\cos\left(2\pi n\frac{\Delta}{T_s}\right)\right).\delta\left(f-\frac{n}{T_s}\right)$$

$$+\frac{1}{2T_s}\left|S(f)\right|^2\left(1-\cos\left(2\pi f\Delta\right)\right) \qquad (7)$$

Replacing S(f) in Eq. (2) by the Fourier transform of the TH sequence when the modulation format is a 2-PPM, the PSD of the modulated signal becomes:

$$G_s(f) = \frac{1}{2T_s^2}\sum_{n=-\infty}^{+\infty}\left[\left|X\left(\frac{n}{T_s}\right)\right|^2\left|\sum_{k=0}^{N_s-1} e^{j2\pi\left(T_f.k+c_kT_c\right).\frac{n}{T_s}}\right|^2 .\left(1+\cos\left(2\pi n\frac{\Delta}{T_s}\right)\right).\delta\left(f-\frac{n}{T_s}\right)\right]$$

$$+\frac{1}{2T_s}\left|X(f)\right|^2.\left|\sum_{k=0}^{N_s-1} e^{j2\pi\left(T_f.k+c_kT_c\right).f}\right|^2.\left(1-\cos\left(2\pi f\Delta\right)\right)$$

(8)

The PSD of this TH-IR signal with 2-PPM is given in Fig. 2. Several spectral lines are above the spectral mask provided by the FCC. For comparison purposes, we fixed the power such that it is equal to the maximum power a signal with the same TH sequence and pulse shape but using BPSK could transmit.

2. OOK

Rewriting Eq. (2) with On Off Keying, with equiprobable input sequence and such that:

$$S_0(f)=\sqrt{2}.S(f) \qquad (9)$$
$$S_1(f)=0$$

we obtain:

$$G_s(f)=\frac{1}{2T_s^2}.\sum_{n=-\infty}^{+\infty}\left(\left|S\left(\frac{n}{T_s}\right)\right|^2\delta\left(f-\frac{n}{T_s}\right)\right)+\frac{1}{2.T_s}.\left|S(f)\right|^2 \qquad (10)$$

The square root of 2 in the definition of $S_0(f)$ allows us to keep the same average symbol energy as for the 2-PPM case.

Replacing S(f) by the Fourier transform of the TH code in Eq. (10), the PSD of the OOK signal becomes:

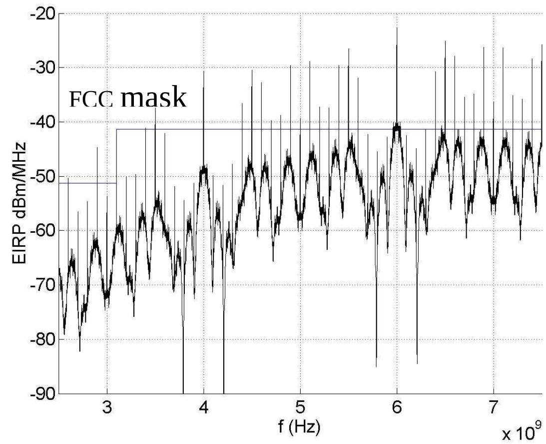


Fig. 3 PSD of a short TH-IR signal with OOK

$$G_s(f)=\frac{1}{2T_s^2}.\sum_{n=-\infty}^{+\infty}\left(\left|X\left(\frac{n}{T_s}\right)\right|^2\left|\sum_{k=0}^{N_s-1}e^{j2\pi\left(T_f.k+c_kT_c\right).\frac{n}{T_s}}\right|^2\delta\left(f-\frac{n}{T_s}\right)\right)$$

$$+\frac{1}{2.T_s}.\left|X(f)\right|^2\left|\sum_{k=0}^{N_s-1}e^{j2\pi\left(T_f.k+c_kT_c\right).f}\right|^2 \qquad (11)$$

The PSD of TH-IR with OOK is given in Fig. 3

We notice that the continuous part of the spectrum contains more energy with OOK than with 2-PPM but the spectral lines remain a problem.

