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### **Abstract**

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# A Two Stage PAPR Reduction Method on Frequency Redundant OFDM System

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**Abstract**—Adding frequency diversity, through subcarrier redundancy, in orthogonal frequency-division multiplexing (OFDM) is a popular approach to improve the robustness of the system. However, frequency redundant OFDM system is prone to high peak-to-average power ratio (PAPR), due to the fact that the same source information is transmitted on multiple subcarriers. Existing schemes such as *Selective Mapping* (SLM) and *partial transmit sequence* (PTS) are effective but difficult to implement due to the high computation complexity. In this paper, we propose a two stage PAPR reduction method. We analyze the computational complexity and extensive simulations on the PAPR and show that our scheme considerably reduces the computational complexity while achieving similar PAPR reduction as SLM and better PAPR reduction than PTS. For instance, in an OFDM system with 2048 subcarriers and diversity of 8, which is the most complicated system simulated, the proposed scheme with 16 random trials can reduce the complex number multiplications by 15.55% with only 1.6 dB PAPR degradation compared to the SLM scheme. In simpler systems with fewer subcarriers and less diversity, the reduction in computational complexity by our scheme is more significant.

**Index Terms**—Frequency Diversity, OFDM, PAPR, SLM, PTS, Low computational complexity.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive technique for achieving high capacity in frequency selective fading channels. However, individual subcarriers in an uncoded OFDM are prone to deep fading. Adding frequency diversity by transmitting the same information bit on multiple interleaved subcarriers is an effective way to further mitigate the effect of frequency-selective fading as well as an enhancement to the system signal to noise ratio (SNR), which leads to a more robust system.

One of the major disadvantages of OFDM systems, especially for the frequency redundant design, is the high peak-to-average power ratio (PAPR) of the transmitted signals, which requires expensive high power amplifiers with large linear ranges. In addition, large PAPR also demands AD converters with large dynamic ranges. In order to reduce the PAPR, a number of approaches have been proposed [1], [2]. Deterministic method such as clipping the OFDM signal before amplification is the most straightforward method that limits the PAPR within a given threshold. However, this method causes performance degradation and creates out-of-band emission [3]. In comparison, probabilistic schemes statistically improve the

characteristics of the PAPR distribution without introducing signal distortion. Selective mapping (SLM) and partial transmit sequence (PTS) belong to this category. Conventional SLM pre-generates a number of statistically independent sequences from the same data, and chooses the one with the lowest PAPR to send out [4]. PTS divides the subcarriers into a set of disjoint subblocks or continuous clusters, each subblock or cluster of subcarriers is multiplied by different phase factors, the subblocks/clusters are then added to form the different OFDM symbols. The phase factor that generates the time domain OFDM symbol with the lowest PAPR is chosen [5]. Both SLM and PTS techniques can be considered multiple signal representation methods as one favorable OFDM symbol is selected from a large set of statistically independent symbols. For both techniques, a large number of IFFT calculations and complex multiplications with associated phase sequences are required, in proportion to the number and length of the phase sequences used. For example, the optimal PTS requires an exhaustive search over all the possible phase factor combinations, whose resulting algorithm complexity is exponential. Then for an OFDM system that has a significantly large number of subcarriers, the required computational load and hardware complexity can become prohibitively high.

In this paper, we propose a two stage PAPR reduction method. In the first stage, we apply the phase rotations to one set of subcarrier clusters and map it strategically to the OFDM subcarriers. In the second stage, we treat each cluster of subcarriers as a group and use a method similar to SLM to generate the favorable OFDM symbol for transmission.

The rest of this paper is organized as follows: We briefly describe the PAPR problem in OFDM system, and introduce the SLM and PTS schemes in Section II. We then discuss frequency redundant OFDM systems, together with the specific PAPR problem that these systems face in Section III. We propose a two-step PAPR reduction method in Section IV. We analyze the complexity of the proposed method in comparison to the SLM and PTS schemes, and we provide the numerical simulation results on the PAPR reduction performance in Section V. Finally, we conclude this paper in Section VI.

