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Abstract

Knowledge of the level crossing rates (LCR) and average outage durations (AOD) of the received signal-to-interference-plus-noise ratio (SINR) is very useful in designing and analyzing the communication system performance in a cellular environment with co-channel interference (CCI). In this paper, we study the analytical LCR and the AOD of the received SINR on Rayleigh fading channels with CCI. A closed-form expression for the LCR and the AOD is obtained for the general case of multiple co-channel interferers, traveling at different speeds, with unequal powers and with additive white Gaussian noise. We also specialize the derived results to the case of both interference-limited and noise-limited scenarios.

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***Abstract*—Knowledge of the level crossing rates (LCR) and average outage durations (AOD) of the received signal-to-interference-plus-noise ratio (SINR) is very useful in designing and analyzing the communication system performance in a cellular environment with co-channel interference (CCI). In this paper, we study the analytical LCR and the AOD of the received SINR on Rayleigh fading channels with CCI. A closed-form expression for the LCR and the AOD is obtained for the general case of multiple co-channel interferers, traveling at different speeds, with unequal powers and with additive white Gaussian noise. We also specialize the derived results to the case of both interference-limited and noise-limited scenarios.**

***Index Terms*—Level crossing problems, average outage duration, signal-to-interference-plus-noise ratio, co-channel interference, cellular systems.**

I. Introduction

Time varying multipath propagation environment together with the interference from other users makes the design and analysis of a multiuser mobile radio system a challenging task [1]. Traditionally, outage probability is considered to be a useful measure of a wireless link performance and is being analyzed extensively for a variety of desired user and interfering users' fading statistics [2]-[5]. The outage probability of the signal-to-interference-plus-noise ratio (SINR) is simply the probability that the received SINR process stays below a predefined threshold, and captures the *static* behavior of the mobile multiuser radio link at any given time instance. However, as argued in [6], it is the duration of the time the SINR stays below a threshold that determines the outage in a cellular environment. Based on an asymptotic level crossing analysis, the authors in [6] have obtained an expression for the minimum duration of the outages in an interference limited Rayleigh fading channel.

The level crossing rate (LCR) and the average outage duration (AOD) of the time-varying SINR process are closely related to the statistics of the error bursts [7]. Specifically, the LCR and AOD can be used in selecting optimal packet lengths to minimize the packet error rate and maintain a relatively small packet overhead [8], [9]. In [10], the authors have considered a noise-limited environment (i.e., no multiuser interference) and obtained expressions for the average LCR and the AOD on Nakagami fading channels with various diversity combining schemes. In [11], a characteristic function approach is proposed to obtain the average LCR in a noise-limited scenario which requires an evaluation of a double integral.

The authors in [12] consider an interference-limited environment with $S(t)$ and $I(t)$ denoting the desired signal and the interference powers, respectively. Instead of calculating the LCR of the SINR process $\gamma(t) = S(t)/I(t)$ at a threshold γ_{th}, the authors define a process $X(t) = S(t) - \gamma_{th} I(t)$ and obtain the average zero crossing rate (ZCR) of $X(t)$. With the above defined $X(t)$, and using the characteristic function approach of [11], in [13] the authors obtained the average outage durations in an interference-limited scenario with Ricean fading.

In this contribution, for arbitrarily correlated stationary signal, $S(t)$, and interference-plus-noise, $I(t)$, processes, in Appendix-A we formally prove that the LCR at the level γ_{th} is identical to the ZCR of $S(t) - \gamma_{th} I(t)$. However, this equivalence is not very useful since, in addition to the knowledge of the joint probability density function (PDF) of $S(t)$, $I(t)$, $\dot{S}(t) = \frac{d}{dt}S(t)$ and $\dot{I}(t) = \frac{d}{dt}I(t)$, the ZCR formulation requires evaluation of a triple integral (please see (48) and (49) in Appendix-A). As a result, we directly work with the SINR process as it requires the joint PDF of only $\gamma(t)$ and $\dot{\gamma}(t) = \frac{d}{dt}\gamma(t)$, and study the analytical LCR and the AOD of the received SINR on Rayleigh fading channels with multiple co-channel interferers (CCIs) and additive noise. Our analysis yields a surprisingly simple closed-form expression for the LCR for the general case of multiple co-channel interferers, traveling at different speeds, with unequal powers and with additive white Gaussian noise. We also specialize the derived results to the case of both interference-limited and noise-limited scenarios.

