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Abstract

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***Index Terms*—Regenerative relaying, cooperative communication, imperfect channel knowledge, nonlinear receivers, log-likelihood ratio.**

EDICS: SPC-DETC, SPC-PERF, SEN-COLB, WIN-PHYL

I. Introduction

Cooperative wireless relaying ideas have become increasingly attractive for their ability to provide distributed spatial diversity [4], [5], [6], reduced transmission power requirements [7], [8], [9], extended coverage [10], and overall capacity improvement [11]. During the past few years, a large number of results are being reported on the receiver performance of various relaying protocols. Using an amplify-and-forward (AF) protocol [12] and with perfect channel state information (CSI) at the receivers, [13] showed that full diversity of $N+1$ is achievable with N relay nodes and a direct link between the source and the destination. With regenerative relay (RR) signal processing (also termed as demodulate-and-forward (DF) relaying), [1] presents receiver structures at the destination with both perfect CSI as well as channel statistics, whereas [2] analyzes the performance of a sub-optimum receiver with perfect CSI.

Estimation of CSI requires training the relay channel, typically by sending known (pilot) symbols from the source as well as the relays, thereby reducing the network throughput. The channel estimates may become outdated if the variation of the channel over time is high relative to the signaling duration [14]. One way to overcome these issues is to employ non-coherent detection techniques which do not require instantaneous channel knowledge. To this end, [15] studies the performance of orthogonal binary frequency shift-keying (BFSK) and on-off keying (OOK) modulations with maximum-likelihood (ML) non-coherent AF receivers. In particular, [15] shows that BFSK achieves full diversity whereas full diversity is not possible OOK. Using a generalized likelihood ratio test receiver [16] with non-coherent BFSK and a single relay, [17] shows that the average bit error rate (BER) decays asymptotically as $(\log \mathrm{SNR})^2/\mathrm{SNR}^2$, where SNR is the average received signal-to-noise ratio. That is, a diversity order of two is possible *only asymptotically*. Considering both short-term as well as long-term average power constraints, approximate receivers with OOK modulation and non-coherent AF protocol are studied in [18]. Assuming channel coherence at least over the duration of two symbols, differential modulation with AF protocol are studied in [19] and [20] whereas [21] considers both AF and DF protocols. In particular, [19] shows that the asymptotic average BER behaves as $(\log \mathrm{SNR})/\mathrm{SNR}^2$. That is, similar to [15] and [17], full diversity of two is possible only asymptotically. In [3], the authors study the performance of non-coherent BFSK signaling with a DF protocol. In particular, with N relays [3] shows that the achievable diversity order is upper and lower bounded by $(N+3)/2$ and $(N+2)/2$, respectively, for odd values of N whereas it is $(N+2)/2$ for even values of N. That is, with a DF protocol non-coherent signaling loses approximately half of the available diversity order.

In this paper, we study the performance of coherent RR protocols with training-based practical channel estimation schemes. Assuming a single-source and a single-destination with multiple relay nodes, we derive ML receiver structure at the destination on frequency-flat and time-varying Rayleigh fading channels. Using binary modulation at the source, our receiver structure takes into account the effects of channel estimation errors as well as a possible fading decorrelation due to node mobility. We show that an exact analysis of the optimal receiver performance is complicated due to the non-linear nature of the log-likelihood ratio (LLR) contribution from the relays to the destination. As a result, we present an approximate receiver that is simple to implement and derive a closed-form expression for the average probability of error with a single relay node. Without requiring any numerical integration, our analytical framework is valid for an arbitrary number of relays and performances of a class of mismatched receivers can be obtained as special cases of the analysis presented in this paper. With perfect CSI, we show that the proposed receiver subsumes the receivers in [1] and [2] whereas with a minimum mean-square error (MMSE) channel estimation it reduces to the non-coherent DF receiver in [3]. An analogy between the approximate receiver in this study and the non-coherent DF receiver in [3] allows us to conclude that the asymptotic diversity order achieved with imperfect CSI is identical to the one with no CSI.

The rest of this paper is organized as follows. In Section II we introduce our system model and present the ML as well as approximate receivers in Section III. Performance analysis of the proposed receivers is detailed in Section IV. An analogy between coherent RR with imperfect CSI and non-coherent DF with no CSI is made in Section V. Numerical and simulation results are presented in Section VI and Section VII concludes our work.

II. System Model

We consider a cooperative wireless system with N relay nodes assisting the communication from source to destination. We employ

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a two-time-slot cooperation protocol with half-duplex relays (i.e., the relays cannot transmit and receive simultaneously) communicating over orthogonal channels. The cooperation protocol is briefly described as follows: In the first time slot the source broadcasts the information to the relays and the destination. In the second time slot the source remains silent while the relays regenerate the source constellation, after demodulation and re-modulation of the received signal from source, and transmit their information, via orthogonal channels, to the destination. The destination appropriately combines the information received from source and the N relays.

We model the fading coefficients on each link as frequency-flat and slowly varying zero-mean complex Gaussian random variables (CGRVs). These random fading gains are also assumed independent across the source→destination ($S \to D$), source→relay ($S \to R_j$) and relay→destination ($R_j \to D$) links. Specifically, we denote by g_0 the fading gain on $S \to D$ link, g_j the fading gain on $S \to R_j$ link, and h_j the fading gain on $R_j \to D$ link, with second moments $E[|g_0|^2] = \Omega_0$, $E[|g_j|^2] = \Omega_j$ and $E[|h_j|^2] = \Lambda_j$, $j = 1, \ldots, N$. The variances, $\{\Omega_j, j = 0, \ldots, N\}$ and $\{\Lambda_j, j = 1, \ldots, N\}$ capture the average path loss across the links and the geometry of relay network.

In this paper, unlike [3], [1] and [2], we consider channel estimation at the receiver for coherent demodulation. The RR protocol with receiver channel estimation and no channel knowledge at the transmitters is described as follows: The overall communication phase is divided into channel estimation phase (CEP) and data transmission phases (DTP). Except for the transmission of known pilot symbols by the source and the relay nodes, the CEP is identical to the DTP. The pilots from the source enable the channel estimation on $S \to D$ and $S \to R_j$ links, whereas the pilots from the relays enable the channel estimation on $R_j \to D$ links. To minimize the performance degradation due to outdated channel knowledge, the sum of the durations of the channel estimation and data transmission phases should equal to or exceed the coherence time of the channel. We denote by $E_{S,T} = E_{S,\text{pilot}} + E_{S,\text{data}}$ the total transmission energy available for source from which a portion, $E_{S,\text{pilot}}$, is spent on pilot transmissions for channel estimation and the remaining portion, $E_{S,\text{data}}$, is allocated for data transmission. In a similar manner, we denote $E_{R,T}(j) = E_{R,\text{pilot}}(j) + E_{R,\text{data}}(j)$, $j = 1, \ldots, N$, where $E_{R,T}(j)$, $E_{R,\text{pilot}}(j)$ and $E_{R,\text{data}}(j)$ are respectively the total energy, the energy available for pilot transmission and the energy available for data transmission at the jth relay.

