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Interference Suppression in Non-coherent Time-Hopping IR-UWB Ranging

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***Abstract*— Ranging requires detection of the first arriving signal component. The accuracy of time-of-arrival-based range estimation is greatly degraded in the presence of multi-user interference (MUI). We develop a method that applies a non-linear filter on received signal energy to suppress multiuser interference. Then, leading signal path is tracked via an iterative searchback algorithm. The method is tested on time-hopping impulse radio ultrawideband systems. Simulations conducted over IEEE 802.15.4a residential line of sight ultrawideband multipath channels indicate that range detection accuracy is significantly improved by the proposed non-linear filtering approach.**

***Index Terms*— Ultrawideband, ranging, impulse radio, time of arrival, multi-user interference**

I. INTRODUCTION

When estimating a range using a radio signal, the range accuracy depends heavily on how well the time of arrival (ToA) can be determined. Identifying multipath components is crucial to be able to determine the ToA. Ultrawideband (UWB) signals provide high time resolution and help identify many individual multipath components better than narrowband signals. However, what makes UWB challenging is the vast number of paths seen by the receiver and a need to search for the first path, which in non-Line-of-Sight (NLoS) propagation, may not be the strongest. Furthermore, in a multiuser network, signals from multiple devices may interfere with a desired signal and increase range error drastically.

A major drawback of ranging via a noncoherent receiver is its poor performance in the presence of multiuser interference. This is due to the fact that interference suppression techniques such as CDMA are not readily applicable to simple noncoherent receivers. In this paper, our scope is to make ranging via non-coherent radios more interference resilient. Specifically, we focus on simple energy detect receivers. We propose a MUI mitigation technique for non-coherent time-hopping impulse-radio ultrawideband (TH-IR UWB) [1] systems to sustain sub-meter range accuracy even under strong MUI. As known, time-hopping provides a certain level of multi-user suppression as long as different users are assigned with different time-hopping codes with low cross-correlation peaks. Typically, some processing gain is obtained by coherently combining received signal energy according to transmitted time-hopping pattern [2]. However, in the case of ranging, even with TH codes with good cross-correlation properties, an interference might still be deleterious. In other words, TOA estimation is much more sensitive to interference than communications applications. This stems from the fact that we are searching for a weak leading edge and even a small amount of interference energy may be construed as a leading edge.

The remainder of this paper is organized as follows. In Section II, we review the literature on UWB ranging. In Section III, we explain the TH-IR UWB signal model and the proposed non-coherent ranging receiver architecture. In Section IV, we give details of our approach. In Section V, we present the simulation settings and discuss simulations results. Finally, we conclude the paper in Section VI with discussion of the future work.

II. LITERATURE ON TOA BASED UWB RANGING

Acquisition of a received signal can be achieved by locking onto the strongest multipath component, which gives a coarse ToA estimate [3]-[8]. However, precise ToA estimation requires identification of the leading path, which may not be the strongest. In [9], a generalized maximum likelihood (GML) approach is proposed to estimate the leading path, which tests the paths prior to the strongest path. A stopping rule is determined based on the statistics of the amplitude ratio and the delay between the strongest path and the leading edge. However, a very high sampling rate on the order of Nyquist rate is required. In [10], the authors relax the sampling rate requirement and propose a simpler threshold-based leading edge estimation technique. In [11], the leading edge detection problem is taken as a break-point estimation of the actual signal itself, where temporal correlation arising from the transmitted pulse is used to accurately partition the received signal.

Acquisition and ToA estimation can generally be achieved using various transceiver types; e.g. matched filters (or stored-reference receivers), transmitted reference receivers, and energy detectors (ED) [3], [12]. Using energy detectors for synchronization and ToA estimation in UWB systems has been investigated in the past [12], [13], [14]. ED receivers using threshold-based ToA estimation techniques are discussed in [15], [16], [17]. A multi-scale product approach that improves the ranging accuracy is investigated in [18], and likelihood based techniques are proposed in [12]. A two-step hybrid ToA estimation via ED and matched filters are also studied in [19], [20], where an energy-detector based step provides a coarse ToA estimate, and a matched-filter-based step refines the range accuracy. In [21], a match filter based receiver's ability to differentiate between desired user signal and interference in TH-UWB during synchronization phase is analyzed.

