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Andreas Molisch, Neelesh Mehta, Jonathan Yedidia, Jin Zhang

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Cooperative communications, where parallel relays forward information to a destination node, can greatly improve the energy efficiency and latency in ad-hoc networks. However, current networks do not fully exploit its potential, as they only use traditional energy-accumulation, which is often used in conjunction with repetition coding or cooperative space-time codes. In this paper, we show that the concept of mutual-information-accumulation can be realized with the help of fountain codes, and leads to a lower energy expenditure and a lower transmission time than energy accumulation. We then provide an analysis of the performance of mutual information accumulation in relay networks with N relay nodes. We first analyze the quasi-synchronuous scenario where the source stops transmitting and the relay nodes start transmitting after L relay nodes have successfully decoded the source data. We show that an optimum L exists and is typically on the order of 3 or 4. We also give closed-form equations for the energy savings that can be achieved by the use of mutual-information-accumulation at the receiver. We then analyze and provide bounds for an alternate scenario where each relay node starts its transmission to the destination as soon as it has decoded the source data, independent of the state of the other relay nodes. This approach further reduces the transmission time, because the transmission by the relay nodes helps the other relay nodes that are still receiving.

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Performance of Fountain Codes in Collaborative Relay Networks

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***Abstract*— Cooperative communications, where parallel relays forward information to a destination node, can greatly improve the energy efficiency and latency in ad-hoc networks. However, current networks do not fully exploit its potential as they only use traditional energy-accumulation, which is often used in conjunction with repetition coding or cooperative space-time codes. In this paper, we show that the concept of mutual-information-accumulation can be realized with the help of fountain codes, and leads to a lower energy expenditure and a lower transmission time than energy accumulation. We then provide an analysis of the performance of mutual information accumulation in relay networks with N relay nodes. We first analyze the quasi-synchronous scenario where the source stops transmitting and the relay nodes start transmitting after L relay nodes have successfully decoded the source data. We show that an optimum L exists, and is typically on the order of 3 or 4. We also give closed-form equations for the energy savings that can be achieved by the use of mutual-information-accumulation at the receiver. We then analyze and provide bounds for an alternate scenario where each relay node starts its transmission to the destination as soon as it has decoded the source data, independent of the state of the other relay nodes. This approach further reduces the transmission time, because the transmission by the relay nodes helps the other relay nodes that are still receiving.**

***Index Terms*— Cooperative communications, energy accumulation, fountain code, radio networks, relay, transmit energy minimization.**

I. Introduction

COOPERATIVE communications, where different nodes in a network work together in order to transmit information from a source to a destination, decreases energy expenditure and improves the reliability of data transmission in a wireless network. For this reason, it has drawn great attention in recent years, see [1]–[5] and references therein. Papers in the area can be broadly classified into two categories: (i) study of large-scale networks, including routing algorithms and limiting behavior, and (ii) study of fundamental building blocks that involve only a small number of nodes.

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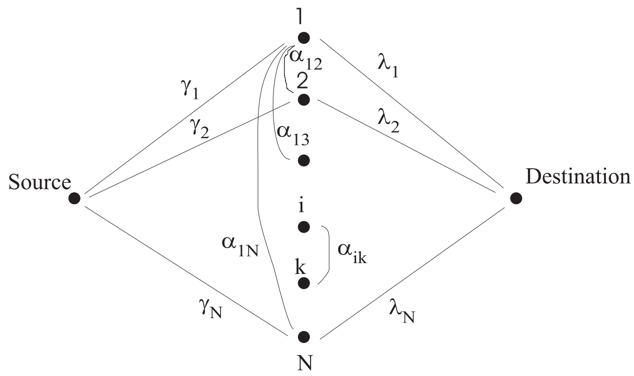

Fig. 1. System setup with source, destination, and N parallel relays. γ_i denotes the channel gain from source to the $i-$th relay, λ_i the channel gain from $i-$th relay to the destination, and α_{ik} the channel gain between i-th and k-th relay.

One of the building blocks that has been analyzed extensively is the transmission from the source to the destination via several parallel decode-and-forward relays (see Fig. 1). The source broadcasts its information, transmitting it to several or all of the available relay nodes (henceforth called "S-R" phase); the relay nodes then cooperate in transmitting the information to the destination ("R-D" phase). If the relay nodes have channel state information about the R-D channels, they can perform "virtual beamforming", i.e., adjusting the amplitude and phases of the transmit signal to minimize the transmit power [6], [7]. However, such a scheme requires frequent feedback and may be sensitive to phase noise and variations of the channel impulse response. In the absence of channel state information at the relay nodes, the receiver can at best collect the energy from the various relay nodes, e.g., through space-time coding [8]–[11].

Energy accumulation can be done not only when multiple relays transmit simultaneously, but also over multiple transmissions of the same information packet by the same (or even different) relays in the network. In energy accumulation, a receiver can recover the original packet so long as the total received energy (from multiple sources or transmissions) exceeds a certain threshold. Energy accumulation was shown to lead to significant energy savings – over conventional relaying – for multicast and broadcast in [12]–[14]. However, energy accumulation using repetition coding, in which each relay node merely retransmits exactly the same packet that it reliably decoded, is capacity achieving only in an asymptotically wideband regime [12].

In this paper, we propose a new approach for the relaying of information using fountain codes, which enables the accumulation of bits directly instead of energy. Fountain codes were introduced by Luby and coworkers in recent years [15]–[17] (see also [18], [19] for an overview). Unlike conventional codes, they encode and transmit the source information in an *infinitely* long codestream. The codes have the special property that a receiver can recover the original information from unordered subsets of the codestream, once *the total obtained mutual information from multiple sources marginally exceeds the entropy of the source information*. Thus, it is certain that the destination can decode the transmitted signal; only the required transmit energy and the transmission time depend on the channel states. Fountain codes were originally designed for erasure channels, but their performance on general discrete memoryless channels, AWGN channels, and Rayleigh-fading channels has since been studied and shown to be good [20]–[23]. They have been suggested for use in wired ethernet-like applications as well as for point-to-point communications, [23], single-relay links [24], [25] and broadcast and multicast applications [26] in wireless networks. However, to the best of our knowledge, their use in cooperative multi-relay wireless networks has not been analyzed yet.

Intuitively, the difference between energy accumulation and mutual information accumulation can be most easily understood from the following simple example. Consider binary signalling using two relays, each with an erasure channel to the destination with an erasure probability p_e. If the two relays use repetition coding (i.e., the receiver accumulates energy), then each bit will be erased with probability p_e^2, so $1-p_e^2$ bits on average are received per transmission of the two relays. On the other hand, if the two relays use different fountain codes, the transmissions are independent and on average $2(1-p_e)$ bits (which exceeds $1-p_e^2$ bits) per transmission of the two relays are received. Information accumulation, and, in particular, the use of fountain codes, can also be seen as a method of approximately achieving the information-theoretic capacity of channels with multiple relays.

In this paper, we investigate how fountain codes can help the relaying of information through several parallel relay nodes. We propose a quasi-synchronous protocol and an asynchronous protocol. In the quasi-synchronous protocol, each relay node after reliably decoding the source information, waits for the source to stop transmitting. After L relay nodes have decoded the information, the source stops its transmission, and all the L relay nodes then simultaneously transmit the information to the destination. If the relays use different fountain codes, the receiver accumulates the mutual information from the different relay nodes, while if all relays use the same fountain code, the destination accumulates the received energy. In the asynchronous protocol, each relay node starts to transmit to the destination as soon as it has decoded the source data. Due to the broadcast nature of the wireless channel and the properties of fountain codes, this speeds up transmission as it provides useful information not only to the destination, but also to the relay nodes that have not finished the decoding process yet.[1] For both protocols, we derive closed-form expressions for their average energy expenditure, and optimize their performance. We also provide an analysis of the quasi-synchronuous protocol with energy-accumulation receivers.