We denote by $\widetilde{g}_0$ the channel estimate on $S \to D$ link, and, for $j = 1, \ldots, N$, by $\widetilde{g}_j$ and $\widetilde{h}_j$ the channel estimates on $S \to R_j$ and $R_j \to D$ links, respectively. Ignoring implementation-specific details of practical channel estimation schemes, in this paper we model the channel estimates as a result of linear filtering of received pilots. As a result, the channel estimates are also complex-Gaussian on a complex-Gaussian fading channel with Gaussian noise. In particular, due to Rayleigh fading assumption, we let $E[\widetilde{g}_j] = 0$ and $E[|\widetilde{g}_j|^2] = \widetilde{\Omega}_j$, $j = 0, \ldots, N$ and, for $j = 1, \ldots, N$, $E[\widetilde{h}_j] = 0$ and $E[|\widetilde{h}_j|^2] = \widetilde{\Lambda}_j$. Let ρ_0, $\rho_1(j)$ and $\rho_2(j)$ denote the correlation coefficients between the true and estimated channels on $S \to D$, $S \to R_j$ and $R_j \to D$ links, respectively. For simplicity, we assume ρ_0, $\{\rho_1(j), \rho_2(j)\}_{j=1}^N$ to take real values. With this, we can express the r.vs g_0, g_j and h_j, conditioned on $\widetilde{g}_0$, $\widetilde{g}_j$ and $\widetilde{h}_j$, as [22]

$$g_0 = \sqrt{\frac{\Omega_0}{\widetilde{\Omega}_0}}\rho_0\widetilde{g}_0 + \sqrt{\Omega_0(1-\rho_0^2)}U_0 \tag{1}$$

$$g_j = \sqrt{\frac{\Omega_j}{\widetilde{\Omega}_j}}\rho_1(j)\widetilde{g}_j + \sqrt{\Omega_j(1-\rho_1^2(j))}U_j, \tag{2}$$

$$\text{and} \quad h_j = \sqrt{\frac{\Lambda_j}{\widetilde{\Lambda}_j}}\rho_2(j)\widetilde{h}_j + \sqrt{\Lambda_j(1-\rho_2^2(j))}V_j, \tag{3}$$

where U_j, $j = 0, \ldots, N$ and V_j, $j = 1, \ldots, N$, are independent CGRVs with zero mean and unit variance. More importantly, U_0 is independent of $\widetilde{g}_0$, U_j is independent of $\widetilde{g}_j$, and V_j is independent of $\widetilde{h}_j$. Using an approach similar to [23, Section II-A], (1)-(3) facilitate modeling additive channel estimation errors, pilot-symbol assisted channel estimation and fading decorrelation as a result of using outdated (stale) channel estimates in a unified manner by appropriately computing the correlation coefficients and the second moments of the channel estimates appearing in them. In what follows, we use the model in (1)-(3) to derive the receiver structures and evaluate the resulting performance with imperfect CSI.

Denote by $X = \pm 1$ the modulation alphabet of source, the received signal at the destination during the first time-slot of DTP is

$$Y_0 = g_0 X\sqrt{E_{S,\text{data}}} + \eta_0 \tag{4}$$

whereas at the jth relay it is

$$Y_j = g_j X\sqrt{E_{S,\text{data}}} + \eta_j \quad j = 1, \ldots, N, \tag{5}$$

where η_j, $j = 0, \ldots, N$, is a zero-mean CGRV with $E[|\eta_j|^2] = N_0$. Let $\widehat{X}_j \in \{-1, +1\}$ denotes the demodulated symbol at the jth relay using Y_j and the channel estimate $\widetilde{g}_j$. That is, $\widehat{X}_j = \text{sign}\left(\text{Real}\{Y_j/\widetilde{g}_j\}\right)$, where $\text{Real}\{x\}$ is the real part of x and $\text{sign}(x) = 1$ for $x \geq 0$ and is equal to -1 for $x < 0$. During the second time-slot of DTP, the received signal at the destination from the jth relay is

$$Z_j = h_j \widehat{X}_j \sqrt{E_{R,\text{data}}(j)} + W_j, \quad j = 1, \ldots, N, \tag{6}$$

where W_j is a zero-mean CGRV with $E[|W_j|^2] = N_0$.

III. Receiver Structures

In this section, we derive both optimum and suboptimum receiver structures at the destination based on the received signals in (4) and (6) and the channel estimates in (1) and (3). We write the LLR at the destination as in (7), shown at the top of next page, where

$$\text{LLR}_j\left(\widetilde{h}_j\right) \triangleq \log \frac{(1-P_{e,j})\, e^{\overline{\text{LLR}}_j(\widetilde{h}_j)} + P_{e,j}}{1 - P_{e,j} + P_{e,j} e^{\overline{\text{LLR}}_j(\widetilde{h}_j)}} \tag{8}$$

$$\text{and} \quad \overline{\text{LLR}}_j\left(\widetilde{h}_j\right) = \log \frac{\text{Prob}\left(Z_j \mid \widehat{X}_j = +1, \widetilde{h}_j\right)}{\text{Prob}\left(Z_j \mid \widehat{X}_j = -1, \widetilde{h}_j\right)}. \tag{9}$$

In (7), $P_{e,j}$ is the average BER at R_j which is derived as [24]

$$P_{e,j} = \frac{1-\rho_1(j)}{2} + \frac{\rho_1(j)}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_{R,\text{data}}(j)}{1+\overline{\gamma}_{R,\text{data}}(j)}}\right) \tag{10}$$

where $\overline{\gamma}_{R,\text{data}}(j) = E_{S,\text{data}}\Omega_j/N_0$. Using the log-max approximation of (8), $\log(e^a + e^b) \approx \max(a, b)$, a tight approximation to (7), leading to a suboptimal implementation, is

$$\text{LLR}_{\text{app}} = \text{LLR}_0\left(\widetilde{g}_0\right) + \sum_{j=1}^{N} \widehat{\text{LLR}}_j\left(\widetilde{h}_j\right), \tag{11}$$

where

$$\widehat{\text{LLR}}_j\left(\widetilde{h}_j\right) = \begin{cases} -T_j & \text{if} \quad \overline{\text{LLR}}_j(\widetilde{h}_j) < -T_j \\ \overline{\text{LLR}}_j(\widetilde{h}_j) & \text{if} \quad -T_j \leq \overline{\text{LLR}}_j(\widetilde{h}_j) \leq T_j \\ T_j & \text{if} \quad \overline{\text{LLR}}_j(\widetilde{h}_j) > T_j \end{cases} \tag{12}$$

