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Xingze He, Man-On Pun, C.-C. Jay Kuo, Ye Zhao

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Abstract

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I. Introduction

It is estimated that power outages and power quality problems could cost at least $\$150$ billion each year in the U.S. [1]. Being motivated by this concern, one of the defining characteristics of the emerging smart grids is their capability of supporting more stable and higher-quality power supply by leveraging state-of-the-art information technology. To assess power quality (PQ), it is a common practice to monitor the quality of voltage and current waveforms by analyzing the real-time information acquired by sensors installed in power distribution networks.

In contrast with the sinusoidal power waveform generated by electric utilities, power waveforms over transmission lines are often distorted. Generally speaking, distortions can be classified into two categories; namely, PQ variations and PQ events [2], [3]. While PQ variations are characterized by small and gradual deviations from the sinusoidal voltage/current waveforms, PQ events incur large waveform deviations. PQ events are more detrimental to the power distribution network since it may potentially inflict more severe damages such as power outages. Consequently, the occurrence of PQ events has to be accurately and timely detected to allow appropriate amending actions. For presentational simplicity, we concentrate on the voltage-based PQ events in this work while its extension to the current-based PQ events can be done in a straightforward manner.

In practice, PQ event monitoring consists of two steps: 1) detection and 2) classification. In the first step, the occurrence of a PQ event is declared when the waveform change is detected to exceed a pre-defined threshold. In the second step, the distorted waveforms are fed into a classifier to identify the cause of the PQ event before further analysis is performed. In this work, we focus on developing novel detection schemes in the first step. For readers interested in the classification step of the PQ event monitoring, we refer to [3] for a very comprehensive treatment.

In this work, we study the PQ monitoring problem in a change-point detection theoretic framework. More specifically, we propose a sequential detection scheme [4] by examining the difference of statistical distributions of power waveforms before and after the PQ event occurrence. Despite the fact that the pre-change signal statistics can be well characterized, the post-change signal statistics are usually unknown, depending on the nature of the underlying PQ event(s). To circumvent this obstacle, we propose to first transform the received signal such that the transformed post-change signal can be modeled as a sum of multiple statistically independent signals. After invoking the central limit theorem, we devise a robust change-point detection approach for PQ monitoring by exploiting change-point detection theory with unknown parameters after change. Since the proposed scheme performs sample-by-sample evaluation, it can achieve the detection task with the finest time resolution.

II. Review of Previous Work

Three conventional PQ event detection methods have been proposed in the current literature. The first one keeps tracking the root mean squared (rms) value of the voltage waveform over a moving window. The likelihood of PQ event occurrence is evaluated based on the rms change across windows. Despite its simplicity, the rms-based method is effective in detecting amplitude-related distortions. The second one detects the distortion in the frequency domain by transforming the time waveform into the frequency waveform using either the wavelet or the short-time Fourier transform (STFT) [3]. The third one decomposes the waveform into a sum of damped sinusoids using super-resolution spectral analysis techniques such as signal estimation via a rotational invariance technique (*e.g.,* ESPRIT) or multiple signal classification (*e.g.,* MUSIC) [5]. The distorted waveform is detected by comparing the decomposed frequency-domain components of a monitored waveform with those of the normal one. Apparently, the latter two are more agile to frequency distortions. Note that a sliding window is also required in the last two methods to segment the waveform into blocks before any transformation

Xingze He and C.-C. Jay Kuo are with the Ming Hsieh Department of Electrical Engineering, University of Southern California, Los Angeles, CA 80089, USA (e-mails: xingzehe@usc.edu; cckuo@sipi.usc.edu).

Man-On Pun is with the Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139, USA (e-mail: mpun@merl.com).

Ye Zhao is with Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115, USA (e-mail: zhao.ye@husky.neu.edu).

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or decomposition is applied [3]. As a result, the time resolution of all three methods is restricted by the sliding window size. Unfortunately, the sliding window size has to be sufficiently large to meet the detection rate and the false alarm rate requirements. Finally, we should emphasize the difference between [6] and our work. Unlike [6] in which the change-point detection theory is employed for denoising purposes in wavelet-based PQ monitoring, our work concentrates on modeling the received signal and subsequently, applying the change-point theory to provide quick and accurate PQ event detection.

III. Problem Formulation

Without loss of generality, a PQ event is assumed to take place at time $t = t_e$. The goal is to detect the PQ event with the minimum delay and the highest detection accuracy. Note that the proposed technique can be straightforwardly extended to the detection of the end of a PQ event. Here, we will focus on the detection of the occurrence of a PQ event.