3. M-PAM

Defining an antipodal modulation of M symbols, we can write:

$$S_0=-S_{M/2}$$
$$\ldots \qquad (12)$$
$$S_{M/2-1}=-S_{M-1}$$

If the input sequence is equiprobable, we derive from Eq. (3):

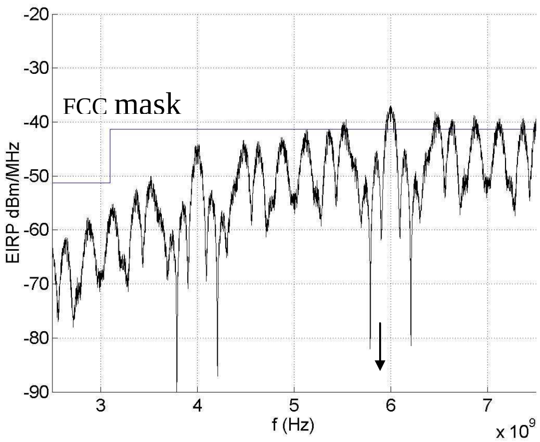
$$G_s(f)=\frac{2}{M.T_s}\left(\sum_{i=0}^{M/2-1}\left|S_i(f)\right|^2\right) \qquad (13)$$

We note that if the waveforms are different, the PSD of the signal is the average of the PSD of each waveform.

Furthermore, if the M signals $s_i(t)$ are scalar multiples such that $s_i(t) = a_i.s(t)$ and

$$\sum_{i=0}^{M/2-1}a_i^2=\frac{M}{2}$$

the PSD of the M-PAM signal becomes:



$$G_s(f) = \frac{1}{T_s}.\left|S(f)\right|^2 \quad (14)$$

Fig. 4. PSD of a short TH-IR signal with M-PPM and symbol-based polarity randomization

We notice that the PSD of such PAM signal using the TH sequence defined by Eq. (1) will have the same continuous envelope as in Fig. 3, but without the spectral lines and with twice the power.

III. - PSD of short TH-IR with polarity randomization

A. PSD of short TH-IR with a constellation of M symbols and Symbol-Based polarity Randomization

The first case we consider randomizes the polarity (sign) of the transmitted pulses of a TH-IR signal on a symbol-by-symbol basis. The constellation of the modulation has M symbols. In the case of M-PPM, this has a formal similarity to a combination of joint PPM and PAM as modulation format (combined with TH for MA). The difference is that in our case, the sign of the transmit pulses does not bear any information, and can thus be easily discarded by simplified receiver structures (envelope detectors). The introduction of pseudorandom polarity serves only to improve the spectral properties of the transmit signal. As we change randomly the polarity of the M symbols S_i ; the signal can be viewed as a pulse train composed of 2M antipodal symbols. Thus, the Eq. (1) can be rewritten as the spectral density of a signal with antipodal modulation:

$$G_s(f) = \frac{1}{M^2 T_s^2}.\sum_{n=-\infty}^{+\infty}\left(\left|\frac{1}{2}\sum_{i=0}^{M-1} S_i\left(\frac{n}{T_s}\right) + \frac{1}{2}\sum_{j=0}^{M-1} S_j\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f-\frac{n}{T_s}\right)\right)$$
$$+\frac{1}{T_s}\left(\sum_{i=0}^{M-1}\frac{1}{M}.\left|S_i(f)\right|^2 - \left|\frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{2}S_i(f) + \frac{1}{M}\sum_{j=0}^{M-1}\frac{1}{2}S_j(f)\right|^2\right)$$

(15).

where S_i is the Fourier Transform of the i^{th} shifted version of the short TH sequence (not a single pulse !). The symbols s_{i+M} are the symbols s_i with an opposite polarity. Hence, $S_{i+M} = -S_i$ for i from 0 to M–1. The continuous and discrete parts of the spectrum then become:

$$G_s(f) = \frac{1}{M^2 T_s^2}.\frac{1}{2}\sum_{n=-\infty}^{+\infty}\left(\left|\sum_{i=0}^{M-1}\left(S_i\left(\frac{n}{T_s}\right) - S_i\left(\frac{n}{T_s}\right)\right)\right|^2 \delta\left(f-\frac{n}{T_s}\right)\right)$$
$$+\frac{1}{T_s}\left(\sum_{i=0}^{M-1}\frac{1}{M}.\left|S_i(f)\right|^2 - \left|\frac{1}{2M}\sum_{i=0}^{M-1}\left(S_i(f) - S_i(f)\right)\right|^2\right)$$
$$G_s(f) = \frac{1}{M.T_s}\sum_{i=0}^{M-1}\left|S_i(f)\right|^2 \quad . \quad (16)$$