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Our literature survey indicates that even though multiuser interference mitigation is investigated extensively for IR-UWB systems for symbol detection [22]-[25], there is no reference that addresses interference mitigation for ToA estimation with non-coherent UWB radios.

III. Signal Model and Receiver Architecture

In this work, we adopt TH-IR UWB signal waveforms, and consider MUI resilient ranging via non-coherent receivers.

A. TH-IR UWB Signal Waveform

In TH-IR signaling, a symbol is divided into virtual time intervals T_f called *frames*. A frame is further split into smaller time slots T_c named *chips*. A single UWB pulse is transmitted in each frame on a chip location specified by a pseudo-random time-hopping code assigned to a user (Fig.1).

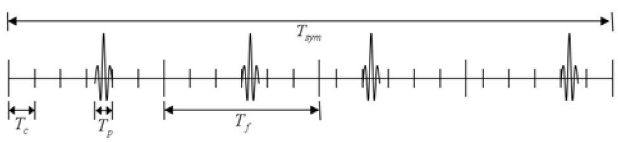

Fig. 1. Illustration of a TH-IR UWB signal waveform, where $N_s = 4$, $\frac{T_f}{T_c} = 6$ and the time-hopping code is ¶4, 4, 3, 5 T_c.

To analytically express a TH-IR waveform, we use the following notations: $E_s^{(k)}$ denotes the symbol energy from the kth user, N_{sym} is the number of transmitted symbols. The ω is the transmitted pulse shape with unit energy, T_{sym} is the symbol duration, T_p is the pulse duration, ϵ_k is the TOA of the kth user's signal, η is the zero-mean AWGN with variance $\sigma_n^2 = \frac{N_0}{2}$, L_k is the total number of multi-path components for the kth user, $\gamma_{l,k}$ and $\tau_{l,k}$ are the amplitudes and delays of the lth multi-path component for the kth user respectively, and N_s is the total number of pulses within a symbol. Then, the received TH-IR signal in a multipath channel from user k is

$$\omega_{mp,k}(t) = \sqrt{\frac{E_s^{(k)}}{N_s}} \sum_{l=1}^{L_k} \gamma_{l,k} \sum_{j=1}^{N_s} d_{j,k} \times \omega\Big(t - (j-1)T_f - c_{j,k}T_c - \tau_{l,k} - \epsilon_k\Big) \ , \quad (1)$$

where $c_{j,k}$ and $d_{j,k}$ are the TH codes and polarity scrambling codes of user k respectively and $b_\lambda \in \{0,1\}$ for the λth symbol. In a multiuser environment with K users, where each user transmits N_{sym} symbols at the same time, the received signal becomes

$$r(t) = \sum_{k=1}^{K} \sum_{\lambda=1}^{N_{sym}} \omega_{mp,k}\big(t - \lambda T_{sym}\big) + \eta(t) \ , \quad (2)$$

B. Non-coherent Receiver Architecture

The ED has the advantages of simplicity, low sampling rate and low cost. In an ED receiver, a received signal is first passed through a low noise amplifier. It is then bandpass filtered and entered into a square-law device. The output of the square-law device is integrated over a time period t_s and then sampled. In what follows, we refer to these energy samples as $z[n]$.

$$z[n] = \int_{(n-1)t_s}^{nt_s} \big|r(t)\big|^2 dt \ , \quad (3)$$

IV. Energy Collection with a Time-hopping Mask

Assume that the receiver knows the time-hopping sequence the desired transmitter uses. Therefore, it can apply a mask onto $z[n]$ to combine energies according to the expected TH code. Assume that M is the number of samples per frame, T_f, N is the number of frames per symbol and $TH(n)$ is a hopping distance between the signals in the n^{th} and $(n-1)^{st}$ frames as an integer multiple of t_s, therefore $TH(1) = 0$.