The rest of this paper is organized as follows. In Section II, we present the system model and derive closed-form expressions for the average LCR and the AOD of the SINR process with interference and additive noise on Rayleigh fading channels. A number of special cases of the derived LCR and AOD expressions are presented in Section III. Conclusions are given in Section IV.

II. System Model

We assume a cellular system with a desired user and K interfereing users transmitting their signals to the base station (BS) (i.e., the uplink). However, we note that the present system model can also be applied to the downlink (i.e., from BS to a mobile), where the desired signal will be that of the BS of interest and the interfering signals are due to neighboring cells. Let Ω_0 denote the average received signal power due to the desired user, and Ω_i, $i = 1, 2, \ldots, K$, correspond to the average received power due to i^{th} CCI. Let N denote the total average noise power at the receiver front end. Let $\alpha_0(t)$ and $\alpha_i(t)$ denote the time varying fading channel coefficients on the path from the desired user to the BS, and the path from the i^{th} interferer to the BS, respectively. With this, the instantaneous received SINR, $\gamma(t)$, at the BS is given by

$$\gamma(t) \quad = \quad \frac{\Omega_0 \alpha_0^2(t)}{N + \sum_{j=1}^{K} \Omega_j \alpha_j^2(t)}. \tag{1}$$

In this paper, we assume that the desired signal and the interferers experience Rayleigh fading. Then, without loss of generality, we assume that $\alpha_j(t)$, $j = 0, 1, \ldots, K$, are independent and identically distributed (i.i.d) with probability density function (PDF) $f_{\alpha_j(t)}(x) = 2xe^{-x^2}$, $x \geq 0$.

The average level crossing rate, LCR, of $\gamma(t)$, at a level γ_{th} is defined as [14]

$$\mathrm{LCR}(\gamma_{th}) = \int_{x=0}^{\infty} x f_{\dot{\gamma}(t),\gamma(t)}(x, \gamma_{th}) dx, \tag{2}$$

where $\dot{\gamma}(t) = \frac{d}{dt}\gamma(t)$ is the time derivative of the SINR process $\gamma(t)$ and $f_{\dot{\gamma}(t),\gamma(t)}(\cdot,\cdot)$ is the joint PDF of $\dot{\gamma}(t)$ and $\gamma(t)$.

The average outage duration, AOD, of $\gamma(t)$ below a level γ_{th} is defined as [14]

$$\mathrm{AOD}(\gamma_{th}) = \frac{\mathrm{Prob}(\gamma(t) \leq \gamma_{th})}{\mathrm{LCR}(\gamma_{th})} = \frac{F_{\gamma(t)}(\gamma_{th})}{\mathrm{LCR}(\gamma_{th})}, \tag{3}$$

where $F_{\gamma(t)}(\gamma_{th}) = \mathrm{Prob}(\gamma(t) \leq \gamma_{th})$ is the cumulative distribution function (CDF) of $\gamma(t)$ at time t. The rest of this section is devoted to obtaining $F_{\gamma(t)}(\gamma_{th})$ and $\mathrm{LCR}(\gamma_{th})$ in closed-form.