## II. PAPR PROBLEM AND CONVENTIONAL SLM AND PTS SCHEMES

An OFDM transmitter reads in data to be transmitted in blocks. Each data block can be represented by a size- $Q$  vector,

$\mathbf{A} = [a_0, a_1, \dots, a_{Q-1}]$ , where  $a_i$ , ( $0 \leq i \leq Q-1$ ) is a complex number representing a modulation alphabet based on a particular modulation scheme (e.g., PSK, QAM, etc.). A mapping function,  $\mathcal{P}(\cdot)$ , maps input data in  $\mathbf{A}$  to a size- $N$  vector,  $\mathbf{S} = [S_0, S_1, \dots, S_{N-1}]$ . Namely,  $\mathbf{S}$  is

$$\mathbf{S} = \mathcal{P}(\mathbf{A}), \quad (1)$$

where  $N$  is the number of subcarriers in an OFDM symbol. In a conventional OFDM systems, there is no subcarrier redundancy, thus  $N = Q$  (for simplicity, we neglect the pilot and null subcarriers),  $S_i = a_i$  ( $0 \leq i \leq N-1$ ).  $\mathbf{S}$  is referred to as the *frequency domain symbol*. The time domain OFDM signal  $s(t)$  is obtained by the inverse fast Fourier transform (IFFT) given by

$$s(t) = \mathcal{F}^{-1}(\mathbf{S}) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \cdot e^{j\frac{2\pi kt}{T}}, \quad 0 \leq t \leq T, \quad (2)$$

where  $T$  is the OFDM symbol duration. In practice, a cyclic prefix (CP) is added to the signal  $s(t)$  in order to avoid the inter-symbol interference (ISI) that occurs in multipath channels. Since the CP does not impact the PAPR, we ignore it [6]. Because of the central limit theorem and the fact that IFFT is a linear operation, the transmitted OFDM signal  $s(t)$  follows a complex Gaussian distribution when the number of subcarriers  $N$  is large.

The PAPR of the transmitted signal is given by

$$PAPR(s(t)) = \frac{\max |s(t)|^2}{E\{|s(t)|^2\}}, \quad (3)$$

where  $E\{\cdot\}$  denotes the expectation or a statistical average operator. In the literature, the complementary cumulative distribution function (CCDF) is used to evaluate the PAPR reduction performance. The CCDF of the PAPR is given in [2] as

$$\Pr(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \quad (4)$$

#### A. Selected Mapping Scheme

SLM is a simple PAPR suppression method for OFDM signals. In the classical SLM technique, frequency domain symbol block  $\mathbf{S}$  is multiplied element by element with  $U$  phase rotation vectors  $p^{(u)} = [e^{j\phi_0^{(u)}}, \dots, e^{j\phi_{N-1}^{(u)}}]$ , ( $u = 1, \dots, U$ ), resulting in a set of  $U$  different sequences with each entry being

$$S_k^{(u)} = S_k e^{j\phi_k^{(u)}}, \quad k = 0, 1, \dots, N-1. \quad (5)$$

All  $U$  sequences are usually oversampled by a factor of  $L$  [1] and then transformed into time domain by IFFT. The time domain sequence with the lowest PAPR is selected for transmission.

#### B. Partial Transmit Sequence Scheme

PTS method [5] divides the input frequency domain symbol  $\mathbf{S}$  into  $M$  disjoint subblocks or clusters consisting of a contiguous set of subcarriers,  $\{\tilde{\mathbf{S}}_m | m = 0, 1, \dots, M-1\}$ . After zero padding at corresponding positions, each subblock  $\tilde{\mathbf{S}}_m$  becomes a length- $N$  vector,  $\tilde{\mathbf{S}}_m = [S_{m,0}, S_{m,1}, \dots, S_{m,N-1}]$ , satisfying

$\mathbf{S} = \sum_{m=0}^{M-1} \tilde{\mathbf{S}}_m$  and  $S_{i,n} \cdot S_{j,n} = 0$  ( $n = 0, 1, \dots, N-1$ ) when  $i \neq j$ , ( $i, j \in \{0, \dots, M-1\}$ ). Through this process, the original vector  $\mathbf{S}$  turns into a  $M \times N$  matrix. Let the partial transmit sequence  $\mathbf{s}_m$  of length- $N$  be the IFFT of subblock  $\tilde{\mathbf{S}}_m$ , we have the time domain transmitted sequence

$$\mathbf{s} = \text{IFFT}(\mathbf{S}) = \sum_{m=0}^{M-1} \mathbf{s}_m. \quad (6)$$