We note that mutual-information accumulation, and, in particular, the use of fountain codes, can be seen as a practical scheme that tries to come closer to the information-theoretic capacity of multiple-relay channels. The capacity of a single-relay channel was first derived in the classical work in [28]. The capacity in Rayleigh-fading channels, taking into account restrictions on the duplexing and synchronization of practical nodes, was derived in [29]. Reference [30] derived the capacity when several relay nodes are used; their underlying system model is similar to the model we describe in Section II. Reference [31] analyzed large Gaussian networks with uniformly distributed nodes.

The rest of the paper is organized as follows: in Section II, we introduce the concept of mutual information accumulation and describe the basic system model and the assumptions underlying our analysis. Section III describes and analyzes the quasi-synchronuous protocol with energy or mutual information accumulation, and Section IV does the same for the asynchronous protocol. A summary and conclusions in Section V wrap up this paper.

II. System Model

The basic system model is shown in Fig. 1. A source needs to transmit an information codeword with bandwidth-normalized entropy H_{target}, given in nats/Hz, to the destination via N parallel decode-and-forward relays. To simplify notation, we assume that the destination cannot obtain information directly from the source, though inclusion of such a direct path in the performance analysis is straightforward. The source as well as the relays use fountain codes for encoding the information (Section V will discuss the advantages of fountain codes versus conventional capacity-achieving codes for that step). Sections III and IV discuss the details of the transmission protocols such as what node transmits what information and when. All nodes operate in half-duplex mode, i.e., they can either transmit or receive, but not do both simultaneously.

In the following, we also assume that transmission is done with a direct-sequence spectrum spreading technique. Such an approach is useful for sensor networks as it allows different information streams to be transmitted in a flexible and decentralized way, and be distinguishable at the receiver. The transmit power of all nodes is P_{T}. The propagation channels between the different nodes are modeled as frequency-flat, block-fading channels. The channel gains are independent and exponentially distributed, which corresponds to Rayleigh fading of their amplitudes. The channel gain, γ_i, between the source and the i-th relay node has the probability density function (pdf)

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\overline{\gamma}_i} \exp\left[-\gamma_i/\overline{\gamma}_i\right], \quad \gamma_i \geq 0 \tag{1}$$

[1]Note that this is somewhat reminiscent of the idea of "cognition" introduced by Tarokh and coworkers (see [27] and references therein). However, in cognition, all nodes have source information they need to transmit, while we are considering a pure relaying scenario.

where $\overline{\gamma}_i$ is the mean channel gain of the i-th channel. Without loss of generality, we assume that the noise and transmit powers are normalized to unity, so that channel gains and SNR become synonymous. Similarly, the channel gain from the i-th relay node to the destination is denoted as λ_i, and has a mean $\overline{\lambda}_i$. The channel gain from the i-th to the k-th relay node is written as α_{ik}, and has a mean $\overline{\alpha}_{ik}$. We assume perfect channel state information (CSI) at the all receivers, but no CSI at the transmitters. This implies, inter alia, that a relay node knows the CSI from the source to itself, but not from itself to the destination.

We consider two receiver types at the destination:

1) Ideal energy accumulation receivers, which can accumulate the energy from the transmissions of different relay nodes. In a CDMA system, a Rake receiver well approximates such a receiver if the information from the different relay nodes arrives with relative delays that are larger than the chip duration. Alternatively, we could also use space-time codes for transmission or assume that orthogonal resources are available for each channel. A packet is reliably decodable at the receiver once

$$T \ln \left(1 + \sum_i \lambda_i \right) \geq H_{\text{target}} \tag{2}$$

where T is the time duration of transmission.

2) Ideal fountain codes and decoders at receivers, so that information streams from different relay nodes can be distinguished, and the mutual information of signals transmitted by relay nodes can be accumulated. Note that for our CDMA system the different nodes must use different spreading codes in order for the destination to resolve the different streams.[2]
In this case, an information packet is reliably decodable at the receiver once

$$T \sum_i \ln(1 + \lambda_i) \geq H_{\text{target}} \tag{3}$$

In order to simplify the analysis, we shall assume that the fountain codes are perfect at all desired rates, i.e., the receiver is capable of correctly identifying the transmitted codeword as soon as the transmission time multiplied with the instantaneous channel capacity is equal to the entropy of the codeword. Note that this assumption is a simplification in two respects:

1) It is impossible to generate "universal" fountain codes that are simultaneously perfect at all possible rates [21]. However, in practice, fountain codes can be found whose overhead compared to perfect codes is bounded and not too large [21]. Including an "overhead factor" $1+\varepsilon$, $\varepsilon \geq 0$, in our computations below can be achieved trivially by replacing H_{target} with $(1+\varepsilon)H_{\text{target}}$. For the decoding at the receiver, belief propagation algorithms can be used [23].

2) A fountain code provides a rate corresponding to the mutual information under a given input distribution, which is not necessarily equal to the channel capacity [32]. Achieving the capacity requires the knowledge of the channel statistics so that the correct input distribution can be chosen. However, under our assumptions of a quasi-static flat-fading channel and a fixed transmit power, this is not a problem since the optimum input distribution is Gaussian for any channel state.

We also assume that the feedback, by which the relay indicates to the source that it has reliably decoded the codeword, is instantaneous. Furthermore, we assume that its impact on the energy budget and spectral efficiency is negligible, which is reasonable if the codewords are long. Finally, we assume that the coherence time of the fading channel, i.e., the time over which the channel realization can be considered constant, is long enough to accomodate the transmission of enough bits of the fountain code so that the receiver decodes the channel. This is a reasonable approximation in many indoor wireless high-data-rate systems, where up to about 10^8 bits can be transmitted within one coherence time of the channel. However, we note that the assumption cannot be $strictly$ fulfilled: due to the Rayleigh-fading nature of the channel, there exist channel realizations that require an (almost) infinite number of bits to be transmitted before the receiver can decide on the correct codeword [20], [23], [33].[3]

III. Quasi-Synchronuous Transmission

A. The Protocol

In the first step, the source encodes the data packet with a fountain code and transmits it. The various relay nodes listen to the source; as soon as they have acquired sufficient energy (entropy) to reliably decode the data, they transmit an acknowledgement to the source that their reception was successful. Once the source has received L acknowledgements, it ceases transmission. At the same time, the relay nodes switch from reception to transmission. For this second phase, we consider two cases:

1) All relay nodes transmit the source data encoded with the same fountain code (which can be the same as that used by the source). Due to the delay difference inherent in the transmission from randomly located relay nodes, the signals arrive at the destination with slightly different delays. We assume in the following that those delays are larger than the chip duration, but much smaller than the symbol duration. This assumption can be well fulfilled in direct-sequence CDMA systems with large spreading factors. At the destination, a Rake receiver is used to accumulate the energy from all the different nodes.
2) Each relay nodes uses a different fountain code, and a different spreading code for the transmission. In that case, the destination distinguishes the signals from the different relay nodes through their different spreading codes, and accumulates the mutual information.