Thus, the spectral lines disappear. Furthermore, from Eq. (16), it appears that the spectrum of the signal is defined by the summation of the spectrum of the symbols. If the symbols have the same waveform, the spectral properties of the signal are identical to the spectral properties of this waveform.

From Eq. (10), it is also clear that the same statement holds for the different modulation formats like On-Off Keying. Furthermore, it is unnecessary to have zero mean information stream to control the spectral characteristics of the modulated signal. That solves the problem of spectral lines caused by non-equiprobable symbols and non-antipodal modulation schemes at the same time.

Using Eq. (4) the PSD of a short TH-IR signal with M-PPM and symbol-based polarity randomization becomes:

$$G_s(f) = \frac{1}{T_s}.\left|S(f)\right|^2 = \frac{1}{T_s}.\left|X(f)\right|^2.\left|\sum_{k=0}^{N_s-1} e^{j2\pi\left(T_f.k + c_k T_c\right).f}\right|^2$$

(17)

Fig. 4 illustrates the elimination of spectral lines using a symbol-based polarity randomization in the case of M-PPM with short TH sequence.

B. PSD of short TH-IR with a constellation of M symbols and long Pulse-Based polarity Randomization

The short TH sequence shown in Fig. 1 contained pulses with the same polarity. However, in addition to the symbol based polarity randomization, we can also use a short pulse based polarity randomization of duration T_s. In that case, the PSD is again given by the previous equation, but now the “basic signal” waveforms have a different, more flexible structure, i.e., the pulses can have different polarities.

This flexibility helps to better design the spectrum of the signal to reduce the spectral crest factor due to the TH structure and, therefore, to limit the power back off.

We now propose to randomize the polarity of each pulse of the short TH sequence on a pulse basis and over an infinite number of symbol periods. This pulse randomization will be called long pulse-based polarity randomization. We will show that the use of this long pulse-based polarity scrambling code will smooth the spectrum such that the PSD of the signal would be given by the spectral characteristics of the basic pulse alone.

This helps to maximize the transmitting power of the signal. The task of shaping the PSD of the signal can be reduced to shaping the pulse only, regardless of the modulation and spreading formats used.

To demonstrate this assertion we will be using the result of the previous chapter with short TH-IR, M symbol modulation and symbol-based polarity randomization, since the long pulse-based polarity randomization leads to the randomization of the polarity on a symbol basis also.

In the previous chapter the modulation format had a constellation of M symbols. The number of symbol waveforms now increases due to the pulse-based polarity randomization sequence. If the short TH sequence contains N_s pulses, the total number of sequences of N_s pulses with a unique polarity pattern is 2^{N_s}. We will call such sequence 'short TH-polarity sequence'. The structure of the signal is then mathematically equivalent to a modulation with $M.2^{N_s}$ antipodal symbols.

Eq. (16) then becomes:

$$G_s(f)=\frac{1}{M.2^{Ns}.T_s}.\sum_{j=0}^{M-1}\sum_{i=0}^{2^{Ns}-1}\left|S_{ij}(f)\right|^2 \quad (18)$$

with $$\left|S_{ij}(f)\right|^2=\left|a_j.S_i(f).e^{-j2\pi.b_j.f}\right|^2 \quad (19)$$

and $$\left|S_i(f)\right|^2=\left|X(f)\right|^2.\left|\sum_{k=1}^{N_s}g_{ki}.e^{-j2\pi T_s d_k f}\right|^2 \quad (20)$$

where a_i and b_i represent the amplitude and delay of the j^{th} symbol. $S_i(f)$ is the Fourier transform of the TH sequence with i^{th} short TH-polarity sequences, $X(f)$ is the Fourier transform of the basic pulse, d_k is the delay of the k^{th} pulse of the TH sequence such that $d_k.T_s=k.T_f+c_kT_c$ (see Eq. (4)) , and g_{ki} is the polarity of the k^{th} pulse of the i^{th} short TH-polarity sequence.