Applying phase factors to subblocks/clusters allows optimization of combining partial transmit sequences. The combined sequence is

$$\mathbf{s} = \text{IFFT}\left(\sum_{m=0}^{M-1} b_m \tilde{\mathbf{S}}_m\right) = \sum_{m=0}^{M-1} b_m \mathbf{s}_m \quad (7)$$

where  $\{b_m = e^{j\phi_m}, m = 0, \dots, M-1\}$  is the phase rotation factor, each factor is applied to one subblock/cluster. Assume  $\phi_m \in \{2\pi\omega/W, \omega = 0, \dots, W-1\}$ , then there will be  $W^M$  possible unique sets of phase factors to choose from. One selection approach is that we exhaustively try all the possible phase rotation factors and choose the sequence generated with the lowest PAPR, but the computational complexity of this method increases exponentially with  $M$ . Another much simpler approach is to randomly generate  $U$  phase rotation vectors  $b^{(u)} = [b_1^{(u)}, \dots, b_{M-1}^{(u)}]$  ( $u = 1, \dots, U$ ) to apply on  $\tilde{\mathbf{S}}_m$  and choose  $\mathbf{s}^{(u)}$  with the lowest PAPR. For fair comparison purpose, the PTS method referred to hereafter uses the latter one.

### III. FREQUENCY REDUNDANT OFDM SYSTEM

The OFDM system we consider here is a frequency redundant system, which utilizes the frequency diversity across OFDM subcarriers. Since the coherent bandwidth in most of the wireless channels is much greater than the subcarrier-spacing and, therefore, each subcarrier is subject to deep fading. Frequency diversity is introduced in OFDM systems to mitigate this. An easy and convenient way to provide such frequency diversity is to map each input symbol,  $a_n$ , to multiple subcarriers [7],

$$S_k = a_n, \quad \forall k \in \mathfrak{S}_n = \{k_0, k_2, \dots, k_{D-1}\}. \quad (8)$$

$D$  is the *degree of frequency diversity*, and  $\mathfrak{S}_n$  is the set of subcarriers assigned to  $a_n$ . To maximize frequency diversity, we find it essential that the subcarriers assigned to the same input data are spread across the entire band. This can be achieved when an *interleaving subcarrier mapping* scheme is used. In a generic OFDM system, which has no non-data subcarriers, for a size- $Q$  input vector, a mapping function would be as follows

$$\begin{aligned} \mathbf{S} = \mathcal{P}(\mathbf{A}) &= [\underbrace{a_0, \dots, a_{Q-1}}_{\text{1st set}}, \underbrace{a_0, \dots, a_{Q-1}}_{\text{2nd set}}, \dots, \underbrace{a_0, \dots, a_{Q-1}}_{\text{Dth set}}] \\ &= [\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2, \dots, \hat{\mathbf{S}}_D] \end{aligned} \quad (9)$$

where  $\hat{\mathbf{S}}_i$ , ( $1 \leq i \leq D$ ) stands for the  $i$ th subcarrier cluster. The mapping function given in (9) maps  $Q$  inputs to  $D \times Q = N$  subcarriers. The  $q$ th input  $a_{q-1}$  is mapped to  $D$  subcarriers,

$\{k_0 = q - 1, k_1 = Q + q - 1, \dots, k_{D-1} = (D - 1)Q + q - 1\}$ . Subcarriers carrying the same data have a minimum separation of  $Q$  subcarriers spacing. The advantage of such a design is that it introduces frequency diversity to mitigate the effects from the frequency selective channel. If the transmitted signals on some frequency subcarriers are affected by the channel fading and can not be detected, the signals on other subcarriers can still be received correctly.

One of the disadvantages of such a frequency redundant OFDM system is the high PAPR. Statistically, if there are  $D$  sets of subcarriers carrying the same data, the probability of having a high peak in time domain is much higher due to the dependency of the signal in frequency domain [1].

Generally, for this frequency redundant OFDM system, the time domain baseband signal can be written as in (2). By sampling the above signal  $s(t)$  with sampling interval  $\Delta t = T_s/N$ , we get discrete time domain signal as

$$\begin{aligned} s(n) &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \cdot e^{j \frac{2\pi kn}{N}} = \frac{1}{\sqrt{N}} \sum_{q=0}^{Q-1} a_q \sum_{d=0}^{D-1} e^{j \frac{2\pi n(dQ+q)}{N}} \\ &= \frac{1}{\sqrt{N}} \underbrace{\sum_{q=0}^{Q-1} a_q \cdot e^{j \frac{2\pi nq}{N}}}_{\check{s}(n)} \underbrace{\sum_{d=0}^{D-1} e^{j \frac{2\pi ndQ}{N}}}_{\zeta}. \end{aligned} \quad (10)$$