$$\text{and} \qquad T_j = \log \frac{1-P_{e,j}}{P_{e,j}}. \tag{13}$$

$$\begin{aligned}\mathrm{LLR} &= \log \frac{\mathrm{Prob}\left(X=+1 \mid Y_0, Z_1, \ldots, Z_N, \widetilde{g}_0, \widetilde{h}_1 \ldots, \widetilde{h}_N\right)}{\mathrm{Prob}\left(X=-1 \mid Y_0, Z_1, \ldots, Z_N, \widetilde{g}_0, \widetilde{h}_1 \ldots, \widetilde{h}_N\right)} = \underbrace{\log \frac{\mathrm{Prob}\left(Y_0 \mid X=+1, \widetilde{g}_0\right)}{\mathrm{Prob}\left(Y_0 \mid X=-1, \widetilde{g}_0\right)}}_{\triangleq \mathrm{LLR}_0(\widetilde{g}_0)} + \sum_{j=1}^{N} \underbrace{\log \frac{\mathrm{Prob}\left(Z_j \mid X=+1, \widetilde{h}_j\right)}{\mathrm{Prob}\left(Z_j \mid X=-1, \widetilde{h}_j\right)}}_{\triangleq \mathrm{LLR}_j(\widetilde{h}_j)} \\ &= \mathrm{LLR}_0\left(\widetilde{g}_0\right) + \sum_{j=1}^{N} \log \frac{(1-P_{e,j})\,\mathrm{Prob}\left(Z_j \mid X=+1, \widehat{X}_j=+1, \widetilde{h}_j\right) + P_{e,j}\mathrm{Prob}\left(Z_j \mid X=+1, \widehat{X}_j=-1, \widetilde{h}_j\right)}{(1-P_{e,j})\,\mathrm{Prob}\left(Z_j \mid X=-1, \widehat{X}_j=-1, \widetilde{h}_j\right) + P_{e,j}\mathrm{Prob}\left(Z_j \mid X=-1, \widehat{X}_j=+1, \widetilde{h}_j\right)} \\ &= \mathrm{LLR}_0\left(\widetilde{g}_0\right) + \sum_{j=1}^{N} \mathrm{LLR}_j\left(\widetilde{h}_j\right) \end{aligned} \tag{7}$$

Implementation of (7) or (11) requires the knowledge of relay probability of error at the destination. In the absence of such knowledge, we also study the performance of the following mismatched receiver:

$$\mathrm{LLR}_{\mathrm{mis}} = \mathrm{LLR}_0\left(\widetilde{g}_0\right) + \sum_{j=1}^{N} \overline{\mathrm{LLR}}_j\left(\widetilde{h}_j\right), \tag{14}$$

a special case of (11) with $T_j = \infty$ in (12). Note that both (11) and (14) require $(\rho_0, \Omega_0, \widetilde{\Omega}_0)$ and $\{\rho_2(j), \Lambda_j, \widetilde{\Lambda}_j\}_{j=1}^{N}$ at the destination.

We now simplify the expressions $\mathrm{LLR}_0(\widetilde{g}_0)$ and $\overline{\mathrm{LLR}}_j(\widetilde{h}_j)$, $j = 1, \ldots, N$. Starting with (4) and (1), conditioned on X and $\widetilde{g}_0$, $\mathrm{LLR}_0(\widetilde{g}_0)$ simplifies to

$$\mathrm{LLR}_0(\widetilde{g}_0) = \sqrt{\frac{E_{S,\mathrm{data}}\Omega_0}{\widetilde{\Omega}_0}} \frac{4\rho_0 \mathrm{Real}\left\{\widetilde{g}_0^* Y_0\right\}}{N_0 + E_{S,\mathrm{data}}\Omega_0(1-\rho_0^2)}. \tag{15}$$

In a similar manner, using (6) and (3) while conditioning on $\widehat{X}_j$ and $\widetilde{h}_j$, $\overline{\mathrm{LLR}}_j(\widetilde{h}_j)$ simplifies to

$$\overline{\mathrm{LLR}}_j(\widetilde{h}_j) = \sqrt{\frac{E_{R,\mathrm{data}}(j)\Lambda_j}{\widetilde{\Lambda}_j}} \frac{4\rho_2(j)\mathrm{Real}\left\{\widetilde{h}_j^* Z_j\right\}}{N_0 + E_{R,\mathrm{data}}(j)\Lambda_j(1-\rho_2^2(j))}. \tag{16}$$

Conditioned on $\widetilde{g}_0$ and X, it is straightforward to show that $\mathrm{LLR}_0(\widetilde{g}_0)$ in (15) is a real Gaussian r.v (RGRV) with mean $4X\gamma_{\mathrm{eff},D}(0)$ and variance $8\gamma_{\mathrm{eff},D}(0)$, where

$$\gamma_{\mathrm{eff},D}(0) = \frac{E_{S,\mathrm{data}}\Omega_0\rho_0^2}{N_0 + E_{S,\mathrm{data}}\Omega_0(1-\rho_0^2)} \times \frac{|\widetilde{g}_0|^2}{\widetilde{\Omega}_0} \tag{17}$$

is the instantaneous received SNR on $S \to D$ link. We note that $\gamma_{\mathrm{eff},D}(0)$ in (17) is exponentially distributed with mean $\overline{\gamma}_{\mathrm{eff},D}(0) = E_{S,\mathrm{data}}\Omega_0\rho_0^2/(N_0 + E_{S,\mathrm{data}}\Omega_0(1-\rho_0^2))$. Following the same lines, conditioned on $\widehat{X}_j$ and $\widetilde{h}_j$, $\overline{\mathrm{LLR}}_j(\widetilde{h}_j)$ is also a RGRV with mean $4\widehat{X}_j\gamma_{\mathrm{eff},D}(j)$ and variance $8\gamma_{\mathrm{eff},D}(j)$, where

$$\gamma_{\mathrm{eff},D}(j) = \frac{E_{R,\mathrm{data}}(j)\Lambda_j\rho_2^2(j)}{N_0 + E_{R,\mathrm{data}}(j)\Lambda_j(1-\rho_2^2(j))} \times \frac{|\widetilde{h}_j|^2}{\widetilde{\Lambda}_j} \tag{18}$$

is the instantaneous received SNR on $R_j \to D$ link which is exponentially distributed with mean $\overline{\gamma}_{\mathrm{eff},D}(j) = E_{R,\mathrm{data}}(j)\Lambda_j\rho_2^2(j)/(N_0 + E_{R,\mathrm{data}}(j)\Lambda_j(1-\rho_2^2(j)))$.