After energies from relevant TH positions are combined via the mask, the mask is shifted by one sample, and the process is repeated (Fig.2). Let us denote as $\mathbf{v}_d^i$ the vector that contains energy values returned by each cell in the mask at shift index i.

$$\mathbf{v}_d^i = \big[z[i + TH(1)], z[i + TH(2)], ..., z[i + TH(N)]\big] \quad (4)$$

where $1 \leq i \leq M$ and $i \in Z^+$. In the conventional coherent energy combining scheme (CONV) [19], the total energy E_i at mask position i would be given as

$$E_i = \sum_{j=1}^{N} \mathbf{v}_d^i(j) \quad (5)$$

However, in the presence of interference, the E_i might contain interference energy, and thus lead to a ToA detection error. As illustrated in Fig.3, interference that arrives at the receiver earlier than the desired signal (see Fig.2) deteriorates ToA estimation.

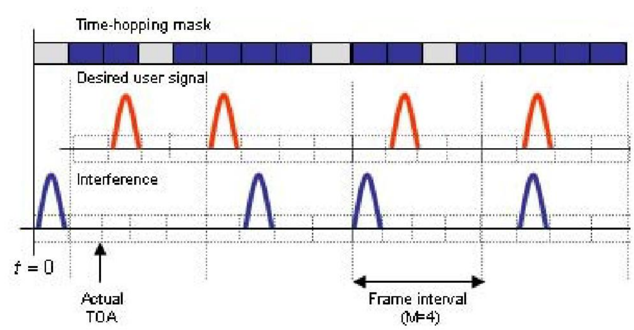

Fig. 2. Illustration of the energy collection operation by using a time-hopping mask. Each block indicates the energy integrated within the time window of length ts. Note that $M = 4$, $N = 4 = N_s$ and $TH(i)$ is the relative time-hopping index for a signal in frame i with respect to the signal in the previous frame. In this illustration, for the desired user $TH(1) = 0$, $TH(2) = 3$, $TH(3) = 5$ and $TH(4) = 3$ and for the interference $TH(1) = 0$, $TH(2) = 6$, $TH(3) = 3$ and $TH(4) = 4$.

In what follows, we introduce a non-linear filtering approach to mitigate interference prior to coherent energy combining.

A. Non-linear Filtering

One well-known non-linear filtering technique is median filtering. Median filters are special cases of stack filters and they have been widely used in digital image and signal processing [26], [27] to remove singularities caused by noise. A non-recursive median filter replaces the center sample of a data set within its window with the median of the set.

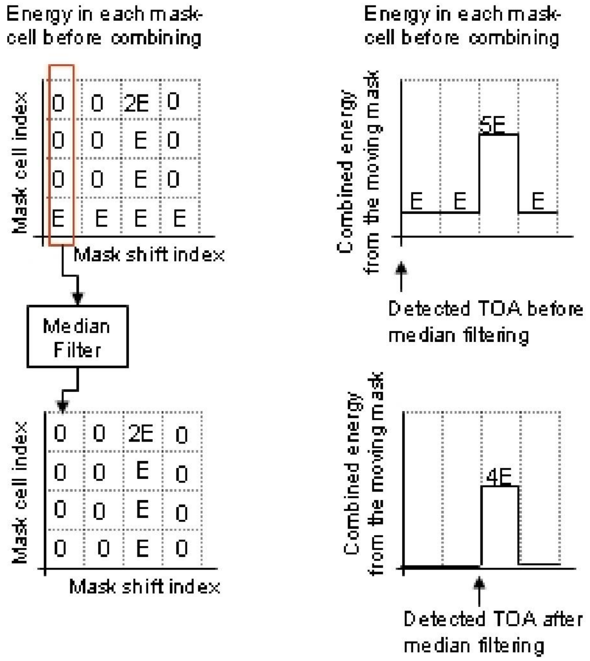

Fig. 3. Illustration of the impact of median filtering in mitigating multiuser interference

Typically, when no interference is present and the channel is stationary during T_{sym}, the elements of $\mathbf{v}_d^i$ either corresponds to only noise energy or noise plus signal energy; and its variance becomes small. On the other hand, interference generates singularities in $\mathbf{v}_d^i$. Median filtering seems to be a natural choice to remove those singularities. When we apply a length-3 median filter onto $\mathbf{v}_d^i$, the elements of the resulting energy vector $\mathbf{y}_d^i$ becomes

$$\mathbf{y}_d^i[j] = \text{median}\left\{ \mathbf{v}_d^i[j-1], \mathbf{v}_d^i[j], ..., \mathbf{v}_d^i[j+1] \right\}, \quad (6)$$