[2]The problem of transmitting different codes from different nodes, where the nodes can help each other, bears a certain similarity to the problem of coded cooperation, as explored, e.g., in [5]. However, there are some key differences, most notably that (i) the underlying source information is the same for all nodes, and (ii) the nodes can start transmission at different times.

[3]Still, we note that with our proposal of using parallel relay links for the data transmission, the probability of such bad channels becomes extremely small.

In both cases, the destination sends a signal to the relay nodes to stop transmission as soon as it has successfuly decoded the source data.

Note that the complexity of the receivers required for the two protocols does not differ significantly. If the sampling rate of the analog-to-digital converter is identical to the symbol rate, then both receivers require L correlators. In the first case, all correlators correlate with the same spreading sequence, and use maximum-ratio-combining to combine the outputs of the integrate and dump filters that follow the correlators [34].[4] In the second case, each correlator is used for the detection of the signal from a different relay node. The main complexity difference lies in the actual decoder, which is more complex if multiple fountain codes are used. The spectral efficiency of this second method is worse, because it uses up multiple spreading codes for the transmission of one source codeword, though the improved coding gain partly offsets this effect.

B. Theory–Rayleigh Fading

In the following, we compute the energy required for the S-R and the R-D transmissions. We can compute those two steps separately, since we assume that the fading of the S-R and R-D channels is independent. In the current section, we assume that all nodes experience Rayleigh fading; shadowing will be treated in Section III-C.

1) Energy Cost of Transmission Until L Nodes Have Reliably Decoded Data: We derive the pdf of the time it takes for L nodes to receive and decode the source data. For this, we first compute the pdf of the time, y_i, required for a relay node, i, to receive and reliably decode the source data, and then derive its order statistics over multiple relays.

From Shannon's capacity equation, we have

$$y_i = \frac{H_{\text{target}}}{\ln\left[1+\gamma_i\right]}, \quad \text{for } \gamma_i \geq 0 \tag{4}$$

where the channel gains γ_i, $1 \leq i \leq N$, have the distribution given in Eq. (1). Using a standard transformation of variables with the Jacobian [35], the pdf of y_i is

$$f_{y_i}(y) = \frac{H_{\text{target}}}{\overline{\gamma} y^2} \exp\left[\frac{1}{\overline{\gamma}} + \frac{H_{\text{target}}}{y} - \frac{e^{H_{\text{target}}/y}}{\overline{\gamma}}\right], \quad \text{for } y \geq 0 \tag{5}$$

and the cumulative distribution function (cdf) is

$$F_{y_i}(y) = \exp\left[\frac{1}{\overline{\gamma}} - \frac{e^{H_{\text{target}}/y}}{\overline{\gamma}}\right], \quad \text{for } y \geq 0 \tag{6}$$

To determine the pdf of the time required for at least L nodes to decode the source data (after which the L relays start transmitting), we order $y_1, \ldots, y_L, \ldots, y_N$ so that $y_{(1)} < y_{(2)} < \ldots. < y_{(L)} < \ldots < y_{(N)}$, where (i) denotes the index of the relay that takes i-th smallest time to decode source data. We thus need to determine the pdf of $y_{(L)}$.

[4] Alternatively, a receiver can use only a single correlator, whose output is sampled L times during each symbol duration. This saves some hardware complexity. However, the signals can arrive from the different relay nodes at irregular intervals, and thus require the ADC to sample at the chip rate. This fast sampling of the ADC increases the energy consumption significantly (by a factor much greater than L), and thus might not be desirable for sensor network applications.

For the general case, in which the S-R channel means are non-identical, the analysis is rather involved. This is because every ordering of the random variables is not equally likely. The expressions for the (L)-th node, even after considerable simplifications, involve a summation of $N!$ terms, one for each ordering [36].

However, the expressions simplify considerably when all the mean S-R channel gains are identical [37]

$$f_{y_{(L)}}(y) = \frac{N!}{(L-1)!(N-L)!} f_y(y) F_y(y)^{L-1} \left[1 - F_y(y)\right]^{N-L} \tag{7}$$

Inserting Eqs. (5) and (6), and using the binomial expansion of the term $\left[1 - F_y(y)\right]^{N-L}$, we obtain, for $y_{(L)} \geq 0$,

$$f_{y_{(L)}}(y) = \frac{H_{\text{target}}}{\overline{\gamma}} \frac{N!}{(L-1)!(N-L)!} \sum_{k=0}^{N-L} \binom{N-L}{k} (-1)^k \times \frac{e^{H_{\text{target}}/y}}{y^2} \exp\left[\frac{L+k}{\overline{\gamma}}\left(1 - e^{H_{\text{target}}/y}\right)\right]. \tag{8}$$

The mean energy expenditure is also the expression for the transmission time (recall that the transmit power is normalized to unity), namely $\Phi_1 = E\{y_{(L)}\}$, since only one node transmits (and thus expends energy), and the transmission stops after L nodes have reliably decoded the source data. From Eq. (8), the mean energy expenditure, Φ_1, can be written for $L < N$ as [38]

$$\Phi_1 = \frac{H_{\text{target}}}{\overline{\gamma}} \frac{N!}{(L-1)!(N-L)!} \lim_{\zeta \to 0} \sum_{k=0}^{N-L} \binom{N-L}{k} (-1)^k \times \exp\left[\frac{L+k}{\overline{\gamma}}\right] R_0\left(\frac{L+k}{\overline{\gamma}}, \zeta\right) \tag{9}$$

where $R_0(x, \zeta)$ is defined as a member of the family of functions

$$R_m(x, \zeta) = \int_{1+\zeta}^{\infty} t^m \exp(-xt)/\ln(t) dt \tag{10}$$

with $m = 0$.

2) Energy Cost of Transmission From Relay Nodes to Destination–Using Single Fountain Code: When the relay nodes use the same fountain code, the receiver accumulates the energy of the relay transmissions. We assume that the relay nodes transmit with equal power, and their signals go through channels with channel gain λ_i. Since the receiver performs energy accumulation, this is equivalent to assuming that the signal is received from one source through an equivalent channel with gain λ. Assuming different mean channel gains for the R-D channels, the pdf of the equivalent channel gain is [39]

$$f_\lambda(\lambda) = \frac{1}{\prod_{i=1}^{L} \overline{\lambda}_i} \sum_i \frac{\exp\left[-\lambda/\overline{\lambda}_i\right]}{\prod_{k \neq i} \left[\frac{1}{\overline{\lambda}_k} - \frac{1}{\overline{\lambda}_i}\right]}, \quad \text{for } \lambda \geq 0 \tag{11}$$

while in the case of equal mean channel gains, it is [34]

$$f_\lambda(\lambda) = \frac{1}{(L-1)!} \frac{\lambda^{L-1}}{\overline{\lambda}^L} \exp\left[-\frac{\lambda}{\overline{\lambda}}\right], \quad \text{for } \lambda \geq 0 \tag{12}$$

The time required for the R-D transmission phase is denoted as z. Performing a variable transformation analogous to Eq. (4), the pdf of z is

$$f_z(z) = \frac{H_{\text{target}}}{z^2 \prod_{i=1}^{L} \overline{\lambda}_i} \sum_{i=1}^{L} \frac{1}{\prod_{k \neq j} \left[\frac{1}{\overline{\lambda}_k} - \frac{1}{\overline{\lambda}_i}\right]} \times \exp\left[\frac{H_{\text{target}}}{z} + \frac{1}{\overline{\lambda}}\left(1 - e^{H_{\text{target}}/z}\right)\right], \qquad \text{for } z \geq 0 \tag{13}$$

for the case of unequal mean R-D channel gains, and

$$f_z(z) = \frac{H_{\text{target}}}{(L-1)!\overline{\lambda}^L z^2} \left(e^{H_{\text{target}}/z} - 1\right)^{L-1} \times \exp\left[\frac{H_{\text{target}}}{z} + \frac{1}{\overline{\lambda}}\left(1 - e^{H_{\text{target}}/z}\right)\right], \qquad \text{for } z \geq 0 \tag{14}$$

for the case of equal mean R-D channel gains.