The continuous-time waveform signal *before* the PQ event is measured and sampled. The k-th sample can be modeled as

$$y[k] = s_{\boldsymbol{\theta}_0}[k] + n[k], \tag{1}$$

where $n[k]$ is the additive white Gaussian noise (AWGN) with zero-mean and variance σ_n^2, denoted by $\mathcal{N}(0, \sigma_n^2)$, and

$$s_{\boldsymbol{\theta}_0}[k] = a_0 \cdot \sin\left(2\pi f_0 T_s k + \phi_0\right), \tag{2}$$

is the undistorted power waveform with T_s being the sampling duration, $\boldsymbol{\theta}_0 \stackrel{\text{def}}{=} \left[a_0, f_0, \phi_0\right]^T$, where $a_0 = 1$ is the signal amplitude gain, and f_0 and ϕ_0 are the fundamental frequency and the initial phase of the power waveform, respectively. Note that we have implicitly assumed the variance of $n[k]$ is independent of k.

Similarly, we can model the power waveform *after* the PQ event as

$$y[k] = s_{\boldsymbol{\theta}_1}[k] + n[k], \quad t \geq t_e, \tag{3}$$

where $\boldsymbol{\theta}_1 \stackrel{\text{def}}{=} \left[a_1, f_1, \phi_1, \boldsymbol{\varphi}^T\right]^T$ and

$$s_{\boldsymbol{\theta}_1}[k] = a_1 \cdot \sin\left(2\pi f_1 T_s k + \phi_1\right) + \xi_{\boldsymbol{\varphi}}[k], \tag{4}$$

with $\xi_{\boldsymbol{\varphi}}[k]$ being the additive distortion parameterized by $\boldsymbol{\varphi}$.

Eq. (3) represents a generalized power waveform by taking typical PQ events into account. For instance, voltage dips can be modeled as sudden drops in the waveform amplitude gain with $a_1 < a_0$ while setting $\xi_{\boldsymbol{\varphi}}[k] = 0$. In contrast, a transient voltage event can be described by non-zero $\xi_{\boldsymbol{\varphi}}[k]$ with $f_0 = f_1$ and $\phi_0 = \phi_1$.

We use $p_{\boldsymbol{\theta}_0}(y)$ and $p_{\boldsymbol{\theta}_1}(y)$ to denote the probability density functions (PDF) of y before and after the PQ event, respectively. Clearly, $p_{\boldsymbol{\theta}_0}(y)$ can be well estimated due to the fact that $\{a_0, f_0, \phi_0\}$ are deterministic whereas σ_n^2 can be accurately measured. In contrast, $p_{\boldsymbol{\theta}_1}(y)$ depends on the specific type of PQ events under consideration. It is generally difficult to fully characterize $p_{\boldsymbol{\theta}_1}(y)$ before the occurrence of the PQ event, which handicaps the conventional statistical hypothesis test methods such as the Neyman-Pearson hypothesis testing.

As a result, most conventional PQ event detection methods are designed to directly exploit the instantaneous changes in the amplitude gain, fundamental frequency or phase without utilizing their long-term statistics. For instance, the conventional rms method concentrates on amplitude changes by sampling and computing the rms of the voltage waveform. Let y_k be the k-th sample of the voltage waveform. The conventional rms method keeps tracking the sample rms over a sliding window of size N, where N usually covers one cycle of the power-system frequency [3]. Mathematically, the q-th rms is given by

$$Y_{\text{rms}}(q) = \sqrt{\frac{1}{N} \sum_{k=q-N+1}^{q} y_q^2}. \tag{5}$$

A PQ event is detected if the current rms value change is larger than a pre-defined threshold of the nominal voltage. Besides the time-resolution problem associated with the sliding window size, conventional methods are sub-optimal due to the fact that they do not exploit the statistical distributions before and after the PQ event.

IV. Proposed Algorithms

In this section, we derive a PQ event detection scheme from the cumulative sum (CUSUM) algorithm, which is most well-known in the change-point detection theory. The pre-event PDF, $p_{\boldsymbol{\theta}_0}(y)$, is assumed to be known while the post-event PDF, $p_{\boldsymbol{\theta}_1}(y)$, is unknown. To circumvent the uncertainty of the post-event PDF, the weighted CUSUM algorithm is employed to replace the conventional log-likelihood ratio (LLR) test. In the following, we assume prior knowledge on $a_0 = 1$, f_0 (*i.e.* either 50 or 60 Hz), ϕ_0 and σ_n^2.