A. Derivation of $F_{\gamma(t)}(x)$

Dropping the time dependence for notational simplicity, and using (1), we have

$$F_\gamma(x) = \mathrm{Prob}\left(\alpha_0^2 \leq x\left(\frac{N}{\Omega_0} + \sum_{k=1}^{K}\frac{\Omega_k}{\Omega_0}\alpha_k^2\right)\right). \tag{4}$$

Denote by $\Gamma_i = \frac{\Omega_i}{N}$ the average signal-to-noise ratio (SNR) of the i^{th} signal in the absense of interference, and $\Lambda_{i,j} = \frac{\Omega_i}{\Omega_j}$ as the average signal-to-interference ratio (SIR) of i^{th} user due to j^{th} user. We also denote

$$X_0 = \alpha_0^2 \tag{5}$$

$$\text{and} \quad Y_0 = \sum_{k=1}^{K}\frac{\Omega_k}{\Omega_0}\alpha_k^2 = \sum_{k=1}^{K}\frac{\alpha_k^2}{\Lambda_{0,k}}. \tag{6}$$

With the above, (4) can be conveniently written as

$$\begin{aligned} F_\gamma(x) &= \mathrm{Prob}\left(X_0 \leq x\left(\frac{1}{\Gamma_0} + Y_0\right)\right) \\ &= 1 - E\left[\exp\left(-x\left(\frac{1}{\Gamma_0} + Y_0\right)\right)\right] \\ &= 1 - \exp\left(-\frac{x}{\Gamma_0}\right) E\left[\exp\left(-\sum_{k=1}^{K}\frac{x}{\Lambda_{0,k}}\alpha_k^2\right)\right] \\ &= 1 - \exp\left(-\frac{x}{\Gamma_0}\right)\prod_{k=1}^{K}\frac{\Lambda_{0,k}}{\Lambda_{0,k}+x} \end{aligned} \tag{7}$$

where the last step in (7) is due to the fact that α_k^2, $k = 1, \ldots, K$, are i.i.d exponential random variables (r.vs) with unit mean.

B. Derivation of $f_{\dot{\gamma}(t),\gamma(t)}(x, \gamma_{th})$

Starting from (1), the time derivative of $\gamma(t)$ is given by (8), as shown at the top of the next page. Since $\alpha_j(t)$, $j = 0, 1, \ldots, K$, is Rayleigh distributed with unit second moment, the derivative $\dot{\alpha}_j(t)$ is Gaussian distributed [15] with zero mean and variance $\sigma_j^2 = \pi^2 f_{\max,j}^2$, where $f_{\max,j} = v_j \cdot f_c/c$ is the maximum Doppler frequency of the j^{th} mobile, v_j is the corresponding mobile speed, f_c is the carrier frequency, and c is the speed of light.

Note that $\gamma(t)$ in (8) is itself a function of $\alpha_0(t), \alpha_1(t), \ldots, \alpha_K(t)$, according to (1). Conditioned on $\alpha_0(t), \ldots, \alpha_K(t)$, it is easy to show that $\dot{\gamma}(t)$ of (8) is Gaussian distributed with zero-mean and variance

$$\sigma^2(\alpha_0^2, \alpha_1^2, \ldots, \alpha_K^2) = 4\sigma_0^2\frac{\gamma^4(t)}{\alpha_0^4(t)}\left(\frac{\alpha_0^2(t)}{\gamma^2(t)} + \sum_{k=1}^{K}\frac{\alpha_k^2(t)\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}\right), \tag{9}$$

and the conditional PDF of $\dot{\gamma}(t)$ can be written as

$$f_{\dot{\gamma}(t)|\alpha_0^2(t),\ldots,\alpha_K^2(t)}(x|\alpha_0^2, \ldots, \alpha_K^2) = \frac{e^{-\frac{x^2}{2\sigma^2(\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2)}}}{\sqrt{2\pi\sigma^2(\alpha_0^2, \alpha_1^2, \ldots, \alpha_K^2)}}. \tag{10}$$