The mean energy expenditure, Φ_2, is L times the mean transmission time, $E\{z\}$, since L nodes (with unit transmit power) are active during the R-D phase. For the case of unequal mean channel gains (and $L > 1$), the mean energy expenditure is

$$\Phi_2 = \frac{L H_{\text{target}} \exp(1/\overline{\lambda})}{\prod_{i=1}^{L} \overline{\lambda}_i} \lim_{\zeta \to 0} \sum_{i=1}^{L} \frac{1}{\prod_{k \neq j} \left[\frac{1}{\overline{\lambda}_k} - \frac{1}{\overline{\lambda}_i}\right]} R_0(1/\overline{\lambda}, \zeta) \ . \tag{15}$$

For the case of equal mean channel gains, the mean energy expenditure (for $L > 1$) is

$$\Phi_2 = \frac{L H_{\text{target}} \exp(1/\overline{\lambda})}{(L-1)!\overline{\lambda}^L} \times \lim_{\zeta \to 0} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^{L-k-1} R_k(1/\overline{\lambda}, \zeta) \ . \tag{16}$$

3) Energy Cost of Transmission From Relay Nodes to Destination–Using Multiple Fountain Codes: When the relay nodes use different fountain codes, the receiver accumulates the mutual information, not the energy, of the relay transmissions. Thus, the total transmission rate is given by the sum of the rates from the relays. The rate is related to the SNR as $r = \ln(1 + \lambda)$. Therefore, $\lambda = e^r - 1$, and the Jacobian is $d\lambda/dr = e^r$. From this, it follows that the pdf of the rate from a single node is

$$f_r(r) = \frac{1}{\overline{\lambda}} \exp\left[\frac{1}{\overline{\lambda}} + r - \frac{e^r}{\overline{\lambda}}\right] \quad , \quad \text{for } r \geq 0 \tag{17}$$

The sum of the rates is most easily computed via its Characteristic Function (CF), which is defined as the Fourier transform of the pdf:

$$M(j\omega) = \int_{-\infty}^{\infty} f_r(r) e^{-j\omega r} dr \tag{18}$$

Substituting $v = e^r$, the CF expression becomes

$$M(j\omega) = \frac{1}{\overline{\lambda}} \exp\left[\frac{1}{\overline{\lambda}}\right] \int_1^{\infty} v^{1-j\omega} e^{-v/\overline{\lambda}} \frac{1}{v} dv \tag{19}$$

From [38, p. 342], $\int_u^{\infty} x^{\nu-1} e^{-\mu x} dx = \mu^{-\nu} \Gamma(\nu, \mu u)$, where the incomplete Gamma function $\Gamma(\alpha, x)$ is defined as $\Gamma(\alpha, x) = \int_x^{\infty} e^{-t} t^{\alpha-1} dt$ [40].

The CF of the rate from a single node can now be written as

$$M(j\omega) = \left[\frac{1}{\overline{\lambda}}\right]^{j\omega} \exp\left[\frac{1}{\overline{\lambda}}\right] \Gamma(1 - j\omega, 1/\overline{\lambda}) \tag{20}$$

Therefore, the CF of the sum rate to the destination node is

$$M(j\omega) = \prod_{i=1}^{L} \left[\exp(1/\overline{\lambda}_i) \left(1/\overline{\lambda}_i\right)^{-j\omega} \Gamma(1 - j\omega, 1/\overline{\lambda}_i)\right] \tag{21}$$

From this, we obtain the pdf of the mutual information. And via a variable transformation, the pdf of the required transmission time, z, simplifies to a single integral

$$f_z(z) = \frac{H_{\text{target}}}{2\pi} \int_{-\infty}^{\infty} M(j\omega) \exp\left[\frac{j\omega H_{\text{target}}}{z}\right] \frac{1}{z^2} d\omega. \tag{22}$$

The total mean expended energy can be computed numerically from this pdf.

C. Theory–Shadowing

In this section, we compute the energy expenditure in the presence of shadowing. To simplify notation, we assume that the shadowing mean and variance are identical for all nodes. Thus, the pdf of the SNR between any two nodes is given as

$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{[\ln(x) - \mu]^2}{2\sigma^2}\right] \tag{23}$$

We also note that the lognormal distribution, Eq. (23), can be used to approximate a Suzuki distribution, which describes the combination of Rayleigh fading and shadowing [41]. Furthermore, even the *sum* of Suzuki random variables can be well approximated by a lognormal. The parameters of this lognormal can be found using the flexible and highly accurate method of [42].

1) Energy Cost of Transmission Until L Nodes Have Reliably Decoded Data: Again, we start out by deriving the time required for an S-R transmission to the i-th node. Applying the variable transformation $r = ln(1 + x)$ to the lognormal pdf in Eq. (23), the pdf of the transmission rate, r, is

$$f_r(r) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^r}{e^r - 1} \exp\left[-\frac{[\ln(e^r - 1) - \mu]^2}{2\sigma^2}\right], \quad \text{for } r \geq 0 \tag{24}$$

For later reference, we note that, for $r \geq 1$, this can be approximated as

$$\tilde{f}_r(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{[r - \mu]^2}{2\sigma^2}\right] \quad , \quad -\infty < r < \infty \tag{25}$$

However, this approximation is not accurate for $r < 1$, and for $r < 0$, it corresponds to nonphysical (negative) transmission times. It should, thus, only be used if μ is sufficiently large.

The pdf of the transmission time, y, is given in closed-form as

$$f_y(y) = \frac{1}{\sqrt{2\pi}\sigma} \frac{H_{\text{target}}}{y^2} \frac{1}{1 - e^{-H_{\text{target}}/y}} \times \exp\left[-\frac{\left[\ln(e^{H_{\text{target}}/y} - 1) - \mu\right]^2}{2\sigma^2}\right] \tag{26}$$

and the cdf is then

$$F_y(y) = Q\left[\frac{\ln(e^{H_{\text{target}}/y} - 1) - \mu}{\sigma}\right] \tag{27}$$

where $Q(x)$ is the Q-function as defined in [40]. Inserting Eqs. (26) and (27) into Eq. (7), we get the pdf of the time it takes to reach at least L nodes in the S-R phase as

$$\begin{aligned} f_{y_{(L)}}(y) = & \frac{H_{\text{target}}}{\sqrt{2\pi}\sigma}\frac{N!}{(L-1)!(N-L)!}\frac{1}{y^2} \\ & \times \frac{1}{1-e^{-H_{\text{target}}/y}} \exp\left[-\frac{\left[\ln(e^{H_{\text{target}}/y}-1)-\mu\right]^2}{2\sigma^2}\right] \\ & \times \sum_{k=0}^{N-L}\binom{N-L}{k}(-1)^k Q^{L-1+k}\left[\frac{\ln(e^{H_{\text{target}}/y}-1)-\mu}{\sigma}\right] \end{aligned} \tag{28}$$