A. Pre-event PDF

Since $\{a_0, f_0, \phi_0\}$ are known, $s_{\boldsymbol{\theta}_0}$ becomes deterministic. We begin with transforming $y[k]$ in (1) into $z[k]$ as

$$z[k] = y[k] - s_{\boldsymbol{\theta}_0}[k] = n[k], \quad 0 \leq t < t_e. \tag{6}$$

Thus, the PDF of z is simply $p_{\boldsymbol{\theta}_0}(z) = \mathcal{N}\left(0, \sigma_n^2\right)$.

B. Post-event PDF

Next, we derive the post-event PDF using results from change-point detection theory with unknown parameters after change. Two solutions have been developed in change-point detection theory [4]; namely, the weighted CUSUM method and the generalized likelihood ratio (GLR) CUSUM method. In this work, the weighted CUSUM method is adopted due to its simplicity.

Similar to (6), we also transform Eq. (3) as

$$z[k] = y[k] - s_{\boldsymbol{\theta}_0}[k] = x[k] + w[k], \tag{7}$$

where

$$\begin{aligned} x[k] &= a_1 \cdot \sin\left(2\pi f_1 t + \phi_1\right), &(8)\\ w[k] &= \xi_{\boldsymbol{\varphi}}[k] - s_{\boldsymbol{\theta}_0}[k] + n[k]. &(9)\end{aligned}$$

Since $\boldsymbol{\theta}_1$ is unknown, rather than evaluating the LLR $\frac{p_{\boldsymbol{\theta}_1}(z_i)}{p_{\boldsymbol{\theta}_0}(z_i)}$ directly, we compute the logarithm of the weighted likelihood ratio with the weighted CUSUM method as

$$s_i = \ln\left[\int_{\boldsymbol{\Theta}_1} \frac{p_{\boldsymbol{\theta}_1}(z_i)}{p_{\boldsymbol{\theta}_0}(z_i)}\, dF_{\boldsymbol{\Theta}_1}\left(\boldsymbol{\theta}_1\right)\right], \tag{10}$$

where $F_R\left(\boldsymbol{r}\right)$ is the cumulative density function (CDF) of the enclosed random variable R.

By invoking the central limit theorem, we can approximate the PDF of w as $\mathcal{N}\left(0, \sigma_w^2\right)$, where $\sigma_w^2 = \sigma_\xi^2 + \sigma_n^2 + \frac{1}{2}a_0^2$. Furthermore, recall that $x[k]$ is approximately uniformly distributed over $\left[-\left|a_1\right|, +\left|a_1\right|\right]$. Thus, it is straightforward to show that

$$p_{\boldsymbol{\theta}_1}(z) = \frac{1}{4\left|a_1\right|}\left[\text{erf}\left(\frac{z+\left|a_1\right|}{\sqrt{2}\cdot\sigma_w}\right) - \text{erf}\left(\frac{z-\left|a_1\right|}{\sqrt{2}\cdot\sigma_w}\right)\right]. \tag{11}$$

With the assumption that $x[k]$ and $w[k]$ are statistically independent, we can express $F\left(\boldsymbol{\theta}_1\right)$ as

$$F_{\boldsymbol{\Theta}_1}\left(\boldsymbol{\theta}_1\right) = F_{A_1}\left(a_1\right)\cdot F_{\Sigma_w}\left(\sigma_w\right). \tag{12}$$

As a result, Eq. (10) becomes

$$s_i = \ln\left[\int_{A_1}\int_{\Sigma_w} \frac{p_{\boldsymbol{\theta}_1}(z_i)}{p_{\boldsymbol{\theta}_0}(z_i)}\, dF_{\Sigma_w}\left(\sigma_w\right)\, dF_{A_1}\left(a_1\right)\right]. \tag{13}$$

The most commonly used distribution of $F\left(\cdot\right)$ includes the uniform and Gaussian distributions [4]. Unless otherwise specified, the Gaussian distribution is employed in our simulation as described in Sec. V.

C. Summary of Weighted CUSUM-based Schemes

The proposed weighted CUSUM-based PQ-event detection scheme is summarized below.