IV. Performance Analysis

Since the input constellation is symmetric, without loss of generality, we assume that $X = +1$ is the transmitted signal point. The probability of error for the approximate receiver is

$$\begin{aligned} P_{e,\mathrm{app}} &= \mathrm{Prob}\left(\mathrm{LLR}_{\mathrm{app}} < 0 \mid X = +1\right) \\ &= \frac{1}{2\pi j} \int_{\Re\{s\}>0} \frac{ds}{s} \mathcal{L}_{\mathrm{LLR}_{\mathrm{app}} \mid X=+1}(s), \end{aligned} \tag{19}$$

whereas for the mismatched receiver it is

$$P_{e,\mathrm{mis}} = \frac{1}{2\pi j} \int_{\Re\{s\}>0} \frac{ds}{s} \mathcal{L}_{\mathrm{LLR}_{\mathrm{mis}} \mid X=+1}(s). \tag{20}$$

In (19) and (20) $\mathcal{L}_Z(s) = E[e^{-sZ}]$ is the Laplace transform (LT) of the probability density function (pdf) of Z.

We recall from Section III that, conditioned on X, $\mathrm{LLR}_0(\widetilde{g}_0)$ and $\widehat{\mathrm{LLR}}_j(\widetilde{h}_j)$, $j = 1, \ldots, N$, are independent r.vs appearing in the approximate receiver of (11) whereas $\mathrm{LLR}_0(\widetilde{g}_0)$ and $\overline{\mathrm{LLR}}_j(\widetilde{h}_j)$, $j = 1, \ldots, N$, are independent r.vs appearing in the mismatched receiver of (14). It then follows that

$$\mathcal{L}_{\mathrm{LLR}_{\mathrm{app}} \mid X=+1}(s) = \mathcal{L}_{\mathrm{LLR}_0 \mid X=+1}(s) \prod_{j=1}^{N} \mathcal{L}_{\widehat{\mathrm{LLR}}_j \mid X=+1}(s) \tag{21}$$

and

$$\mathcal{L}_{\mathrm{LLR}_{\mathrm{mis}} \mid X=+1}(s) = \mathcal{L}_{\mathrm{LLR}_0 \mid X=+1}(s) \prod_{j=1}^{N} \mathcal{L}_{\overline{\mathrm{LLR}}_j \mid X=+1}(s) \tag{22}$$

where, for brevity, in (21) and (22) we have suppressed the dependence of LLR_0 on $\widetilde{g}_0$, and $\widehat{\mathrm{LLR}}_j$ and $\overline{\mathrm{LLR}}_j$ on $\widetilde{h}_j$. Since $\mathrm{LLR}_0(\widetilde{g}_0)$, conditioned on X and $\widetilde{g}_0$, is a RGRV with mean $4\gamma_{\mathrm{eff},D}(0)$ and variance $8\gamma_{\mathrm{eff},D}(0)$, with the help of [24, App. C], we have

$$\mathcal{L}_{\mathrm{LLR}_0 \mid X=+1}(s) = \mathcal{L}_Z\left(s; 4, 8, \overline{\gamma}_{\mathrm{eff},D}(0)\right), \tag{23}$$

where

$$\mathcal{L}_Z\left(s; a, b, c\right) = \frac{1}{1 + cs\left(a - \frac{sb}{2}\right)}. \tag{24}$$

In a similar manner, upon averaging over $\widehat{X}_j$, we arrive at

$$\begin{aligned} &\mathcal{L}_{\widehat{\mathrm{LLR}}_j \mid X=+1}(s) = (1-P_{e,j})\,\mathcal{L}_W\left(s; 4, 8, \overline{\gamma}_{\mathrm{eff},D}(j), -T_j, T_j\right) \\ &+P_{e,j}\mathcal{L}_W\left(s; -4, 8, \overline{\gamma}_{\mathrm{eff},D}(j), -T_j, T_j\right) \end{aligned} \tag{25}$$

$$\begin{aligned} &\text{and } \mathcal{L}_{\overline{\mathrm{LLR}}_j \mid X=+1}(s) = (1-P_{e,j})\,\mathcal{L}_Z\left(s; 4, 8, \overline{\gamma}_{\mathrm{eff},D}(j)\right) \\ &+P_{e,j}\mathcal{L}_Z\left(s; -4, 8, \overline{\gamma}_{\mathrm{eff},D}(j)\right). \end{aligned} \tag{26}$$

In (25)

$$\begin{aligned} &\mathcal{L}_W\left(s; a, b, c, d, e\right) = e^{-sd}\Xi(d; a, b, c) + e^{-se}\left(1 - \Xi(e; a, b, c)\right) + \\ &\frac{1 - e^{d\left(\frac{a}{b} + \frac{1}{b}\sqrt{\frac{2b+a^2c}{c}} - s\right)}}{\frac{a}{b} + \frac{1}{b}\sqrt{\frac{2b+a^2c}{c}} - s} + \frac{e^{e\left(\frac{a}{b} - \frac{1}{b}\sqrt{\frac{2b+a^2c}{c}} - s\right)} - 1}{\frac{a}{b} - \frac{1}{b}\sqrt{\frac{2b+a^2c}{c}} - s} \end{aligned} \tag{27}$$

and $\Xi\left(z; a, b, c\right)$ in (27) is defined as

$$\Xi\left(z; a, b, c\right) = \begin{cases} \frac{1}{2}\left(1 - \sqrt{\frac{a^2c}{2b+a^2c}}\right) e^{\frac{az}{b} + \frac{z}{b}\sqrt{\frac{2b+a^2c}{c}}} & z \le 0 \\ 1 - \frac{1}{2}\left(1 + \sqrt{\frac{a^2c}{2b+a^2c}}\right) e^{\frac{az}{b} - \frac{z}{b}\sqrt{\frac{2b+a^2c}{c}}} & z \ge 0 \end{cases}. \tag{28}$$

In [24], we show that (25) reduces to (26) as $T_j \to \infty$. Upon substituting (23) and (25) in (21), and (23) and (26) in (22), we arrive at

$$\mathcal{L}_{\text{LLR}_{\text{app}}\,|X=+1}(s) = \mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right) \times \prod_{j=1}^{N} \left\{(1-P_{e,j})\,\mathcal{L}_W\left(s;4,8,\overline{\gamma}_{\text{eff},D}(j),-T_j,T_j\right) + P_{e,j}\mathcal{L}_W\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(j),-T_j,T_j\right)\right\} \quad (29)$$

and
$$\mathcal{L}_{\text{LLR}_{\text{mis}}\,|X=+1}(s) = \mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right) \times \prod_{j=1}^{N} \left\{(1-P_{e,j})\,\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(j)\right) + P_{e,j}\mathcal{L}_Z\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(j)\right)\right\}. \quad (30)$$

From (24) and (27), we note that all the poles appearing in (29) and (30) are real. As a result, by invoking Cauchy's residue theorem [25], (19) and (20) can be expressed, in non-integral form, as

$$P_{e,\mathsf{S}} = -\sum_{n=1}^{K} \text{Residue}\left(\frac{\mathcal{L}_{\text{LLR}_{\mathsf{S}}\,|X=+1}(s)}{s}, s_n > 0\right), \quad (31)$$

where $\mathsf{S} \in \{\text{app}, \text{mis}\}$ and $s_n > 0$ denotes the nth of the K poles on the positive real axis. For an arbitrary number of relay nodes, average BER of sub-optimum and mismatched receivers can be numerically obtained by using (29) and (30), respectively, in (31) at the locations of poles given in Table I.