2) Energy Cost of Transmission From Relay Nodes to Destination - Using Single Fountain Code: In the case that a single fountain code is used for the R-D stage, the effective channel consists of the sum of L lognormally distributed random variables. As mentioned at the beginning of Section III-C, such a sum can be accurately modeled by a single lognormally distributed random variable. The parameters μ and σ of this random variable can be obtained from the following set of nonlinear equations [42]:

$$\begin{aligned} & \sum_{i=1}^{N_G}\frac{w_i}{\sqrt{\pi}}\exp\left[-s_m\exp\left(\frac{\sqrt{2}\sigma a_i+\mu}{\xi}\right)\right] \\ & \qquad = \prod_{i=1}^{K}\widehat{\Psi}_X(s_m;\mu_i,\sigma_i), \quad m=1,2 \end{aligned} \tag{29}$$

where

$$\widehat{\Psi}_X(s;\mu,\sigma) = \sum_{i=1}^{K}\frac{w_i}{\sqrt{\pi}}\exp\left[-s\exp\left(\frac{\sqrt{2}\sigma a_i+\mu}{\xi}\right)\right] \tag{30}$$

and the weights, w_i, and abscissas, a_i, of the Gaussian quadrature for different orders, N_G, are tabulated in [40]. The parameters s_1 and s_2 are the arguments of the moment-generating function at which the exact distribution of the sum of lognormal variables should match the distribution of the equivalent lognormal variable. As we are mainly interested in high SNRs, small values of s_1 and s_2 should be used, e.g., $s_1 = 1.0$ and $s_2 = 0.2$ [42]. Once the parameters of the equivalent channel have been determined, the pdf of the transmission time is obtained from Eq. (26).

3) Cost of Transmission From Relay Nodes to Destination – Using Multiple Fountain Codes: If multiple fountain codes are used, then the rates of the different users add up. The rate for a single node is given by Eq. (24). The CF of the composite rate is obtained as the L-th power of the CF of a single node's rate. After some manipulations, it can be written as

$$M(j\omega) = \left[\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}[e^z+1]^{-j\omega}\exp\left[-\frac{[z-\mu]^2}{2\sigma^2}\right]dz\right]^L \tag{31}$$

This integral cannot be solved in closed form. However, using the *approximate* pdf for the single-user rate in Eq. (25), leads to the following approximation to the CF of the sum rate

$$M(j\omega) = \exp(-j\omega L\mu)\exp\left[-\omega^2\sigma^2 L/2\right]. \tag{32}$$

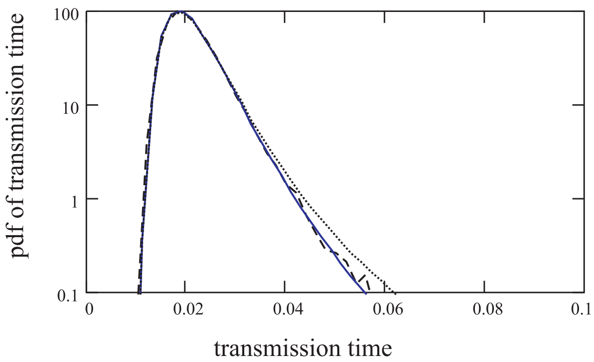

Fig. 2. Pdf of the R-D transmission time with multiple fountain codes in the presence of shadowing. ($N = 10$, $L = 5$, $\mu = 10$, $\sigma = 5$). Solid line: exact computation; dashed line: Monte Carlo simulations; dotted line: results using the Gaussian approximation of the characteristic function.

Inserting Eq. (31) or Eq. (32) into Eq. (22), we obtain the pdf of the R-D transmission time. Fig. 2 shows the accuracy of the approximation. It displays the pdf of the energy expenditure in the R-D link, as computed exactly, and with the approximation Eq. (32). We see that the approximation leads to a discrepancy only in the far tails of the pdf, so that for most practical applications, the approximation is sufficiently good.

D. Results

The results derived in the previous sections allow us to evaluate the performance of relay networks for different values of available relay nodes, N, and used relay nodes, L. We can optimize L by evaluating the performance (either transmission time or energy consumption), and choosing the value of L that gives the best performance for either of those performance criteria. Fig. 3 shows the mean energy expenditure as a function of L for different values of N in Rayleigh fading. Both the single fountain code (energy accumulation) case and multiple fountain code (mutual information accumulation) case are simulated. In both cases, we find that there is a pronounced minimum, which depends on N, but usually lies between $L = 2$ and $L = 5$. Further analysis (not shown here for space reasons) shows that the energy expenditure for the S-R phase sharply increases as L increases in both cases. This is intuitive because a larger L means that the information has to be transmitted to nodes with progressively worse S-R channels. While the energy expenditure for the R-D transmission drops sharply for mutual information accumulation as L increases from 1 to 5 and saturates thereafter, it shows a clear minimum for energy accumulation. [5]

It is also interesting to investigate the pdf of the total energy expenditure. Fig. 4 shows the pdf for $N = 10$, and $L = 2$ as well as $L = 5$ in Rayleigh fading. Here we find that – as expected – the concentration of the pdf around its mean value increases with increasing L for both energy and mutual information accumulation. For the same L, mutual information accumulation requires less total energy than energy accumulation. We see that mutual information accumulation achieves a

[5]Note that a similar figure, Fig. 2 in [43], plotted the mean energy after ignoring all transmissions that required $y > 1000$ or $z > 1000$.

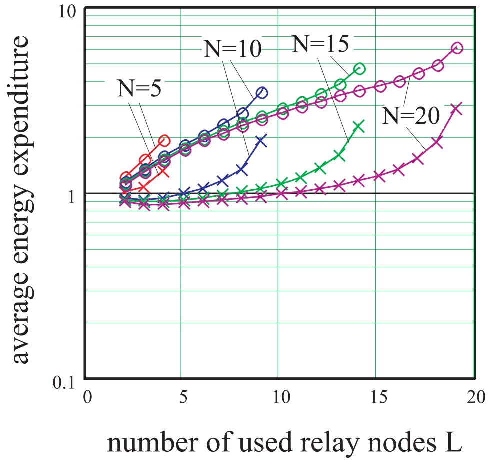

Fig. 3. Mean energy expenditure as a function of the number of active relay nodes L for different numbers of available relay nodes, N. Lines with crosses: multiple fountain codes (mutual information accumulating receiver); lines with circles: single fountain code (energy accumulating receiver), $\overline{\gamma} = \overline{\lambda} = 10$, $H_{\text{target}} = 1$.

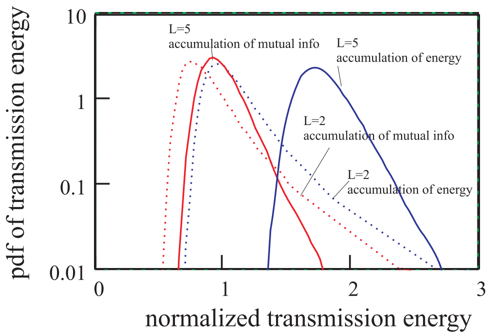

Fig. 4. Pdf of transmission energy expenditure for $N = 10$, $L = 2$, $L = 5$, and $\overline{\gamma} = \overline{\lambda} = 10$.