Algorithm 1 Weighted CUSUM-based PQ-event detection

Inputs: samples $\{y_k\}$ and a preset threshold h
States: Initialize $t_e = 0$
Procedure:
  for $k = 1, 2, \cdots, \infty$ **do**
    $z_k = y_k - s_{\boldsymbol{\theta}_0}(t_k)$;
    $s_k = \ln\left[\int_{A_1}\int_{\Sigma_w} \frac{p_{\boldsymbol{\theta}_1}(z_k)}{p_{\boldsymbol{\theta}_0}(z_k)}\, dF_{\Sigma_w}\left(\sigma_w\right)\, dF_{A_1}\left(a_1\right)\right]$;
    $S_k = \sum_{i=1}^{k} s_i$;
    $m_k = \min_{1\le j\le k} S_j$;
    $g_k = S_k - m_k$;
    if $g_k \ge h$ **then**
      $\hat{t}_e = t_k$;
      break;
    end if
  end for
  Declare the detection of a PQ event at time $\hat{t}_e$ if $\hat{t}_e \neq 0$.

V. Simulation Results

In this section, simulation results are provided to compare the performance of the proposed CUSUM and several previous detection schemes. We use ATP to simulate two types of PQ events, *i.e.,* voltage transients and dips generated with the IEEE 14-bus test setup specified in [7]. For a fair comparison, we employ the same sampling rate of $T_s = 10$ ms for all detection methods under consideration without individually optimizing T_s for each method. The PQ event is set to take place at $t_e = 0.06$s in the simulation. Furthermore, we define the signal-to-noise ratio (SNR) as $\frac{1}{\sigma_n^2}$ while fixing $a_0 = 1$.

A. Voltage Transients

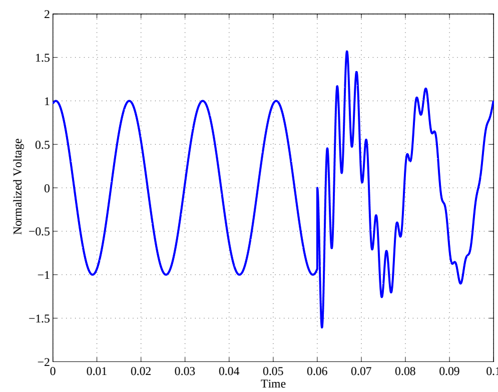

Fig. 1. Illustration of a voltage transient event.

Fig. 1 depicts the power waveform distorted by a PQ transient event at $t = 0.06$s due to switching "in" the capacitor at bus9 at 0.06s. Figs. 2 and 3 show the temporal-frequency plot using STFT and spectral estimates using the MUSIC over multiple windows, respectively. As shown in Figs. 2 and 3, it is difficult to detect the PQ event directly from these plots.

In contrast, Figs. 4 and 5 show the sample-by-sample rms generated by the conventional rms scheme and the logarithm of the weighted likelihood ratio by the proposed CUSUM scheme at SNR of 20 dB. Apparently, the proposed CUSUM scheme has much stronger indication on the PQ event occurrence at $t = 0.06$s.

To compare the performance of the rms scheme and the proposed CUSUM scheme quantitatively, we define the following mean squared error (MSE) of the event detection as the performance metric:

$$\text{MSE} = E\left\{\left(\hat{t}_e - 0.06\right)^2\right\}. \tag{14}$$

Note that more systematic evaluation can be performed in terms of the false alarm rate and detection delay as shown in [8]. To optimize the threshold employed in the RMS and the proposed CUSUM schemes, we first establish the optimal threshold for each scheme by exhaustive search. Fig. 6 shows an example of the MSE performance as a function of threshold h for the CUSUM scheme at $SNR = 20$ dB, which suggests

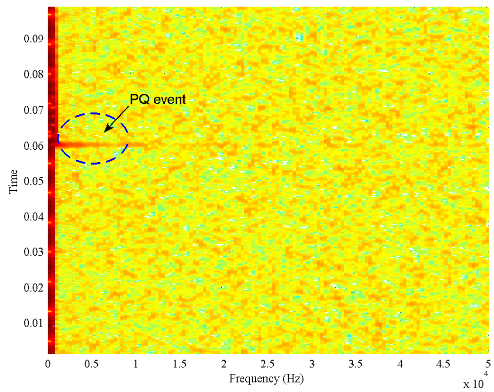

Fig. 2. The temporal-frequency plot using STFT w.r.t. a transient event.