With one relay node, we now present a closed-form solution to $P_{e,\text{app}}$. With $N=1$, $\mathcal{L}_{\text{LLR}_{\text{app}}\,|X=+1}(s)$ in (29) and $\mathcal{L}_{\text{LLR}_{\text{mis}}\,|X=+1}(s)$ in (30) are given by (32) and (33), shown at the top of the next page. Upon using (32) in (31), and after some algebra, the average BER of approximate receiver with one relay simplifies to (34), shown in the next page, where

$$\alpha_0 = \frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(0)}{\overline{\gamma}_{\text{eff},D}(0)}} - 1\right) \qquad \beta_0 = 1+\alpha_0 \quad (35)$$

$$\alpha_1 = \frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(1)}{\overline{\gamma}_{\text{eff},D}(1)}} - 1\right) \qquad \beta_1 = 1+\alpha_1 \quad (36)$$

$$\Phi(\alpha,\beta,T) = \left(1 - \frac{\alpha e^{-\beta T}}{\alpha+\beta}\right)1_{T\ge 0} + \frac{\beta e^{\alpha T}}{\alpha+\beta}1_{T<0} \quad (37)$$

$$\Psi(\alpha_0,\beta_0,\alpha_1,\beta_1,T) = 1 - \frac{(\alpha_0\alpha_1)\left(\frac{1-e^{(\alpha_1+\beta_0)T}}{1-e^{\alpha_1 T}}\right)}{(\alpha_0+\beta_0)(\alpha_1+\beta_0)} \quad (38)$$

$$\Upsilon(\alpha_0,\beta_0,\alpha_1,\beta_1,T) = \frac{\beta_0\beta_1\left(1-e^{-(\alpha_0+\beta_1)T}\right)}{(\alpha_0+\beta_0)(\alpha_0+\beta_1)\left(1-e^{-\beta_1 T}\right)} \quad (39)$$

and $\mathbf{1}_{\mathsf{A}}$ in (37) is the indicator function that evaluates to 1 when A is true and evaluates to 0 otherwise. The average BER of the mismatched receiver is obtained as a special case of the average BER of the approximate receiver in (34) by taking the limit as $T_1 \to \infty$.

We are also interested in the asymptotic behavior of (34) at high average SNRs on $S \to D$, $S \to R_1$ and $R_1 \to D$ links. To proceed forward, we introduce the positive scaling parameters $\mathtt{t}_D(0)$, $\mathtt{t}_D(1)$ and $\mathtt{t}_R(1)$ such that $\overline{\gamma}_{\text{eff},D}(0) = \mathtt{t}_D(0)\overline{\Gamma}$, $\overline{\gamma}_{\text{eff},D}(1) = \mathtt{t}_D(1)\overline{\Gamma}$ and $\overline{\gamma}_{\text{eff},R}(1) = \mathtt{t}_R(1)\overline{\Gamma}$, where $\overline{\Gamma}$ is the common SNR. By letting $\overline{\Gamma} \to \infty$ we let $\overline{\gamma}_{\text{eff},D}(0) \to \infty$, $\overline{\gamma}_{\text{eff},D}(1) \to \infty$ and $\overline{\gamma}_{\text{eff},R}(1) \to \infty$ simultaneously. Using these in (35)-(36), (10) and (13), as $\overline{\Gamma} \to \infty$, we have $\alpha_0 \approx 1/(4\mathtt{t}_D(0)\overline{\Gamma})$, $\beta_0 \approx 1/(4\mathtt{t}_D(0)\overline{\Gamma})$, $\alpha_1 \approx 1/(4\mathtt{t}_D(1)\overline{\Gamma})$, $\beta_1 \approx 1/(4\mathtt{t}_D(1)\overline{\Gamma})$, and $T_1 = \log(\sqrt{1+\overline{\gamma}_{\text{eff},R}(1)} + \sqrt{\overline{\gamma}_{\text{eff},R}(1)}) - \log(\sqrt{1+\overline{\gamma}_{\text{eff},R}(1)} - \sqrt{\overline{\gamma}_{\text{eff},R}(1)}) = 2\log(\sqrt{1+\overline{\gamma}_{\text{eff},R}(1)} + \sqrt{\overline{\gamma}_{\text{eff},R}(1)}) \approx \log\overline{\Gamma} + \log \mathtt{t}_R(1)$. Using these approximations, and after some algebra, (34) as $\overline{\Gamma} \to \infty$ simplifies to

$$P_e \approx \frac{f_1\left(\mathtt{t}_D(0),\mathtt{t}_D(1),\mathtt{t}_R(1)\right) + f_2\left(\mathtt{t}_D(0),\mathtt{t}_D(1),\mathtt{t}_R(1)\right)\log\overline{\Gamma}}{\overline{\Gamma}^2}, \quad (40)$$

where $f_1\left(\mathtt{t}_D(0),\mathtt{t}_D(1),\mathtt{t}_R(1)\right) = 3/(16\mathtt{t}_D(0)\mathtt{t}_D(1)) + 1/(8\mathtt{t}_D(0)\mathtt{t}_R(1)) + \log\mathtt{t}_R(1)/(4\mathtt{t}_D(0)\mathtt{t}_R(1))$ and $f_2\left(\mathtt{t}_D(0),\mathtt{t}_D(1),\mathtt{t}_R(1)\right) = 1/(4\mathtt{t}_D(0)\mathtt{t}_R(1))$. From (40), due to the presence of $\log\overline{\Gamma}$, we conclude that full diversity of order 2 is not possible with coherent RR protocol.

TABLE I
POLES ON THE POSITIVE REAL AXIS OF FUNCTIONS IN (29) AND (30).

Function	Locations of poles $s_n > 0$
$\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right)$	$\frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(0)}{\overline{\gamma}_{\text{eff},D}(0)}}+1\right)$
$\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(j)\right)$	$\frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(j)}{\overline{\gamma}_{\text{eff},D}(j)}}+1\right)$
$\mathcal{L}_Z\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(j)\right)$	$\frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(j)}{\overline{\gamma}_{\text{eff},D}(j)}}-1\right)$
$\mathcal{L}_W\left(s;4,8,\overline{\gamma}_{\text{eff},D}(j),-T_j,T_j\right)$	$\frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(j)}{\overline{\gamma}_{\text{eff},D}(j)}}+1\right)$
$\mathcal{L}_W\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(j),-T_j,T_j\right)$	$\frac{1}{2}\left(\sqrt{\frac{1+\overline{\gamma}_{\text{eff},D}(j)}{\overline{\gamma}_{\text{eff},D}(j)}}-1\right)$