(10) shows how redundancy affects the OFDM signal's PAPR.  $\frac{1}{\sqrt{N}}\check{s}(n)$  is the scaled periodic extension of the IFFT of  $\hat{S}_i$  and  $\zeta_D$  is the IFFT of a length- $N$  vector  $[1, \underbrace{0, \dots, 0}_{Q-1}, \dots, 1, \underbrace{0, \dots, 0}_{Q-1}]$ . Fig. 1 shows one example of the amplitude of  $\zeta$ . We can see that due to the dependency of the subcarriers,  $\zeta$  periodically raise the amplitude of  $\check{s}(n)$ . Clearly this subcarrier dependency affects the PAPR of the OFDM signal. In the following section, we propose a two stage phase rotation method to change the probabilistic behavior of the PAPR of this design.

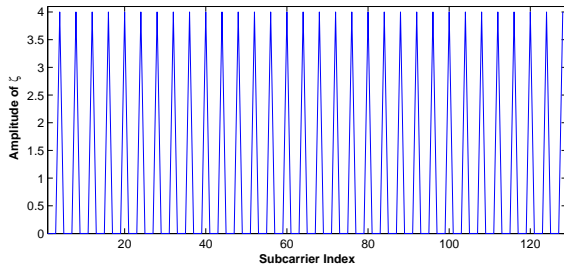


Fig. 1. Amplitude of  $\zeta$  ( $N=128, D=4, Q=32$ )

#### IV. A TWO STEP PAPR REDUCTION METHOD

We propose a two stage PAPR reduction method. As shown in Fig. 2, after the modulation, we have a vector of length- $Q$ . This is a set in (9), that carry one time of the original input data. Due to the frequency redundancy in our design, the same set of input data will be mapped on  $D$  clusters of subcarriers to compose the length- $N$  OFDM symbol. In our PAPR reduction

method, we first apply the phase rotation on this subblock before mapping it on the  $D$  subcarrier clusters. In this case, the chosen phase rotation vector  $p^{(u)}$  only needs to have  $Q$  components. The phase rotation sequence is generated using the unit-magnitude complex number. For convenience, binary ( $\{\pm 1\}$ ) or quaternary elements ( $\{\pm 1, \pm j\}$  or  $\{\pm \sqrt{2} \pm j\sqrt{2}\}$ ) are usually used for elements of  $p^{(u)}$ .

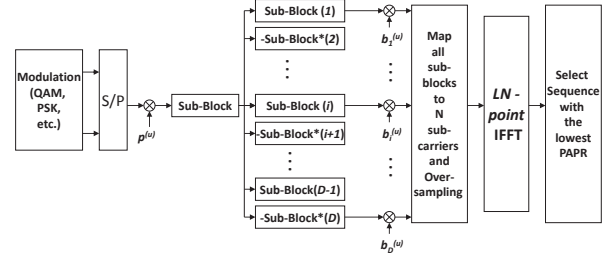


Fig. 2. Block diagram of the proposed design

Different subcarrier clusters contain the same information, hence, in order to avoid accumulated components of particular phase which might produce excessive peak power signal in time domain, we use a simple alternative signal allocation method. We convert one of the adjacent clusters to the conjugate of themselves ( $\hat{S}_d, \hat{S}_{d+1} = -\hat{S}_d^*$ ). By doing this, the phase difference between two adjacent clusters varies with respect to the original input data symbols themselves. Thus, the dependency between the different clusters is reduced.

After this stage, the input frequency symbol  $\mathbf{S}$  in (9) turns to

$$\tilde{\mathbf{S}}^{(u)} = p^{(u)} \cdot [\hat{S}_1, -\hat{S}_2^*, \dots, \hat{S}_{D-1}, -\hat{S}_D^*]. \quad (11)$$

The output,  $\tilde{\mathbf{S}}^{(u)}$ , is further manipulated in the second stage. In this stage, we treat each cluster as a group and rotate every cluster by one rotation factor. Now  $\tilde{\mathbf{S}}^{(u)}$  can be expressed as

$$\tilde{\mathbf{S}}^{(u)} = p^{(u)} \cdot [\hat{S}_1 \cdot b_1^{(u)}, -\hat{S}_2^* \cdot b_2^{(u)}, \dots, \hat{S}_{D-1} \cdot b_{D-1}^{(u)}, -\hat{S}_D^* \cdot b_D^{(u)}]. \quad (12)$$