Note that $\{g_k\}_i$, the sequence of polarity of the N_s pulses of the i^{th} short TH-polarity sequence, is unique: $\forall\{i,j\},i\neq j,\{g_k\}_i\neq\{g_k\}_j$ and g_{ki} = 1 or -1

Furthermore, the sequence of delays $\{d_k\}$ is the same for all 2^{Ns} short TH-polarity sequences.

Equations (18) and (19) combined with the condition $\sum_{i=0}^{M-1}a_i^2=M$ result in:

$$\begin{aligned}G_s(f)&=\frac{1}{2^{Ns}.T_s}.\sum_{i=0}^{2^{Ns}-1}\left|S_i(f)\right|^2\\&=\frac{1}{2^{Ns}.T_s}.\left|X(f)\right|^2.\sum_{i=0}^{2^{Ns}-1}\left|\sum_{k=1}^{N_s}g_{ki}.e^{-j2\pi T_s d_k f}\right|^2\end{aligned} \quad (21)$$

We can simplify the summations as follow:

$$\begin{aligned}A&=\sum_{i=0}^{2^{N_s}-1}\left|\sum_{k=1}^{N_s}g_{ki}.e^{-j2\pi T_s d_k f}\right|^2\\
&=\sum_{i=0}^{2^{N_s}-1}\left|\sum_{k=1}^{N_s}g_{ki}.e^{j\omega d_k f}\right|^2 \qquad with \quad \omega=2\pi T_s\\
&=\sum_{i=0}^{2^{N_s}-1}\left|\sum_{k=1}^{N_s}g_{ki}.\cos(\omega d_k f)+j.g_{ki}.\sin(\omega d_k f)\right|^2\\
&=\sum_{i=0}^{2^{N_s}-1}\left[\left(\sum_{k=1}^{N}g_{ki}.\cos(\omega d_k f)\right)^2+\left(\sum_{k=1}^{N}g_{ki}.\sin(\omega d_k f)\right)^2\right]\\
&=\sum_{i=0}^{2^{N_s}-1}\left[\begin{array}{l}\sum_{k=1}^{N_s}\sum_{k'=1}^{N_s}g_{ki}.g_{k'i}.\cos(\omega d_k f).\cos(\omega d_{k'} f)\\+\sum_{k=1}^{N_s}\sum_{k'=1}^{N_s}g_{ki}.g_{k'i}.\sin(\omega d_k f).\sin(\omega d_{k'} f)\end{array}\right]\\
&=\sum_{i=0}^{2^{N}-1}\left[\sum_{k=1}^{N_s}\sum_{k'=1}^{N_s}g_{ki}.g_{k'i}.\left[\begin{array}{l}\cos(\omega d_k f).\cos(\omega d_{k'} f)\\+\sin(\omega d_k f).\sin(\omega d_{k'} f)\end{array}\right]\right]\\
&=\sum_{i=0}^{2^{N_s}-1}\left[\begin{array}{l}\sum_{k'=k=1}^{N_s}g_{ki}.g_{k'i}.\cos(\omega(d_k-d_{k'})f)\\+\sum_{k=1}^{N_s}\sum_{\substack{k'=1\\k'\neq k}}^{N_s}g_{ki}.g_{k'i}.\cos(\omega(d_k-d_{k'})f)\end{array}\right]\\
&=\sum_{i=0}^{2^{N_s}-1}\left[\sum_{k=1}^{N_s}g_{ki}^2+\sum_{k=1}^{N_s}\sum_{\substack{k'=1\\k'\neq k}}^{N_s}g_{ki}.g_{k'i}.\cos(\omega(d_k-d_{k'})f)\right]\\
&=\sum_{i=0}^{2^{N_s}-1}N_s+\sum_{i=0}^{2^{N_s}-1}\sum_{k=1}^{N_s}\sum_{\substack{k'=1\\k'\neq k}}^{N_s}g_{ki}.g_{k'i}.\cos(\omega(d_k-d_{k'})f)\\
&=2^{N_s}.N_s+\sum_{k=1}^{N_s}\sum_{\substack{k'=1\\k'\neq k}}^{N_s}\sum_{i=0}^{2^{N_s}-1}g_{ki}.g_{k'i}.\cos(\omega(d_k-d_{k'})f)\\
&=2^{N_s}.N_s \qquad (22)\end{aligned}$$