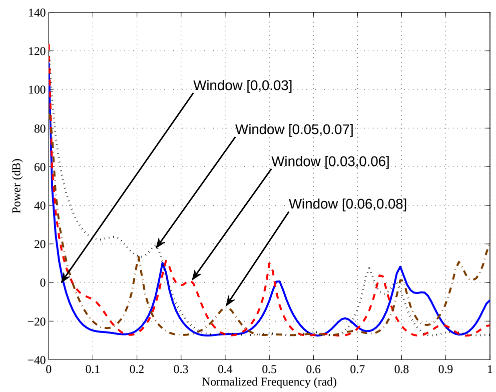

Fig. 3. Spectral estimates with MUSIC w.r.t. a transient event.

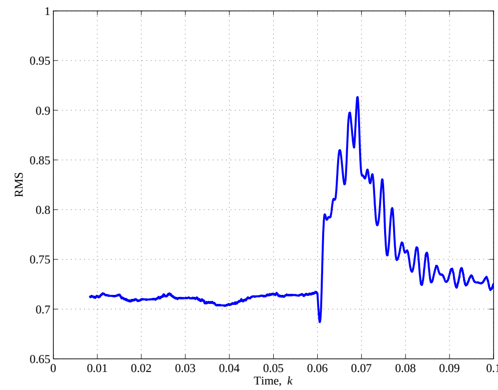

Fig. 4. Sample-by-sample RMS of a transient event as a function of time ($SNR = 20$ dB).

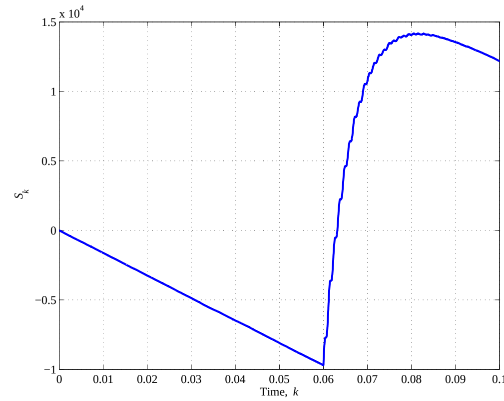

Fig. 5. The logarithm of the weighted likelihood ratio of a transient event using CUSUM ($SNR = 20$ dB).

that the optimal threshold for the CUSUM scheme is about 10 for this SNR value.

Fig. 7 compares the MSE performance of the CUSUM and RMS schemes as a function of SNR. As shown in this figure, the CUSUM scheme outperforms the RMS scheme by a large margin. We would also like to point out that the RMS scheme shown in Fig. 7 is performed in the sample-by-sample fashion (rather than the typical cycle-by-cycle fashion). Thus, the MSE performance depicted in Fig. 7 is the optimal performance that the RMS scheme can achieve.

B. Voltage Sags

Voltage sags are another type of PQ events. Fig. 8 illustrates a voltage sag event at bus9 due to a temporary ground fault. We can evaluate its temporal-frequency plot using STFT and its spectral estimates using the MUSIC scheme over multiple windows. Being similar to Figs. 2 and 3, it is difficult to detect the event occurrence using these two schemes. Due to space limitations, we will not show the corresponding plots here.

Figs. 9 and 10 show the sample-by-sample rms generated by the rms scheme and the logarithm of the weighted likelihood ratio generated by the CUSUM scheme. Being similar to Figs. 4 and 5, the occurrence of the sag event is easier by observing the waveforms shown in Figs. 9 and 10.

Finally, Fig. 11 shows the MSE performance of the proposed CUSUM and the conventional RMS schemes as a function of SNR. Inspection of Fig. 11 reveals that the CUSUM scheme outperforms the RMS scheme by a significant margin.

VI. Conclusion and Future Work

A change-point detection approach to PQ event detection in smart grids was examined in this research. The proposed CUSUM-based scheme computes the logarithm of the weighted likelihood ratio by exploiting both the instantaneous and the long-term information of the power waveform. The superior performance of the proposed CUSUM-based schemes in the presence of PQ voltage transient and sag events was shown by computer simulation.

There are several extensions of this study that can be further explored. First of all, rather than exhaustively searching for the optimal threshold h as shown in Fig. 6, it will be of great practical interest to analytically derive the optimal threshold. Second, the impact of the event duration and the presence of multiple closely located PQ events on the performance of the proposed approach deserves further investigation.