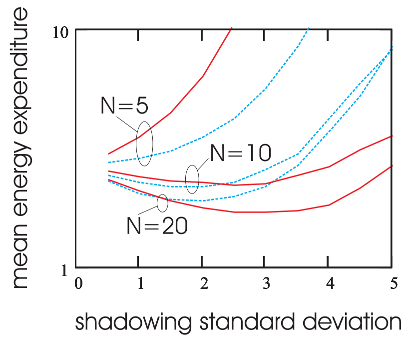

Fig. 5. Mean energy expenditure of the synchronous protocol with multiple fountain codes for different numbers of nodes, N. Number of used nodes, L: 4 (solid red lines), 3 (dashed blue lines). Mean μ of shadowing: 0.1. Independent fading at from-relays to-relays links.

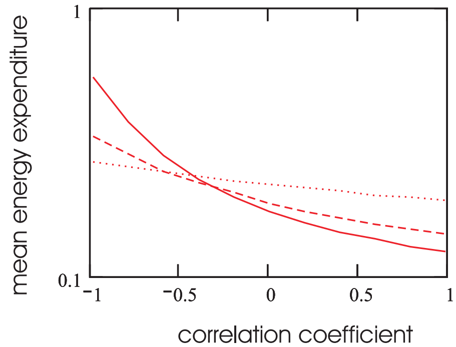

Fig. 6. Mean energy expenditure as a function of the correlation of the shadowing in uplink and downlink with $\mu = 10$, $\sigma = 5$, $L = 3$, and $N = 20$ (solid line), 10 (dashed line), or 5 (dotted line).

high diversity order without an excessive penalty in the mean expended energy.

Fig. 5 shows the mean energy expenditure as a function of the shadowing variance for different numbers of available nodes, N. For small N, a high shadowing variance leads to a much larger average energy expenditure. This can be explained by the fact that a small N also engenders a small L. This, in turn, leads to a high probability that all of the L active nodes for the R-D transmission are in a fading dip. For larger values of N, we see that the energy expenditure even decreases and reaches a minimum at a certain shadowing variance. This is caused by the increased probability of higher mean channel gains due to the variations in shadowing among multiple nodes.

Finally, Fig. 6 shows the effect of correlation between lognormal fading of the S-R and the R-D channels. As we can see, a positive correlation decreases the energy expenditure; this fact is intuitive since it means that the nodes selected during the S-R phase (i.e., the nodes that are the first to receive the complete information) also have good R-D channels. We furthermore see that the effect is the more pronounced the larger the ratio N/L is. Again, that can be easily explained: the quality of the actually selected S-R channels (and thus of the R-D channels as well) increases with N/L.

IV. Asynchronous Transmission

A. The Protocol

In the protocol of Section III, the relay nodes receive their information *only* from the source node. However, with fountain codes, the relay nodes can also help each other to receive the information faster, and thus accelerate the information transmission process. The key idea here is that a relay node starts to transmit information to the destination as soon as it has received sufficient information to decode the source codeword. This transmission can also be heard and used by relay nodes that are still receiving.

The protocol uses the following steps (see also Fig. 7):

1) We establish an ensemble of $M \geq N + 1$ spreading codes and fountain codes. The source node and each of the relay nodes is assigned one of those code pairs.
2) The source node starts to transmit information to all of the relay nodes using its assigned spreading code and fountain code
3) All relay nodes constantly receive signals from the relays (and the source) that have reliably decoded the codeword so far, and accumulate the mutual information. The protocol signals when the transmission on a given

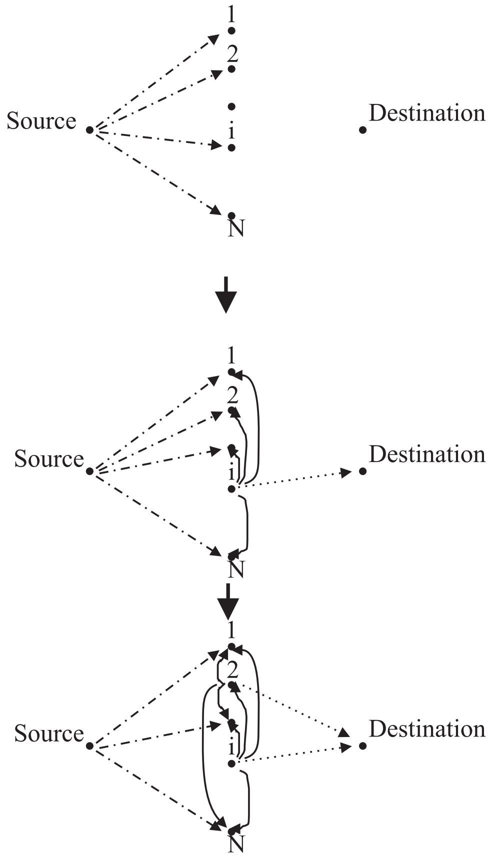

Fig. 7. Phases of the asynchronous transmission protocol: phase one (top): no relay has decoded the source information. Phase 2 (middle) relay i has decoded source information; Phase 3: two relays (i and 2) have decoded source information.

spreading code starts, to avoid unnecessary reception of noise by the receivers.

4) As soon as a relay node has sufficient information to decide on a codeword, it switches from reception mode to transmission mode. It transmits the information using its assigned spreading and fountain code. Because the relay nodes that are in reception mode observe all spreading codes simultaneously, they can receive the information from the source node and all the transmitting relay nodes, and accumulate the mutual information from all those nodes.
5) The destination node is constantly receiving on all possible spreading codes, and thus accumulating the mutual information from the various relay nodes. As mentioned, the direct contribution from the source is neglected.

B. Theoretical Formulation

First, let us assume in the following that all channel gains are deterministic. In that case, a closed-form equation for the total transmission time is feasible. $\widetilde{\tau_1}$ denotes the time until one relay node has gathered sufficient information:

$$\widetilde{\tau_1} = \frac{H_{\text{target}}}{\log\left[1+\gamma_{k_1}\right]} \tag{33}$$

where k_1 is the index of the relay node that finishes the decoding first, i.e., has the highest channel gain to the source node. Next, we determine the time until a second relay node has sufficient information. The mutual information that has arrived at the i-th node by time T is $H_i = T\log\left[1+\gamma_i\right] + (T - \widetilde{\tau_1})\log\left[1+\alpha_{k_1 i}\right]$ so that the time at which a second node decodes the codeword is

$$\widetilde{\tau_2} = H_{\text{target}} \min_{i \neq k_1} \frac{1+\log\left[1+\alpha_{k_1 i}\right] / \log\left[1+\gamma_{k_1}\right]}{\log\left[1+\gamma_i\right] + \log\left[1+\alpha_{k_1 i}\right]} \tag{34}$$

We denote the index of the node that achieves this minimum as k_2. The time during which the source node and *exactly* one relay node k_1 transmit the codeword (using different fountain codes), is denoted as $\tau_1 = \widetilde{\tau}_2 - \widetilde{\tau}_1$, and so on. Generally, the time by which i relay nodes have collected sufficient mutual information is denoted as $\widetilde{\tau}_i$, and the time that exactly i relay nodes (plus the source node) are transmitting is denoted as τ_i; note that $\tau_0 = \widetilde{\tau}_1$ and $\tau_i = \widetilde{\tau}_{i+1} - \widetilde{\tau}_i$ for $i > 0$. Transmission stops at time t when

$$\sum_{i=1}^{N} (t - \widetilde{\tau}_i)\mathcal{H}(t - \widetilde{\tau}_i)\log\left[1+\lambda_{k_i}\right] = H_{\text{target}} \tag{35}$$

where $\mathcal{H}(x)$ is the Heaviside step function. The total transmission energy is $\sum_{i=0}^{N}(i+1)\tau_i$, as transmission during time τ_i involves transmission from i relay nodes plus the source node. Note that $\tau_i = 0$ if the transmission to the destination is complete before relay i has decoded the message. We assume here that the source node will continue to transmit until the destination has successfuly received the message; additional energy can be saved if the source can monitor the N relay nodes, and stops transmitting as soon as all of them start transmitting. In any case, all nodes shall cease transmission after the destination has decoded the sourceword.