Our immediate goal is to obtain an expression for $f_{\dot{\gamma}(t),\gamma(t)}(\cdot,\cdot)$ in order to successfully compute the LCR in (2). We proceed as follows: removing the time index and using the following transformation of r.vs

$$\dot{\gamma} = \dot{\gamma} \tag{11}$$

$$u_k = \alpha_k^2, \; k = 1, \ldots, K \tag{12}$$

$$\gamma = \frac{\Omega_0\alpha_0^2}{N + \sum_{k=1}^{K}\Omega_k\alpha_k^2} = \frac{\Omega_0\alpha_0^2}{N + \sum_{k=1}^{K}\Omega_k u_k} \tag{13}$$

and noting that

$$f_{\dot{\gamma},\gamma,u_1,\ldots,u_K}(\cdot,\cdot,\cdot,\ldots,\cdot) = \frac{f_{\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2}(\cdot,\cdot,\cdot,\ldots,\cdot)}{\left|\frac{\partial(\dot{\gamma},\gamma,u_1,u_2,\ldots,u_K)}{\partial(\dot{\gamma},\alpha_0^2,\alpha_1^2,\alpha_2^2,\ldots,\alpha_K^2)}\right|}, \tag{14}$$

where $\frac{\partial(\cdots)}{\partial(\cdots)}$ is the standard Jacobian operation defined as (15), shown in the next page. With (15), (14) can be simplified as (16), shown in the next page. The expression $f_{\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2}(\dot{\gamma}, \alpha_0^2, \alpha_1^2, \ldots, \alpha_K^2)$ can be written as (17), also shown in the next page. With the substitution $\alpha_k^2 = u_k$, $k = 1, \ldots, K$, $\alpha_0^2 = \frac{\gamma}{\Omega_0}(N + \sum_{k=1}^{K}\Omega_k u_k)$, the conditional variance, $\sigma^2(\alpha_0^2, \alpha_1^2, \ldots, \alpha_K^2)$, of (9) becomes (18), as shown in the next page. Using (2), the LCR at a level γ_{th} can be obtained as

$$\begin{aligned} \mathrm{LCR}(\gamma_{th}) &= \int_{x=0}^{\infty} x f_{\dot{\gamma},\gamma}(x, \gamma_{th}) dx = \int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty}\int_{x=0}^{\infty} \times \\ & x f_{\dot{\gamma},\gamma,u_1,\ldots,u_K}(x, \gamma_{th}, u_1, \ldots, u_K) du_1, \ldots u_K dx. \end{aligned} \tag{19}$$

We can express $f_{\dot{\gamma},\gamma,u_1,\ldots,u_K}(x, \gamma_{th}, u_1, \ldots, u_K)$ as (20), shown above. The expression $f_{\dot{\gamma}|\gamma,u_1,\ldots,u_K}(x|\gamma_{th}, u_1, \ldots, u_K)$ is already shown to be Gaussian with zero-mean and variance $\sigma^2(\gamma_{th}, u_1, \ldots, u_K)$ which is given in (18). Using (13), it is easy to show that the conditional PDF $f_{\gamma|u_1,\ldots,u_K}(\gamma_{th}|u_1, \ldots, u_K)$ is given by

$$\begin{aligned} & f_{\gamma|u_1,\ldots,u_K}(\gamma_{th}|u_1, \ldots, u_K) = \\ & \left(\frac{N + \sum_{k=1}^{K}\Omega_k u_k}{\Omega_0}\right) e^{-\frac{\gamma_{th}}{\Omega_0}(N + \sum_{k=1}^{K}\Omega_k u_k)}. \end{aligned} \tag{21}$$