V. CONNECTIONS WITH NON-COHERENT DF RECEIVER IN [3]

In this section, we show that there is a close connection between the coherent RR system with imperfect CSI studied in this paper and the non-coherent DF receiver considered by Chen and Laneman in [3]. With MMSE channel estimation at the relay nodes, the correlation coefficient between the true and the estimated channel at the jth relay is given by [23, Section V] $\rho_1(j) = \sqrt{\overline{\gamma}_{R,\text{pilot}}(j)/(1+\overline{\gamma}_{R,\text{pilot}}(j))}$ where $\overline{\gamma}_{R,\text{pilot}}(j) = E_{S,\text{pilot}}\Omega_j/N_0$ is the average received pilot SNR. The effective SNR at the jth relay is then

$$\overline{\gamma}_{\text{eff},R}(j) = \frac{\overline{\gamma}_{R,\text{data}}(j)\rho_1^2(j)}{1+\overline{\gamma}_{R,\text{data}}(j)\left(1-\rho_1^2(j)\right)} = \frac{1}{4}\times\frac{\overline{\gamma}_{R,T}^2(j)}{1+\overline{\gamma}_{R,T}(j)} \quad (41)$$

with $E_{S,\text{pilot}} = E_{S,\text{data}} = E_{S,T}/2$, where $\overline{\gamma}_{R,\text{data}}(j) = E_{S,\text{data}}\Omega_j/N_0$ and $\overline{\gamma}_{R,T}(j) = E_{S,T}\Omega_j/N_0$. Upon substituting (41) in (10), we obtain

$$P_{e,j} = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}_{R,T}^2(j)}{4+4\overline{\gamma}_{R,T}(j)+\overline{\gamma}_{R,T}^2(j)}}\right) = \frac{1}{2+\overline{\gamma}_{R,T}(j)}, \quad (42)$$

which is the average BER with binary orthogonal modulation and non-coherent detection [26].

Next, with MMSE channel estimation on $S \to D$ and $R_j \to D$ links, we have $\rho_0 = \sqrt{\overline{\gamma}_{S,\text{pilot}}(0)/(1+\overline{\gamma}_{S,\text{pilot}}(0))}$ and $\rho_2(j) = \sqrt{\overline{\gamma}_{R,\text{pilot}}(j)/(1+\overline{\gamma}_{R,\text{pilot}}(j))}$, where $\overline{\gamma}_{S,\text{pilot}}(0) = E_{S,\text{pilot}}\Omega_0/N_0$ and $\overline{\gamma}_{R,\text{pilot}}(j) = E_{R,\text{pilot}}(j)\Lambda_j/N_0$. Using these with $E_{R,\text{pilot}}(j) = E_{R,\text{data}}(j) = \frac{E_{R,T}(j)}{2}$ and $E_{S,\text{pilot}} = E_{S,\text{data}} = \frac{E_{S,T}}{2}$, the pdfs of the LLRs on the $S \to D$ and $R_j \to D$ links simplify to

$$f_{\text{LLR}_0\,|X}(z) = \frac{\left(1+\overline{\gamma}_{D,T}(0)\right)e^{\frac{Xz}{2}-\frac{|z|\left(2+\overline{\gamma}_{D,T}(0)\right)}{2\overline{\gamma}_{D,T}(0)}}}{\overline{\gamma}_{D,T}(0)\left(2+\overline{\gamma}_{D,T}(0)\right)} \quad (43)$$

and
$$f_{\overline{\text{LLR}}_j\,|\widehat{X}_j}(z) = \frac{\left(1+\overline{\gamma}_{D,T}(j)\right)e^{\frac{\widehat{X}_j z}{2}-\frac{|z|\left(2+\overline{\gamma}_{D,T}(j)\right)}{2\overline{\gamma}_{D,T}(j)}}}{\overline{\gamma}_{D,T}(j)\left(2+\overline{\gamma}_{D,T}(j)\right)}, \quad (44)$$

$$\mathcal{L}_{\text{LLR}_{\text{app}}|X=+1}(s) = (1-P_{e,1})\,\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right)\mathcal{L}_W\left(s;4,8,\overline{\gamma}_{\text{eff},D}(1),-T_1,T_1\right)+P_{e,1}\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right)\mathcal{L}_W\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(1),-T_1,T_1\right) \quad (32)$$

$$\mathcal{L}_{\text{LLR}_{\text{mis}}|X=+1}(s) = (1-P_{e,1})\,\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right)\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(1)\right)+P_{e,1}\mathcal{L}_Z\left(s;4,8,\overline{\gamma}_{\text{eff},D}(0)\right)\mathcal{L}_Z\left(s;-4,8,\overline{\gamma}_{\text{eff},D}(1)\right). \quad (33)$$

$$\begin{aligned}P_{e,\text{app}} \;=\;& (1-P_{e,1})\Phi\left(\beta_0,\alpha_0,T_1\right)\Phi\left(\beta_1,\alpha_1,-T_1\right)+P_{e,1}\Phi\left(\beta_0,\alpha_0,T_1\right)\Phi\left(\alpha_1,\beta_1,-T_1\right)+\\ & (1-P_{e,1})\Phi\left(\beta_0,\alpha_0,-T_1\right)\left\{1-\Phi\left(\beta_1,\alpha_1,T_1\right)\right\}+P_{e,1}\Phi\left(\beta_0,\alpha_0,-T_1\right)\left\{1-\Phi\left(\alpha_1,\beta_1,T_1\right)\right\}+\\ & (1-P_{e,1})\Psi\left(\beta_0,\alpha_0,\beta_1,\alpha_1,-T_1\right)\left\{\Phi\left(\beta_1,\alpha_1,0\right)-\Phi\left(\beta_1,\alpha_1,-T_1\right)\right\}+P_{e,1}\Psi\left(\beta_0,\alpha_0,\alpha_1,\beta_1,-T_1\right)\left\{\Phi\left(\alpha_1,\beta_1,0\right)-\Phi\left(\alpha_1,\beta_1,-T_1\right)\right\}+\\ & (1-P_{e,1})\Upsilon\left(\beta_0,\alpha_0,\beta_1,\alpha_1,T_1\right)\left\{\Phi\left(\beta_1,\alpha_1,T_1\right)-\Phi\left(\beta_1,\alpha_1,0\right)\right\}+P_{e,1}\Upsilon\left(\beta_0,\alpha_0,\alpha_1,\beta_1,T_1\right)\left\{\Phi\left(\alpha_1,\beta_1,T_1\right)-\Phi\left(\alpha_1,\beta_1,0\right)\right\},\end{aligned} \quad (34)$$

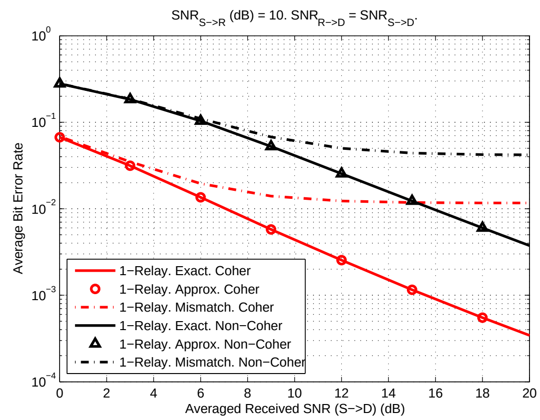
respectively. In (43) and (44), $\overline{\gamma}_{D,T}(0) = E_{S,T}\Omega_0/N_0$ and $\overline{\gamma}_{D,T}(j) = E_{R,T}(j)\Lambda_j/N_0$. It is interesting to know that the LLR pdfs in (43) and (44) are identical to the pdfs of the non-coherent detector output test statistics $\mathtt{t}_0$ and $\mathtt{t}_1$, respectively, derived in [3].