When the channel gains are random variables, each step in the above formulation requires the repeated use of order statistics, as it includes the minimization over the receiving relay nodes. Thus, a closed-form evaluation of the statistics of the total required transmission time does not seem possible. We therefore derive, in the following sections, upper and lower bounds on the achievable transmission time, and compare them with Monte Carlo-based averaging over the fading statistics. These bounds are physically motivated and are relatively close together for some typical parameter settings.

C. Bounds

A lower bound on the transmission time can be obtained by considering a scenario with extremely strong inter-relay channels. In this situation, all relays obtain the source information as soon as the relay with the strongest link can decode the source information; due to the strong inter-relay links, the time required to forward the message from this relay to the others is negligible. Thus, the transmission time is the S-R time (whose pdf given in Eq. (8), with $L = 1$) plus the R-D time (with pdf given in Eq. (22), with $L = N$).

An upper bound for the transmission time corresponds to the case of extremely weak inter-relay channels, i.e., the relays do not help each other. However, we still allow that each relay starts transmission as soon as it has received the message from the source. Since the relays are decoupled, the accumulated mutual information that has arrived by time T at the destination via the i-th relay is

$$H(T)=\begin{cases}\ln(1+\lambda)\left[T-\frac{H_{\text{target}}}{\ln(1+\gamma)}\right], & \text{for} \quad \gamma\geq e^{H_{\text{target}}/T}-1\\ 0, & \text{otherwise}\end{cases} \tag{36}$$

where we assume that $\overline{\gamma}_i=\overline{\gamma}$ and $\overline{\lambda}_i=\overline{\lambda}$. This equation reflects the fact that a relay transmits information to the destination only after the relay has decoded the message (for which it requires time $H_{\text{target}}/\ln(1+\gamma)$). The next step is the computation of the CF of Eq. (36). We obtain, for each T,

$$\begin{aligned}M_{\text{onelink}}(j\omega;T)&=E\{e^{-j\omega H}\}\\ &=\int_{e^{H_{\text{target}}/T}-1}^{\infty}\int_{0}^{\infty}\exp\left\{-j\omega\ln(1+\lambda)\left[T-\frac{H_{\text{target}}}{\ln(1+\gamma)}\right]\right\}\\ &\quad\times\frac{\exp(-\gamma/\overline{\gamma})}{\overline{\gamma}}\frac{\exp(-\lambda/\overline{\lambda})}{\overline{\lambda}}d\lambda d\gamma\\ &\quad+\int_{0}^{\exp(H_{\text{target}}/T)-1}\int_{0}^{\infty}\frac{\exp(-\gamma/\overline{\gamma})}{\overline{\gamma}}\frac{\exp(-\lambda/\overline{\lambda})}{\overline{\lambda}}d\lambda d\gamma\end{aligned} \tag{37}$$

Note that the CF is parameterized by the considered time T. It can be rewritten as

$$\begin{aligned}M_{\text{onelink}}(j\omega;T)=&a+\int_{\exp(H_{\text{target}}/T)-1}^{\infty}\frac{\exp(-\gamma/\overline{\gamma})}{\overline{\gamma}}\\ &\frac{\exp\left[1/\overline{\lambda}\right]\Gamma\left[-j\omega\left[T-\frac{H_{\text{target}}}{\ln(1+\gamma)}\right]+1,1/\overline{\lambda}\right]}{\overline{\lambda}^{j\omega\left[T-\frac{H_{\text{target}}}{\ln(1+\gamma)}\right]}}d\gamma\end{aligned} \tag{38}$$

where $\Gamma(x,y)$ is again the incomplete Gamma function, and $a=1-\exp\left\{\frac{1-e^{H_{\text{target}}/T}}{\overline{\gamma}}\right\}$.

The CF of the total information at the destination is simply $M^N_{\text{onelink}}(j\omega;T)$, from which the pdf and cdf of the received information at time T can be obtained numerically by the inverse transformation. The cdf of the transmission time is then obtained as the probability that the information at the destination, received by time T, is smaller than or equal to H_{target}.

D. Simulation Results

Fig. 8 shows the mean time required to transmit the message to the destination as a function of the number of available relay nodes, N. We find that most of the benefits can be realized with about five relay nodes. Note that now the results strongly depend on the channel gains *between* the relay nodes. From top to bottom, the three curves show the results for the mean channel gains being 0, 10 (the same as the mean S-R and R-D channel gains), and 100. As the relay nodes that have already performed the decoding help the other relay nodes that have not yet decoded, the total transmission time decreases as the channel gains between the relays increase.

Fig. 8 also shows the mean energy expenditure as a function of N. We see that the total energy expenditure decreases as the number of available relay nodes increases, but that

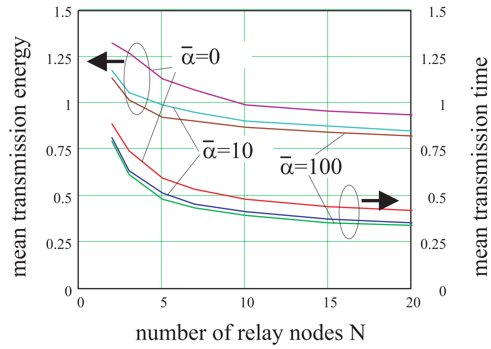

Fig. 8. Mean transmission time and mean expended energy for the asynchronous protocol as a function of the number of available relay nodes. Mean link gain between the relay nodes, $\overline{\alpha}$, is 0, 10, and 100.and $\overline{\gamma}=\overline{\lambda}=10$.

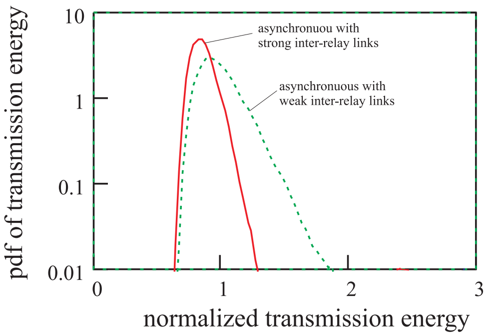

Fig. 9. Pdf of transmission energy expenditure with $N=10$ relay nodes for weak and strong inter-relay links. $\overline{\gamma}=\overline{\lambda}=10$.

this decrease is relatively smaller than the decrease of the transmission time. This can be explained by the fact that the number of transmitting nodes is not fixed, but, rather, it can increase as more and more nodes become available. Further investigations show that the energy expenditure for the relaying saturates very quickly as N increases if the channel gains between the relay links are sufficiently strong.

However, it is noteworthy that in the case of delay-constrained applications, having a large number of nodes has definite advantages. Not only does the mean transmission time and energy decrease with the number of nodes, but also the pdf of the transmission times is more concentrated around its mean value, which improves the quality of service for such applications. This is shown in Fig. 9, which plots the pdf of the total transmission energy when the number of available relay nodes is $N=10$, for the cases of weak and strong inter-relay links. We see that these pdfs are more concentrated than the pdfs for the quasi-synchronous case in Fig. 4.