Using (20) and (21) in (19), we can simplify $\mathrm{LCR}(\gamma_{th})$ as (22), shown in the next page. Using (18) in (22), and noting that $\Gamma_k = \frac{\Omega_k}{N}$, $k = 0, 1, \ldots, K$, we obtain

$$\begin{aligned} \mathrm{LCR}(\gamma_{th}) &= \frac{\sigma_0 e^{-\frac{\gamma_{th}}{\Gamma_0}}\sqrt{2\gamma_{th}}}{\sqrt{\Gamma_0\pi}}\int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty} du_1 \ldots du_K \\ & e^{-\sum_{k=1}^{K}u_k\left(1+\frac{\gamma_{th}}{\Lambda_{0,k}}\right)}\sqrt{1 + \sum_{k=1}^{K}u_k\left(\Gamma_k + \frac{\gamma_{th}\Gamma_0\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}\right)}. \end{aligned} \tag{23}$$

$$\dot{\gamma}(t) = \frac{d}{dt}\gamma(t) = \frac{(N+\sum_{k=1}^{K}\Omega_k\alpha_k^2(t))2\Omega_0\alpha_0(t)\dot{\alpha}_0(t) - 2\Omega_0\alpha_0^2(t)\sum_{k=1}^{K}\Omega_k\alpha_k(t)\dot{\alpha}_k(t)}{(N+\sum_{k=1}^{K}\Omega_k\alpha_k^2(t))^2}$$

$$= \frac{2\gamma(t)}{\alpha_0(t)}\dot{\alpha}_0(t) - \frac{2\gamma^2(t)}{\alpha_0^2(t)}\sum_{k=1}^{K}\frac{\alpha_k(t)}{\Lambda_{0,k}}\dot{\alpha}_k(t). \quad (8)$$

$$\frac{\partial(\dot{\gamma},\gamma,u_1,u_2,\ldots,u_K)}{\partial(\dot{\gamma},\alpha_0^2,\alpha_1^2,\alpha_2^2,\ldots,\alpha_K^2)} = \begin{vmatrix} \frac{\partial\dot{\gamma}}{\partial\dot{\gamma}} & \frac{\partial\dot{\gamma}}{\partial\alpha_0^2} & \frac{\partial\dot{\gamma}}{\partial\alpha_1^2} & \cdots & \frac{\partial\dot{\gamma}}{\partial\alpha_K^2} \\ \frac{\partial\gamma}{\partial\dot{\gamma}} & \frac{\partial\gamma}{\partial\alpha_0^2} & \frac{\partial\gamma}{\partial\alpha_1^2} & \cdots & \frac{\partial\gamma}{\partial\alpha_K^2} \\ \frac{\partial u_1}{\partial\dot{\gamma}} & \frac{\partial u_1}{\partial\alpha_0^2} & \frac{\partial u_1}{\partial\alpha_1^2} & \cdots & \frac{\partial u_1}{\partial\alpha_K^2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial u_K}{\partial\dot{\gamma}} & \frac{\partial u_K}{\partial\alpha_0^2} & \frac{\partial u_K}{\partial\alpha_1^2} & \cdots & \frac{\partial u_K}{\partial\alpha_K^2} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \frac{\gamma}{\alpha_0^2} & -\frac{\Omega_1\gamma^2}{\Omega_0\alpha_0^2} & \cdots & -\frac{\Omega_K\gamma^2}{\Omega_0\alpha_0^2} \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{vmatrix} = \frac{\gamma}{\alpha_0^2}. \quad (15)$$

$$f_{\dot{\gamma},\gamma,u_1,\ldots,u_K}(\dot{\gamma},\gamma,u_1,\ldots,u_K) = \frac{\alpha_0^2}{\gamma} f_{\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2}(\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2)\Big|_{\dot{\gamma}=\dot{\gamma},\alpha_k^2=u_k,\forall k=1,\ldots,K,\ \alpha_0^2=\frac{\gamma}{\Omega_0}(N+\sum_{k=1}^{K}\Omega_k u_k)}. \quad (16)$$