The impact of median filtering is illustrated in Fig.3. It successfully removes even strong interference as long as the interference does not exist in more than half of the number of cells in the mask. Median filtering is followed by coherent energy combining and the ToA detection; and leading edge search is performed on $\mathbf{E}^m$.

$$\mathbf{E}^m = \left[E_1^m, E_2^m, ..., E_M^m\right] \quad (7)$$

$$E_i^m = \sum_{j=1}^{N-2} \mathbf{y}_d^i(j) \quad (8)$$

B. Computational Complexity and Memory Requirement

Median filtering increases the complexity of an energy-detection receiver. Assume that $z[n]$ are provided by a 16-bit ADC. Then, the memory requirement for storing $M \times N$ samples would be 2MN Bytes. Sorting of W samples has inherent computational complexity of $O(WlogW)$. Thus, the complexity of applying MWF would be $M(N - W + 1)O(WlogW)$.

C. Uncertainty of ToA

Each column represents a time interval of t_s. Assume that we consider the ToA of a signal as the mid-point of the time window indicated by the detected column. Then, if the column that corresponds to the true ToA is detected, the ToA error range would be $[0, \frac{t_s}{2}]$. If the detection is off by p columns, the error range would be $[pt_s - \frac{t_s}{2}, pt_s + \frac{t_s}{2}]$, $p \geq 1$. In calculation of the mean absolute errors (MAE), we take the time difference between the actual ToA and the mid-point of the detected time window. Therefore, it is likely for the MAE to be less than t_s.

D. Searchback Algorithm

We incorporate an adaptive search-back algorithm on $\mathbf{E}^m$ as follows. Let n_m is the index of $max(\mathbf{E}^m)$ and w_{sb} the search-back window length. Then, the part of $\mathbf{E}^m$) under search is denoted as $\mathbf{E}^{ms}$ such that $\mathbf{E}^{ms} = [E^m_{n_m - w_{sb}}, E^m_{n_m - w_{sb}+1}, ...E^m_{n_m}]$. In IEEE 802.15.4a channels, multi-path components arrive in clusters; and between the first two clusters an only noise region might exist. Assume that w_{cls} denotes the expected number of noise-only samples between two clusters. In [16], w_{cls} is reported to be between 2 and 3 for CM1 channel model, when $t_s = 4$ns.

The leading edge index with respect to the first sample of $\mathbf{E}^{ms}$ is given by

$$\hat{n} = \text{First}\left\{ n' \middle| \mathbf{E}^{ms}[n'] > \xi \text{ and } \max\left\{ \mathbf{E}^{ms}[n'-1], \mathbf{E}^{ms}[n'-2], ..., \mathbf{E}^{ms}[\max(n'-w_{cls}, 1)] \right\} < \xi \right\}, \quad (9)$$

where the hypothesis $n' \in \{1, 2, ..., w_{sb}\}$ is tested backwards starting from $n' = w_{sb}$ down to $n' = 1$. The threshold ξ that corresponds to a fixed P_{fa} is given by[1] [16]

$$\xi = \sigma_{ed} Q^{-1}\left(1 - (1 - P_{fa})^{\frac{1}{w_{cls}}}\right) + \mu_{ed}\ , \quad (10)$$

where μ_{ed} and σ_{ed} are the mean and the variance of noise-only samples. The optimal threshold is a function of w_{cls}.

When the signal samples are processed with MWF the noise statistics change. It becomes cumbersome to derive P_{fa} for a given threshold. Therefore, we use the threshold that corresponds to the P_{fa} for the interference-free case.