since $$\sum_{i=0}^{2^{N}-1}g_{ki}.g_{k'i}=0 \quad (23)$$

This last propriety comes from the fact that we sum across the entire set of 2^{Ns} short TH-polarity sequences. As a reminder, they form the complete set of combinations of polarities of N_s pulses.

Therefore, considering positions k and k' only, we have 2^{Ns-1} couples of pulses with same polarity ($g_{ki}.g_{k'i}=1$) and 2^{Ns-1} couples of pulse with opposite polarity ($g_{ki}.g_{k'i}=-1$), and the sum is zero.

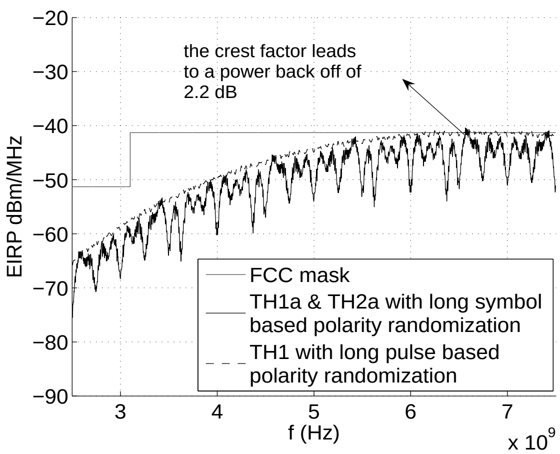


Fig. 5: effect of short & long pulse-based random polarity with PPM

Hence:

$$G_s(f) = \frac{1}{2^{Ns}.T_s}.\sum_{i=0}^{2^{Ns}-1}\left|S_i(f)\right|^2$$
$$= \frac{1}{2^{Ns}.T_s}.\left|X(f)\right|^2.Ns.2^{Ns}$$
$$= \frac{Ns}{T_s}.\left|X(f)\right|^2 \qquad (24)$$

The factor N_s is eliminated normalizing the TH sequence and the PSD of a short TH-IR signal with a M-symbol modulation and a long pulse-based polarity randomization becomes:

$$G_s(f) = \frac{1}{T_s}.\left|X(f)\right|^2 \quad (25)$$

The PSD of the signal is given by the basic pulse shape (Fig. 5).

We also note that this result is independent of the values of $\{d_k\}$. Therefore, there is no need to assume a subdivision of the symbol and Time Hopping sequence into chips: the pulses can take any position within the symbol. This is the case, for example, when a jitter is considered to randomize the position of the pulses.

IV. - Remarks on finite sequences

In this document a complete polarity randomization over infinitely long sequences was assumed when considering symbol-based or pulse-based polarity randomization. We checked by simulation that finite sequences provide similar results when their length increases. Fig. 6 illustrates the smoothing of the spectrum and, therefore, the reduction of the Crest factor when the length of the pulse-based polarity randomization sequence changes from one symbol duration to one thousand. A short TH sequence is used and the

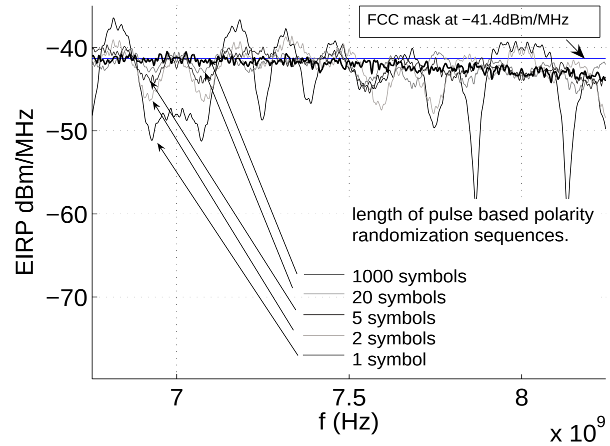


Fig. 6: effect of the length of the pulse based polarity randomization sequence on the PSD of a BPSK signal with short TH sequence.

modulation format for this simulation is BPSK. The simulation results are similar for PPM and OOK with a long symbol-based polarity randomization sequence.