$$f_{\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2}(\dot{\gamma},\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2) = f_{\dot{\gamma}|\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2}(\dot{\gamma}|\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2)\prod_{j=0}^{K} f_{\alpha_j^2}(\alpha_j^2) = \frac{e^{-\frac{\dot{\gamma}^2}{2\sigma^2(\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2)}}e^{-\sum_{j=0}^{K}\alpha_j^2}}{\sqrt{2\pi\sigma^2(\alpha_0^2,\alpha_1^2,\ldots,\alpha_K^2)}}. \quad (17)$$

$$\sigma^2(\gamma,u_1,\ldots,u_K) = \frac{4\sigma_0^2\gamma^2\Omega_0^2}{(N+\sum_{k=1}^{K}\Omega_k u_k)^2}\left(\frac{1}{\gamma\Omega_0}\left(N+\sum_{k=1}^{K}\Omega_k u_k\right)+\sum_{k=1}^{K}\frac{u_k\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}\right) = \frac{4\sigma_0^2\gamma\Omega_0}{(N+\sum_{k=1}^{K}\Omega_k u_k)^2}\left(N+\sum_{k=1}^{K}u_k\left(\Omega_k+\frac{\gamma\Omega_0\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}\right)\right). \quad (18)$$

$$f_{\dot{\gamma},\gamma,u_1,\ldots,u_K}(x,\gamma_{th},u_1,\ldots,u_K) = f_{\dot{\gamma}|\gamma,u_1,\ldots,u_K}(x|\gamma_{th},u_1,\ldots,u_K)f_{\gamma,u_1,\ldots,u_K}(\gamma_{th},u_1,\ldots,u_K)$$

$$= f_{\dot{\gamma}|\gamma,u_1,\ldots,u_K}(x|\gamma_{th},u_1,\ldots,u_K)f_{\gamma|u_1,\ldots,u_K}(\gamma_{th}|u_1,\ldots,u_K)\prod_{k=1}^{K}f_{u_k}(u_k)$$

$$= f_{\dot{\gamma}|\gamma,u_1,\ldots,u_K}(x|\gamma_{th},u_1,\ldots,u_K)f_{\gamma|u_1,\ldots,u_K}(\gamma_{th}|u_1,\ldots,u_K)\exp\left(-\sum_{k=1}^{K}u_k\right). \quad (20)$$

$$\mathrm{LCR}(\gamma_{th}) = \int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty}\exp\left(-\sum_{k=1}^{K}u_k\right)f_{\gamma|u_1,\ldots,u_K}(\gamma_{th}|u_1,\ldots,u_K)du_1\ldots du_K\int_{x=0}^{\infty}xf_{\dot{\gamma}|\gamma,u_1,\ldots,u_K}(x|\gamma_{th},u_1,\ldots,u_K)dx$$

$$= \frac{\exp\left(-\frac{\gamma_{th}N}{\Omega_0}\right)}{\Omega_0}\int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty}\exp\left(-\sum_{k=1}^{K}u_k\left(1+\frac{\gamma_{th}}{\Lambda_{0,k}}\right)\right)\left(N+\sum_{k=1}^{K}\Omega_k u_k\right)du_1\ldots du_K\int_{x=0}^{\infty}\frac{x\exp\left(-\frac{x^2}{2\sigma^2(\gamma_{th},u_1,\ldots,u_K)}\right)dx}{\sqrt{2\pi\sigma^2(\gamma_{th},u_1,\ldots,u_K)}}$$

$$= \frac{\exp\left(-\frac{\gamma_{th}N}{\Omega_0}\right)}{\Omega_0\sqrt{2\pi}}\int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty}\exp\left(-\sum_{k=1}^{K}u_k\left(1+\frac{\gamma_{th}}{\Lambda_{0,k}}\right)\right)\left(N+\sum_{k=1}^{K}\Omega_k u_k\right)\sigma(\gamma_{th},u_1,\ldots,u_K)du_1\ldots du_K. \quad (22)$$