We can see that compared to conventional schemes such as SLM and PTS this scheme rotates the subcarriers twice in two stages instead of only once. The increased freedom of rotation can further randomize the phase of different subcarriers. After the two phase rotation stages, the subcarrier clusters are then mapped in cascade to form the length- $N$  OFDM symbol. Note that in order to obtain an improved approximation of the true PAPR in the discrete-time signal, we need to oversample the candidate signals. An oversampling rate of  $L$  for the system can be achieved by inserting  $(L-1) \cdot N$  zeros in the middle of the encoded symbol vectors. Thus,  $\tilde{\mathbf{S}}^{(u)}$  becomes

$$\begin{aligned} \tilde{\mathbf{S}}^{(u)} &= p^{(u)} \cdot [\hat{S}_1 \cdot b_1^{(u)}, -\hat{S}_2^* \cdot b_2^{(u)}, \dots, -\hat{S}_{D/2}^* \cdot b_{D/2}^{(u)}, \underbrace{0, \dots, 0}_{(L-1) \cdot N}, \\ &\quad \hat{S}_{D/2+1} \cdot b_{D/2+1}^{(u)}, \dots, \hat{S}_{D-1} \cdot b_{D-1}^{(u)}, -\hat{S}_D^* \cdot b_D^{(u)}]. \end{aligned} \quad (13)$$

TABLE I  
COMPLEXITY COMPARISON OF DIFFERENT ALGORITHMS

	SLM	PTS	Proposed
# Complex multiplications	$ULN/2\log_2 N + ULN + N(U-1)$	$(U-1)MNL + ULN/2$	$(U-1)[(N/D) + LN/2\log_2 N] + UD + ULN/2$
# Complex additions	$ULN(\log_2 N + 1/2)$	$(M-1/2)ULN$	$(U-1)LN\log_2 N + ULN/2$

To follow SLM signal representations, we get the time-domain signal by using IFFT on  $\tilde{\mathbf{S}}^{(u)}$

$$\tilde{\mathbf{s}}^{(u)} = \text{IFFT}(\tilde{\mathbf{S}}^{(u)}). \quad (14)$$

Then the selecting can be mathematically expressed as

$$\check{\mathbf{s}} = \arg \min_{1 \leq u \leq U} \{PAPR(\tilde{\mathbf{s}}^{(u)})\} \quad (15)$$

Finally, the transmitter selects the most favorable time domain signal  $\check{\mathbf{s}}$  with the lowest PAPR for transmission.

## V. ANALYSIS OF COMPUTATIONAL COMPLEXITY AND SIMULATION RESULTS

### A. Complexity comparison

It is expected that the proposed method shows reduction of the number of complex multiplications and complex additions since the length of the phase rotation sequences are notably shortened. For fair comparison, we assume SLM, PTS and the proposed method all use  $U$  times random phase rotation trials to obtain the sequence with the lowest PAPR. According to convention, the first of these  $U$  signal mappings is just the original OFDM symbol. It should be noted that, the numbers of complex multiplications and additions of the  $LN$ -point IFFT for the oversampled case are  $(LN/2)\log_2 N + N/2$  and  $(LN)\log_2 N$  respectively, instead of  $(LN/2)\log_2(LN)$  and  $(LN)\log_2(LN)$  due to the sparseness of the length- $LN$  vector [8].

For SLM, the complex multiplication (CM) and complex addition (CA) are needed in three places. i) Frequency-domain phase rotation in (5) needs  $N(U-1)$  CMs; ii)  $U$  length- $LN$  IFFTs on oversampled vector requires  $U[(LN/2)\log_2 N + LN/2]$  CMs and  $(LN)\log_2 N$  CAs; iii) Calculating the length- $LN$  time domain signal  $\mathbf{s}$  to determine PAPR requires  $2ULN$  real multiplications equally as  $ULN/2$  CMs, meanwhile  $ULN/2$  CAs are needed.

For PTS, if we divide the oversampled input vector  $\mathbf{S}$  to  $M$  subblocks, we need  $M$  length- $LN$  IFFTs to create  $\mathbf{s}$ . According to [8] and [9], the phase rotation and IFFT process together require  $MLN(U-1)$  CMs. The PAPR calculation needs  $ULN/2$  CMs. Correspondingly, we need  $(M-1)ULN$  CMs and  $ULN/2$  CMs in these two steps, respectively.