Finally, upon using $P_{e,1} = 1/\left(2+\overline{\gamma}_{R,T}(1)\right)$, $T_1 = \log\left(1+\overline{\gamma}_{R,T}(1)\right)$, $\alpha_0 = 1/\overline{\gamma}_{D,T}(0)$, $\beta_0 = 1+1/\overline{\gamma}_{D,T}(0)$, $\alpha_1 = 1/\overline{\gamma}_{D,T}(1)$, and $\beta_1 = 1+1/\overline{\gamma}_{D,T}(1)$ in (34), the resulting expression coincides with the average BER of binary orthogonal signaling with non-coherent DF relaying in [3, Eqns. (14) and (15)]. With N relays, [3] shows that the diversity order is lower and upper bounded by $(N+2)/2$ and $(N+3)/2$, respectively, for an odd value of N whereas it is $(N+2)/2$ for an even value of N. Since our coherent RR system with imperfect CSI is different from the non-coherent DF in [3] only in terms of the average effective SNRs, and as far as the diversity order analysis is considered the average effective SNRs in both the systems differ only up to multiplicative constants, it follows immediately that the diversity order of coherent RR system with imperfect CSI is identical to that of [3].

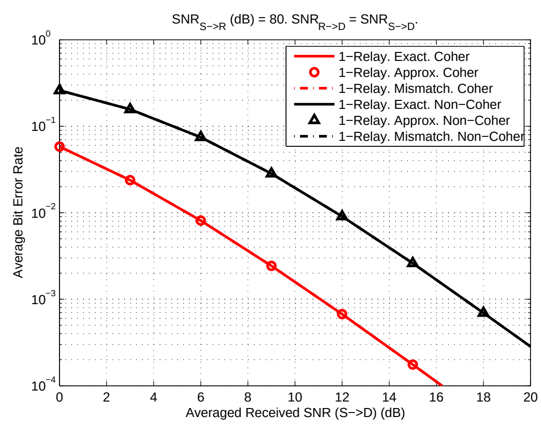
VI. Results and Discussion

In this section, we present some numerical and simulation results on the performance of regenerative relays with imperfect channel knowledge. For simplicity, we restrict our results to the case of a single relay node. Figs. 1(a) and 1(b) show the average BER performances at the destination as a function of the total average SNR on the $S \rightarrow D$ link, $\overline{\gamma}_{D,T}(0)$. In Fig. 1(a) we set $\overline{\gamma}_{R,T}(1) = 10$ dB whereas it is set to 80 dB (i.e., almost error-free $S \rightarrow R$ links) in Fig. 1(b). In both Figs. 1(a) and 1(b), we set $\overline{\gamma}_{D,T}(1) = \overline{\gamma}_{D,T}(0)$. The average BER of coherent RR system with perfect channel knowledge is compared against the performance of a non-coherent RR system with no channel knowledge. In Figs. 1(a) and 1(b), the exact performance curves are obtained by Monte-Carlo simulation of the receiver in (7) (by simulating 40 million bits per SNR point), whereas the approximate and mismatched receiver performances are obtained via the analytical results derived in Section IV. From the excellent match between the simulated exact performance and analytical approximate performance in Figs. 1(a) and 1(b) we conclude that there is *almost* no loss of performance incurred by the receiver in (11). Figs. 1(a) and 1(b) also show the impact of relay probability of error on the overall error performance of approximate and mismatched receivers. The positive impact of LLR clipping of approximate receiver is conspicuous in Fig. 1(a), where, at $\overline{\gamma}_{R,T}(1) = 10$, the mismatched receiver suffers from severe error floor. On the other hand, as the average received SNR on $S \rightarrow R$ link improves, there is less need to clip the LLRs from the relays, and the mismatched receiver yields performance close to the approximate receiver.

The average BER as a function of the normalized squared correlation coefficient on $S \rightarrow D$ link, ρ_0^2, is plotted in Fig. 2 for various values of $\rho_1^2(1)$ and $\rho_2^2(1)$ on $S \rightarrow R_1$ and $R_1 \rightarrow D$ links,



(a) One relay with $\overline{\gamma}_{R,T} = 10$ dB



(b) One relay with $\overline{\gamma}_{R,T} = 80$ dB

Fig. 1. Average BER as a function of the total average SNR $\overline{\gamma}_{D,T}(0) = E_{S,T}\Omega_0/N_0$ on $S \rightarrow D$ link. Regenerative receivers with perfect and no CSI are compared.

respectively. In Fig. 2, we set $\rho_1^2(1) = \rho_2^2(1)$, $E_{S,\text{data}}\Omega_0/N_0 = 40$ dB, $E_{S,\text{data}}\Omega_1/N_0 = 20$ dB and $E_{R,\text{data}}\Lambda_1/N_0 = 30$ dB. From Fig. 2, we conclude that a five percent degradation in $\rho_1^2(1)$, from perfect CSI to $\rho_1^2(1) = 0.95$, introduces a degradation in average BER by an order of magnitude, whereas for the same percentage change in ρ_0^2 leads to a BER degradation by more than two orders of magnitude.

The impact of mobility on the average BER is investigated in Fig. 3. The source and the relay nodes are assumed to be stationary, the receivers are assumed to have perfect CSI, and Jakes correlation model [14] is employed to model the time variations on the $S \rightarrow D$ and $R \rightarrow D$ links. In Fig. 3, we set $\overline{\gamma}_{R,T}(0) = \overline{\gamma}_{D,T}(1) = 10$ dB. The average BER is shown in Fig. 3 as a function of the

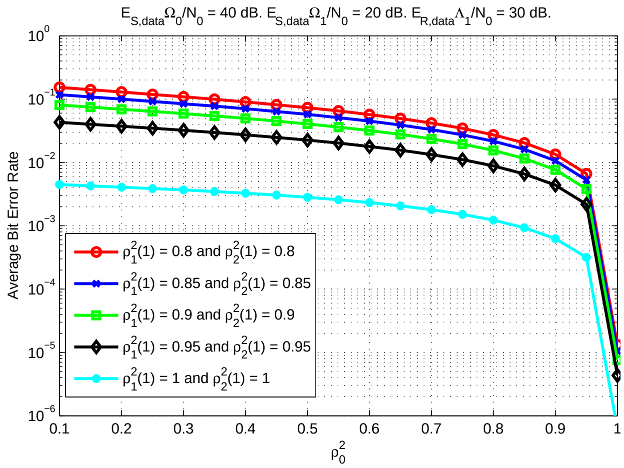


Fig. 2. Average BER as a function of the normalized correlation coefficient, ρ_0, on $S \to D$ link with a single relay node.