Fig. 10 shows how the transmission time changes with the strength of the inter-relay links. We see that the upper and lower bounds derived in the previous section are quite close to each other. We also observe again that the slope of the cdf is steeper in the asynchronous case compared to the synchronous case.

Fig. 11 compares the asynchronous and the quasi-synchronous protocol when the nodes are placed at random in a given area, and experience path loss, according to a d^{-4} law, as well as Rayleigh fading. We see that the asynchronous

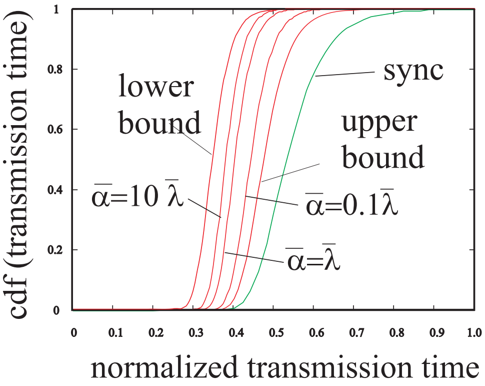

Fig. 10. Cdf of the transmission time for the asynchronous case for various strengths of the inter-relay links $\overline{\alpha}$ and $\overline{\gamma} = \overline{\lambda} = 10$. Also shown are the analytical upper and lower bounds (coinciding with the cases $\overline{\alpha} = 0, \overline{\alpha} = \infty$), and the transmission time for the quasi-synchronous case with $L = 3$.

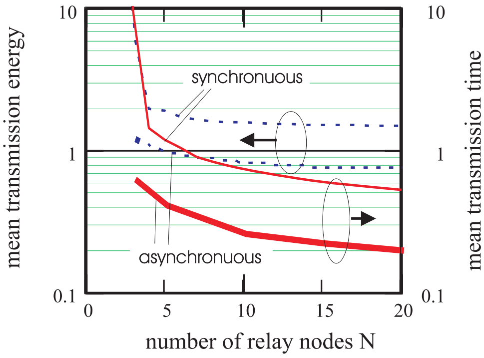

Fig. 11. Mean energy expenditure and transmission time of system with N relay nodes, which are placed randomly in the area $-1 < x < 1$, $-0.5 < y < 0.5$, where the source is at $(-1, 0)$ and the destination is at $(1, 0)$. Path gains are determined by distance law $10d^{-4}$, and superimposed on the pathloss is Rayleigh fading. Solid lines: mean transmission time. Dashed lines: mean transmission energy. Curves given for synchronous protocol with optimum L chosen for each N, and for asynchronous protocol

protocol outperforms the quasi-synchronous protocol to a higher degree than for the situation where all nodes experience the same pathloss. In terms of mean energy, the asynchronous protocol requires almost 50% less energy than the quasi-synchronous protocol. The savings in terms of transmission time are even larger. These effects can be explained by the help that some relay nodes render to the other nodes.

The above considerations assumed that the relay nodes, as well as the destination, can perfectly separate the different fountain codes, and thus truly add up the mutual information stemming from those codes. This is obviously an idealization. The spreading codes are not orthogonal to each other because of the asynchronous nature of relay transmission. In order to simplify the discussion, we assume that the cross-talk between codes acts like additive white Gaussian noise [44], so that the possible rate for each data stream (fountain code) in the R-D links can be computed as

$$r_i = \log\left[1 + \frac{\lambda_i}{1 + \frac{1}{SF}\sum_{k \neq i} \lambda_k}\right] \quad (39)$$

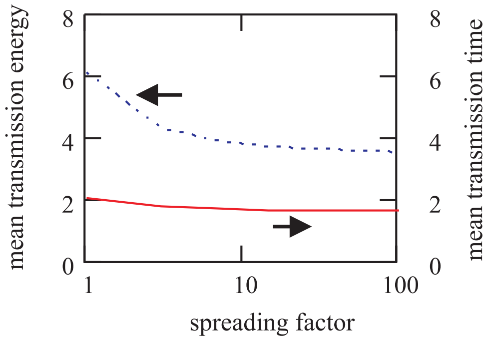

Fig. 12. Energy expenditure for the asynchronous protocol with $N = 10$ nodes if there is cross-talk between the spreading codes of the different nodes. $\overline{\gamma} = \overline{\lambda} = 1$; weak inter-relay links.

where SF is the spreading factor. Fig. 12 shows the energy required for transmission for different values of the spreading factor. We find that even a rather small spreading factor of 10 is sufficient to give almost ideal performance.

V. Summary and Conclusions

In this paper, we proposed the use of fountain codes for relaying in cooperative sensor networks, and analyzed their performance. We compared a quasi-synchronous protocol, in which all relay nodes switch from receiving to transmitting at the same time, and an asynchronous protocol, in which each relay node switches to transmission as soon as it has decoded the source data. We found that for quasi-synchronous transmission, there is a distinct optimum for the number of relay nodes that should transmit the information to minimize total energy expenditure; this optimum typically lies between $L = 2$ and $L = 4$. Furthermore, we found that the asynchronous protocol reduces the latency of the transmission and can lead to additional savings in the transmit energy.

It is worthwhile to discuss whether fountain codes are strictly necessary for the schemes presented in this paper. For the quasi-synchronous scheme, it would be possible to achieve the same performance without fountain codes if all the nodes know all the channel gains. The transmitters would use capacity-achieving codes specifically suitable for the current link strengths. Note, however, that in this case a feedback of all the channel gains to the transmitters is required, which (for the R-D part) is less spectrally efficient than the single bit that informs the relays of the successful decoding. For the asynchronous scheme, fountain codes have a unique advantage, in that they allow the "better" relay nodes to help the weaker ones. The transmission of ordinary capacity-achieving codes, e.g., LDPC codes, from a relay node would not help the other relay nodes with their decoding until *all* the coded information bits have been received. However, the design of the LDPC code transmitted from the i-th relay node is governed by the channel gain to the destination, and not the inter-relay gains. By the time the other relay nodes can decode the information from the i-th relay node, the destination has already finished the decoding.

It is also interesting to discuss the impact of the multiple-access scheme on the feasibility of the scheme when using different fountain codes from each relay to the destination.

In the paper, we assumed a CDMA scheme with a sufficient number of available spreading codes so that each relay node can use a different fountain code. It is also feasible to use different fountain codes in a TDMA scheme, with the different relay nodes taking turns in transmitting. However, there are two drawbacks to a TDMA scheme: (i) the delay dispersion of the channel leads to intersymbol interference, necessitating an equalizer at the receiver (which is more difficult to implement than a Rake receiver), (ii) the energy accumulation is not continuous, because (in order to keep the overhead of timing advance measurements and guard period low) each relay node transmits $several$ bits in a TDMA frame. FDMA can easily realize our proposed scheme, as the different fountain codes are transmitted on different frequencies. Note that the traditional disadvantage of FDMA, namely the sensitivity to fading, is not a problem in our case since the mutual information from different frequency channels is added up.

Future work will concentrate on how the schemes of this paper can be generalized to multi-hop relay networks. The setup discussed here, where there is only a single hop (with multiple relays) between the source and destination, obviously is just a building block for larger networks. The promising results obtained here motivate a further study especially of asynchronuous protocols where all relay nodes can help each other in the decoding and forwarding of the information. At the same time, smart algorithms need to be found that prevent the participation of an excessive number of nodes.

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