V. Simulation Results

In this section, we present simulation results for the ranging performance of the noncoherent TH-IR system outlined in the previous sections. As performance metrics, we use mean absolute error (MAE) of ToA estimations and their confidence level. The confidence level indicates the percentage the range estimation error is below a given threshold. We adopt the IEEE 802.15.4a TG settings of 90% for the confidence level, and $3ns$ for the threshold. Note that 3ns ToA error corresponds to 90cm range error. The time-hopping codes assigned to the desired user and interference are $th_{desired} = [1, 1, 4, 2] \times T_c$ and $th_{desired} = [1, 4, 2, 1] \times T_c$. Note that three hopping positions are the same for both desired user and the interference. The

[1] We define P_{fa} to be the probability of identifying a noise-only sample as a signal sample.

chip positions are selected to avoid inter-pulse-interference. The TH-IR symbol waveform is repeatedly transmitted over 40us duration. In our simulations,the other settings are $T_{sym} =$ 512ns, $T_f = 128$ns, $T_p = T_c = 4$ns, $w_{cls} = 2$ and $t_s = 4ns$.

Two methods are compared: non-linear filtering *length* 3 *median filtering* (M3F), and CONV, which is the conventional coherent energy combining without interference mitigation [19]. The $E_b^{(2)}/N_0$ denotes the interference energy to noise ratio (INR), and E_b/N_0 for the desired user signal energy to noise ratio. In the simulations, we vary E_b/N_0 from $4dB$ to $22dB$ while $E_b^{(2)}/N_0 = \{none, 0dB, 5dB, 10dB\}$, and look at changes in MAE and confidence level performance of CONV and M3F.

The MAE results are plotted in Fig.4-5. In Fig.4, the top plot shows the MAE performance of M3F without interference. The MAE of M3F is less than 3ns above $E_b/N_0 = 13dB$. On the other hand, CONV also achieves 3ns MAE at 13dB, but as the SNR improves the MAE gets worse. Due to imperfect autocorrelation properties of TH codes and sub-optimality of threshold settings, at high SNRs the the TH code side lobes become more likely to be identified as the leading edge. Also, it is observed that the M3F is worse than CONV at low SNR, because essentially M3F penalizes the signal at very low SNRs.

Fig.4b-5 show the performance of CONV and M3F under various levels of INR. It is shown that CONV is not capable of sub-meter ranging performance under all interference test cases, and the MAE of CONV never falls below 6ns, whereas M3F successfully maintains the MAE of 3ns until INR becomes as strong as 10dB.

In Fig.6-7, the 3ns confidence levels are provided. The introduction of interference that is $E_b^{(2)}/N_0 = 0$dB or higher drastically degrades also the confidence level of CONV, and the CONV can't exceed 70%. On the other hand, M3F maintains 3ns confidence level at 90% at INR levels of $E_b^{(2)}/N_0 =$ $0, 5$dB, and proves to be resilient against interference.

VI. Conclusion

We introduce a method that uses non-linear filtering to mitigate interference in ToA estimation via non-coherent receivers. The effectiveness of this approach is proven by simulations conducted using IEEE 802.15.4a channel models. Knowing that non-linear filtering changes noise and signal characteristics, the impact of non-linear filtering on the receiver detection performance will be studied in another article. Due to space limitations, we don't include it in this article.

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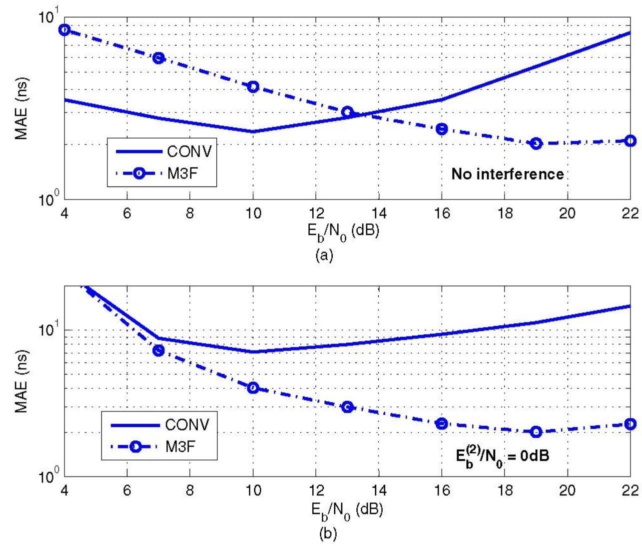

Fig. 4. MAEs for TH-IR: a) no interference, and b) $E_b^{(2)}/N_0 = 0$dB.