Simulations with a long TH sequence provide qualitatively similar results as shown in Fig. 5 and Fig. 7. Fig. 7 illustrates the case of a long TH sequence used with different pulse-based polarity randomization sequence lengths. The longer the pulse-based polarity randomization sequence, the smoother the spectrum. The PSD of the signal is then exclusively given by the spectral characteristics of the pulse shape $X(f)$.

Combining a finite pulse-based polarity randomization sequence of P symbol periods and a short TH sequence is mathematically equivalent to a signal with a higher order modulation composed by symbols of duration $P.T_s$. If the modulation is not antipodal or the antipodal pairs of symbols are not equiprobable, spectral lines will occur. The use of a long symbol-based polarity randomization sequence solves this problem and the spectral characteristics of these symbols will be averaged.

In the same manner, increasing the length of the finite pulse-based polarity randomization will eliminate the spectral lines and smooth the spectrum.

The FCC regulations require a certain power spectral density within each 1 MHz band. Thus, fine structures of the spectrum within this bandwidth do not have an impact on the compliance with the FCC regulations. Hence, any periodicity that is significantly larger than 10^{-6}s can be ignored.

In Fig. 5, two different TH sequences, TH1a and TH2a, are used randomly and concurrently with a long polarity-based polarity randomization. TH1a and TH2a are two short TH sequences with different short pulse-based polarity randomization sequences on top of them to improve the crest factor of their respective Fourier Transform.

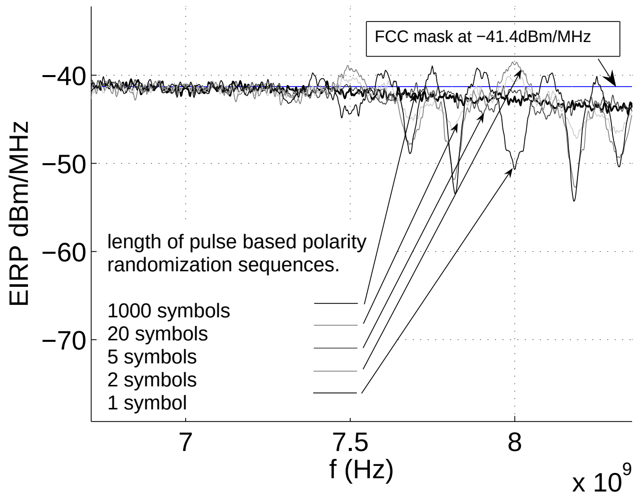


Fig. 7: effect of the length of the pulse based polarity randomization sequence on the PSD of a BPSK signal with long TH sequence.

This simulation shows that increasing the number of basic waveforms can help smooth the spectrum as noted in section II B.3. The reduction of the peak to average ratio should be compared to Fig. 4 where only one symbol waveform is used (TH) without short pulse-based polarity randomization.

Conclusion

This document provided demonstrations and simulation results to explain the influence of the modulation format, multiple access format, and pulse shape on the spectral characteristics of impulse radio. We showed that PPM and time hopping led to similar spectral characteristics, namely spectral lines. The results of this document suggested a remedy to solve this problem:

- Randomization of the polarity of the transmitted symbols or pulses eliminates the spectral lines.
- Polarity randomization on a symbol-by-symbol basis, combined with the use of *several* short TH sequences or a long TH sequence, can lead to the reduction of spectral crest factors.
- The use of long pulse-based polarity randomization codes ensures that the spectrum of the composite signal is equal to the spectrum of the constituent pulse.

The last case (spectrum of the signal identical to the spectrum of the underlying pulse) is desirable for practical applications. Shaping this waveform in order to get a better power efficiency or to reduce interference and get better coexistence with other systems such as 802.11a can then be achieved without any further consideration for the modulation scheme. It is possible to design the PSD of the UWB signal using the results of [7] that considers shaping a waveform to avoid interference and [8] that gives general methods to shape the spectrum.

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