For the proposed method, as shown in Fig. 2, only  $UN/D$  CMs are needed at the first phase rotation step  $\hat{\mathbf{S}}_0 \cdot p^{(u)}$ . The second step needs  $UD$  CMs to finish the cluster rotation. The  $LN$  point IFFT procedure needs  $U[(LN/2)\log_2 N + N/2]$  CMs and  $U(LN)\log_2 N$  CAs, respectively. Finally, the PAPR calculation needs  $ULN/2$  CMs and the same number of CAs.

The comparison of computational complexity between the SLM, PTS and the proposed method is summarized in Table

TABLE II  
COMPARISON OF COMPLEXITY WITH  $U = 8, N = 128, M = 16, D = 4, L = 4$

	SLM	PTS	Proposed
# Complex multiplications	19328	59392	14876
# Complex additions	30720	63488	27136

TABLE III  
COMPARISON OF COMPLEXITY WITH  $U = 16, N = 2048, M = 16, D = 8, L = 4$

	SLM	PTS	Proposed
# Complex multiplications	882688	2031616	745464
# Complex additions	1507328	2031616	1417216

I. We show two extreme cases in Table II and III. We can see that the proposed method brings down the complexity significantly. For example, in a simpler system shown in Table II, the proposed method reduces the CM and CA number by 23.2% and 11.84% compared to SLM and by 75% and 57.26% compared to the PTS method with  $M = 16$ .

### B. PAPR Simulation Results

We conduct a series of simulations to evaluate the proposed scheme's PAPR reduction performance. The simulation system is set up as follows: QPSK modulation is used, number of subcarriers  $N$  span from 128 to 2048, and the OFDM signal is oversampled by a factor of  $L = 4$ . For simplicity, the elements of the phase sequence  $p^{(u)}$  in SLM and  $b^{(u)}$  in PTS and in the proposed method are randomly chosen from set  $\{\pm 1, \pm j\}$ . We ignore the cyclic prefix and non-data tones in the OFDM subcarriers. The frequency diversity  $D$  of 4 and 8 are used, respectively. We also simulate regular OFDM systems with same number of subcarriers, which have no frequency redundancy to compare with our frequency redundant designs. Fig. 3 and Fig. 4 show the CCDF of PAPR for different methods. Obviously, the main target for the proposed method to compare with is the SLM method. The reason that the PTS method performs badly in this design is that we only treat each of the  $M$  subblocks as a group and do the phase rotation randomly. Without any intelligent phase selection optimization method that appear in most PTS method, this PTS method can not perform well in this specific OFDM system. As illustrated in Fig. 3, when diversity is 4 in the OFDM system, both the SLM and proposed method can reduce the PAPR by 8dB at CCDF of  $10^{-4}$ . The proposed method is less than 0.5dB worse when  $N = 128$  and  $N = 256$ . It performs better than SLM using  $U = 8$  random trials. When the subcarrier number grows ( $N = 1024$  and  $N = 2048$ ), the proposed method has 0.6-0.7dB reduction in the PAPR axis.