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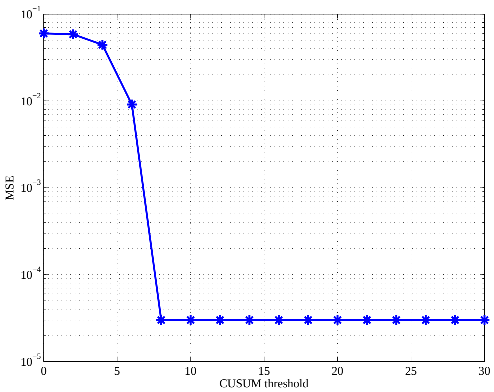

Fig. 6. The MSE versus the CUSUM threshold in a transient event ($SNR =$ 20 dB).

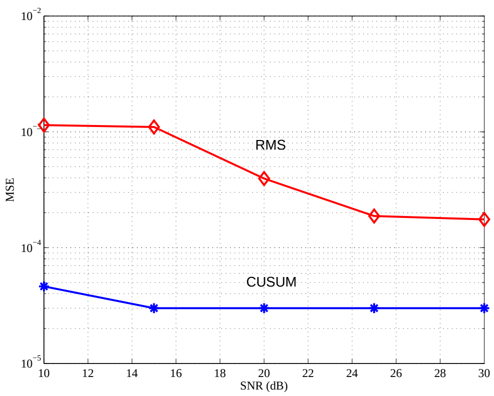

Fig. 7. The MSE as a function of the SNR value for CUSUM and RMS in a transient event.

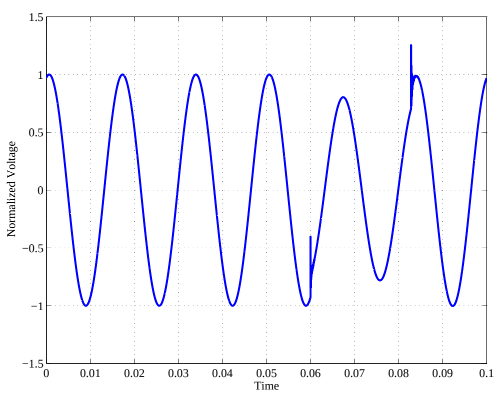

Fig. 8. Illustration of a voltage sag event.

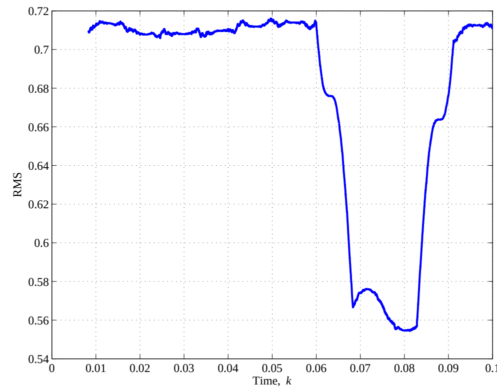

Fig. 9. Sample-by-sample RMS of a sag event as a function of time ($SNR =$ 20 dB).

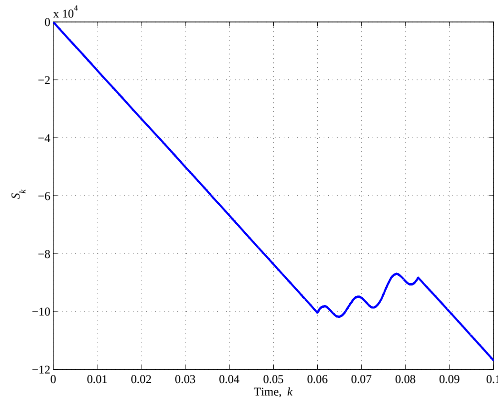

Fig. 10. The logarithm of the weighted likelihood ratio of a sag event using CUSUM ($SNR = 20$ dB).

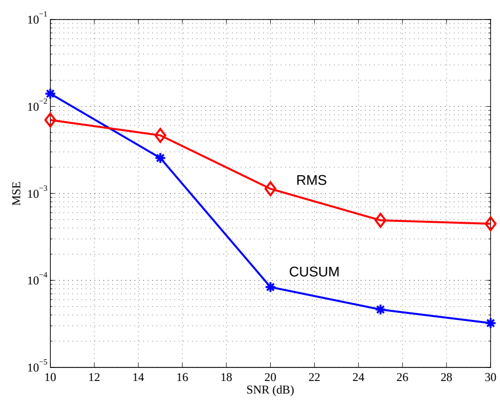

Fig. 11. MSE performance as a function of SNR for CUSUM and RMS in a sag event.

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