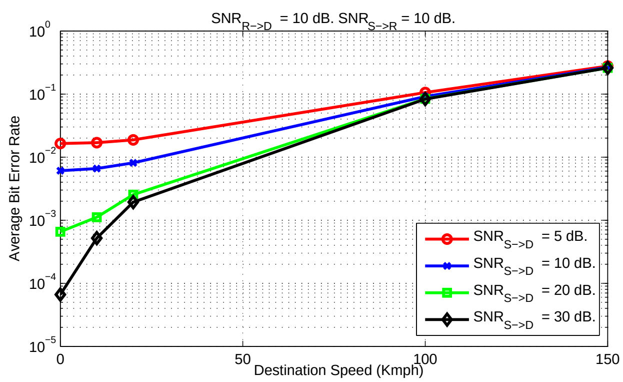


Fig. 3. Impact of mobility on the average BER with a single relay node.

destination speed, parameterized by the average received on $S \to D$ link, $\overline{\gamma}_{D,T}(0)$. From Fig. 3, we conclude that the degradation in average BER is more pronounced at higher values of the average received SNR on $S \to D$ link. For example, with $\overline{\gamma}_{D,T}(0) = 30$ dB, the average BER increases by approximately 1000 folds when the destination speed increases from 1 to 100 Kmph, whereas it degrades by about 10 folds with $\overline{\gamma}_{D,T}(0) = 5$ dB.

VII. Conclusion

In this paper, we studied the performance of coherent regenerative relaying on time-varying Rayleigh fading channels with imperfect CSI. Using a two-hop orthogonal multiple-access protocol with multiple relay nodes, we derived optimum and suboptimum receiver structures with binary modulation. For specific values of correlation coefficients between true and estimated channels, our receiver structures subsume coherent and non-coherent receivers studied in [1], [2] and [3], respectively. For an arbitrary number of relay nodes, employing Laplace transform techniques, we derived analytical expressions for the average BER at the destination. With one relay our closed-form expression for the average BER was shown to subsume the results in [3], whereas with multiple relays the achievable diversity order was shown to be identical to that [3].

References

[1] M. N. Khormuji and E. G. Larsson, "Receiver design for wireless relay channels with regenerative relays," in *Proc. IEEE ICC'07*.

[2] W. Su, "Performance analysis for a suboptimum ML receiver in decode-and-forward communications," in *Proc. IEEE GLOBECOM'07*, pp. 2962-2966, Nov. 2007.

[3] D. Chen and J. N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Communications,* vol. 5, no. 7, pp. 1785-1794, July 2006.

[4] J. Boyer, D. D. Falconer and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.,* vol. 52, no. 10, Oct. 2004, pp. 1820-1830.

[5] J. N. Laneman, D. Tse and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Info. Theory,*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[6] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Comm. Magazine,* vol. 42, no. 10, Oct. 2004, pp. 74-80.

[7] N. Ahmed, M. Khojastepour, and B. Aazhang, "Outage minimization and optimal power control for the fading relay channel," in *Proc. IEEE Info. Theory Workshop*, San Antonio, USA, Oct. 2004.

[8] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Info. Theory,* vol. 51, no. 6, June 2005, pp. 2020-2040.

[9] R. Annavajjala, P. C. Cosman and L. B. Milstein, "Stastical channel knowledge-based optimum power allocation for relaying protocols in the high-SNR regime," *IEEE Journal on Selected Areas in Communications,* vol. 25, no. 2, Feb. 2007.

[10] R. Pabst *et al*, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Comm. Magazine,* vol. 42, no. 9, Sept. 2004, pp. 80-90.

[11] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity-Part I: System description, Part II: Implementation aspects and performance analysis," in *IEEE Transactions on Communications,* vol. 51, no. 11, November 2003, pp. 1927-1948.

[12] S. Berger *et al*, "Recent advances in amplify-and-forward two-hop relaying," *IEEE Communications Magazine,* July 2009, pp. 50-56.

[13] A. Ribeiro, X. Cai and G. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Communications,* vol. 4, no. 8, pp. 1389-1398, Aug. 2003.

[14] W. C. Jakes, *Microwave Mobile Communications,* John Wiley & Sons, Inc., 1974.

[15] R. Annavajjala, P. C. Cosman, L. B. Milstein, "On the performance of optimum noncoherent amplify-and-forward reception for cooperative diversity," in *Proc. IEEE MILCOM'05*, Atlantic City, NJ, Oct. 2005.

[16] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory,* Prentice Hall International, Inc., 1993.

[17] M. R. Souryal, "Non-coherent amplify-and-forward generalized likelihood ratio test receiver," in *Proc. IEEE Globecom 2008.*

[18] Y. Zhu, P-Y. Kam and Y. Xin, "Non-coherent detection for amplify-and-forward relay systems in a Rayleigh fading enviroment," in *Proc. IEEE GLOBECOM'07*, pp. 1658-1662, Nov. 2007.

[19] Q. Zhao and H. Li, "Performance of differential modulation with wireless relays in Rayleigh fading channels," *IEEE Commun. Lett.,* vol. 9, no. 4, pp. 343-345, Apr. 2005.

[20] T. Himsoon, W. Su and K. J. R. Liu, "Differential transmission for amplify-and-forward cooperative communications," *IEEE Signal Processing Letters,* vol. 12, no. 9, pp. 597-600, Sept. 2005.

[21] W. Cho and L. Yang, "Distributed differential schemes for cooperative wireless networks," in *Proc. IEEE ICASSP'06*, vol. IV, pp. 61-64, May 2006.

[22] T. W. Anderson, *An Introduction to Multivariate Statistical Analysis,* Wiley Series in Probability and Statistics, 2003.

[23] R. Annavajjala, P. C. Cosman and L. B. Milstein, "Performance analysis of linear modulation schemes with generalized diversity combining on Rayleigh fading channels with noisy channel estimates," *IEEE Trans. Info. Theory,* vol. 53, no. 12, pp. 4701-4727, Dec. 2007.

[24] R. Annavajjala, "On optimum regenerative relaying with imperfect channel knowledge," *Technical Report*. Available at http://www.box.net/annavajjala.

[25] Wilbur R. LePage, *Complex Variables and the Laplace Transform for Engineers,*, Dover Publications, Feb. 1980.

[26] J. G. Proakis, *Digital Communications,* Fourth Edition, McGraw-Hill, 2001.