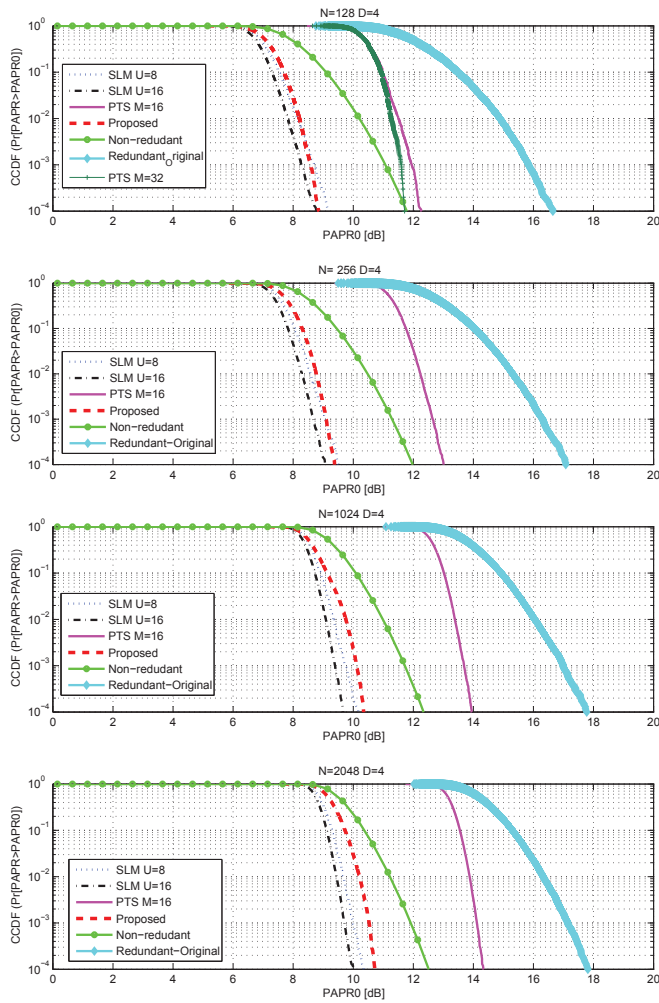


Fig. 3. Comparison of PAPR reduction performance, when  $D = 4$  and  $N = \{128, 256, 1024, 2048\}$ .

## VI. CONCLUSION

In an OFDM system using subcarrier redundancy, same source information is carried on multiple clusters of subcarriers. Such a system is robust to noise and channel fading, but prone to high PAPR. We propose a two stage phase rotation method to reduce the PAPR. At the first stage, we apply a random phase rotation on one cluster of subcarriers, and then we use a strategy to determine the phase rotations on other clusters of subcarriers. At the second stage, we treat each cluster of subcarriers as a group to apply a second round phase rotation on each group. Multiple random phase rotations are tried, and the most favorable OFDM symbol in terms of PAPR is selected for transmission. Our scheme reduces the computational complexity substantially compared to SLM or PTS schemes. Simulation results show that our scheme incurs little PAPR performance loss when compared to SLM scheme, and better PAPR performance than PTS scheme.

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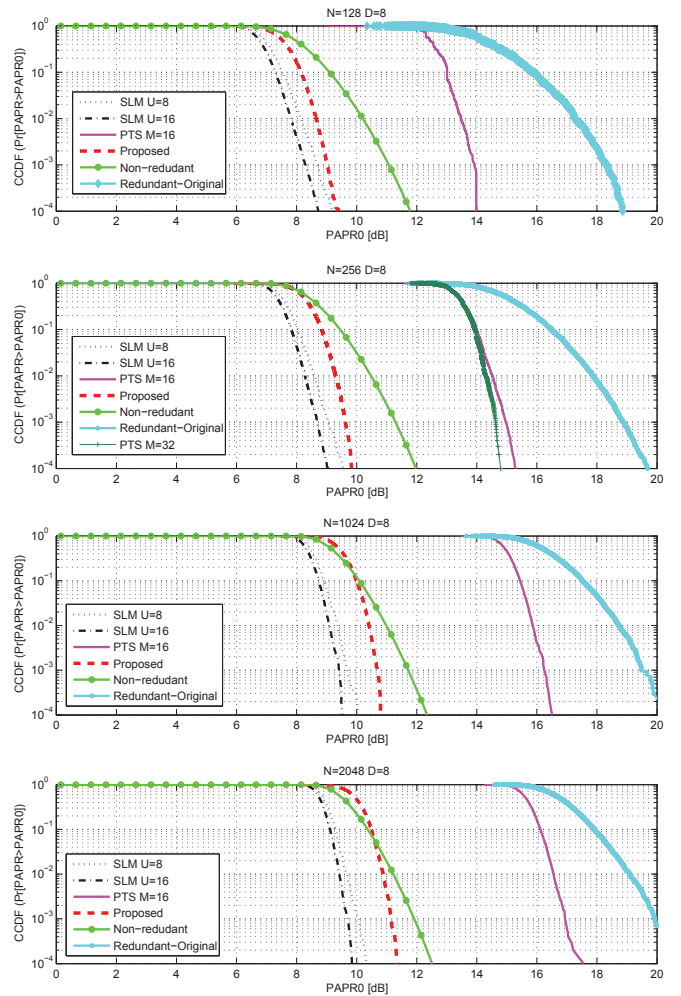


Fig. 4. Comparison of PAPR reduction performance, when  $D = 8$  and  $N = \{128, 256, 1024, 2048\}$ .

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