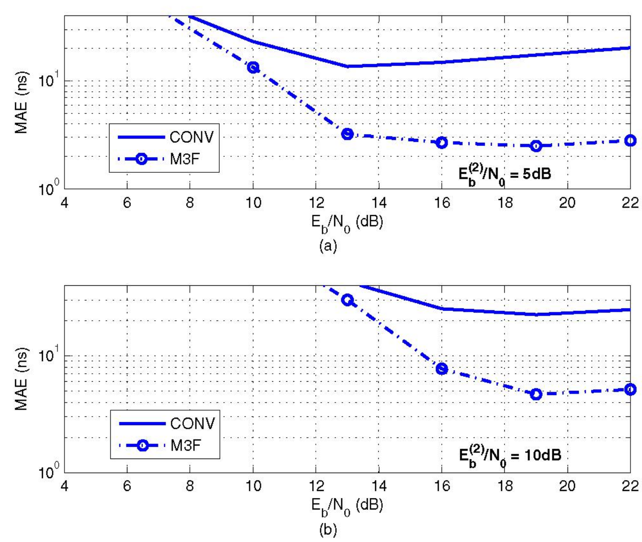

Fig. 5. MAEs for TH-IR: a) $E_b^{(2)}/N_0 = 5$dB, and b) $E_b^{(2)}/N_0 = 10$dB.

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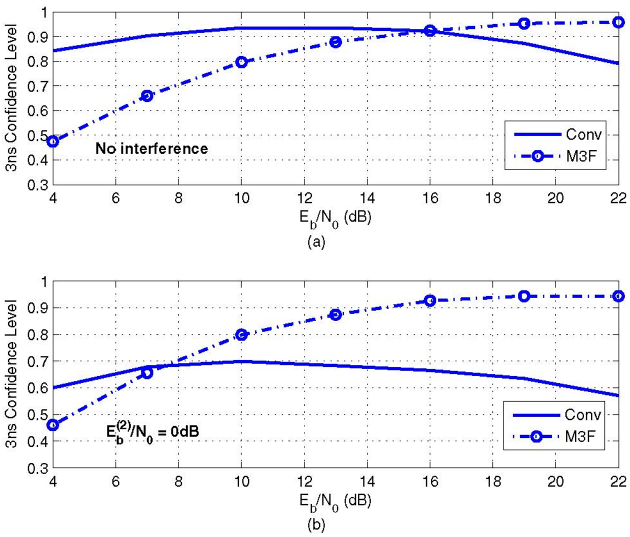

Fig. 6. Confidence levels of 3ns for TH-IR: a) no interference, and b) $E_b^{(2)}/N_0 = 0$dB.

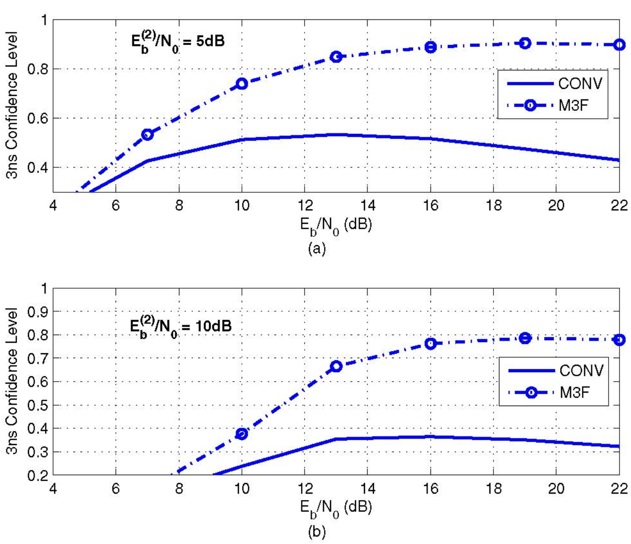

Fig. 7. Confidence levels of 3ns for TH-IR: a) $E_b^{(2)}/N_0 = 5$dB, and b) $E_b^{(2)}/N_0 = 10$dB.

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