In order to obtain a closed-form expression for $\mathrm{LCR}(\gamma_{th})$ we change the integration variables u_k, $k=1,\ldots,K$, of (23) to ν_k, $k=1,\ldots,K$ through the transformation $u_k(1+\frac{\gamma_{th}}{\Lambda_{0,k}})=\nu_k$. We also define, for $k=1,\ldots,K$,

$$\mathcal{W}_k = \frac{1+\frac{\gamma_{th}}{\Lambda_{0,k}}}{\Gamma_k+\frac{\gamma_{th}\Gamma_0\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}} = \frac{f_{\max,0}^2(\Lambda_{0,k}+\gamma_{th})}{\Gamma_k(f_{\max,0}^2\Lambda_{0,k}+\gamma_{th}f_{\max,k}^2)}. \quad (24)$$

Using (24), (23) reduces to

$$\mathrm{LCR}(\gamma_{th}) = \frac{\sigma_0\exp\left(-\frac{\gamma_{th}}{\Gamma_0}\right)\sqrt{2\gamma_{th}}}{\sqrt{\Gamma_0\pi}}\left\{\prod_{k=1}^{K}\frac{\Lambda_{0,k}}{\Lambda_{0,k}+\gamma_{th}}\right\}\times$$

$$\int_{\nu_1=0}^{\infty}\cdots\int_{\nu_K=0}^{\infty}d\nu_1\ldots d\nu_K e^{-\sum_{k=1}^{K}\nu_k}\sqrt{1+\sum_{k=1}^{K}\frac{\nu_k}{\mathcal{W}_k}}. \quad (25)$$

The above integral is in the form of (50) in Appendix-B, whose closed-form solution is given in (54). Using (50) and (54) of Appendix-B, the final expression for $\mathrm{LCR}(\gamma_{th})$ is given by

$$\mathrm{LCR}(\gamma_{th}) = \frac{\sigma_0\exp\left(-\frac{\gamma_{th}}{\Gamma_0}\right)\sqrt{2\gamma_{th}}}{\sqrt{\Gamma_0\pi}}\left\{\prod_{k=1}^{K}\frac{\Lambda_{0,k}}{\Lambda_{0,k}+\gamma_{th}}\right\}$$

$$\times \sum_{k=1}^{K} \delta_k \frac{\exp(\mathcal{W}_k)}{\sqrt{\mathcal{W}_k}} \Gamma_{\mathsf{inc}}\left(\mathcal{W}_k, \frac{3}{2}\right), \tag{26}$$

where $\Gamma_{\mathsf{inc}}(u, n) = \int_{x=u}^{\infty} e^{-x} x^{n-1} dx$ is the complementary incomplete Gamma function [16] and

$$\delta_k = \prod_{i=1, i \neq k}^{K} \frac{\mathcal{W}_i}{\mathcal{W}_i - \mathcal{W}_k}. \tag{27}$$

To the best of the authors' knowledge, (26) is new, and has not been reported in the literature.

III. Some Special Cases of Interest

In this section, we specialize the LCR and the AOD expressions derived in the previous section for various cases of interest.

A. Multiple Equal Power Interferers

When all the interferers have equal average received power we have $\Lambda_{0,k} = \Lambda_{0,1}$, $k = 1, 2 \ldots, K$. With this, (7) reduces to

$$F_{\gamma(t)}(\gamma_{th}) = 1 - \exp\left(-\frac{\gamma_{th}}{\Gamma_0}\right)\left(\frac{\Lambda_{0,1}}{\Lambda_{0,1} + \gamma_{th}}\right)^K. \tag{28}$$

Additionally, if we assume equal Doppler frequency for each user, from (24), we have $\mathcal{W}_k = \mathcal{W}_1 = 1/\Gamma_1$, $k = 1, 2 \ldots, K$. Then, using (57) of Appendix-B together with (25), (26) can be simplified to

$$\mathrm{LCR}(\gamma_{th}) = \frac{f_{\max,0} e^{\mathcal{W}_1} e^{-\frac{\gamma_{th}}{\Gamma_0}}}{\Gamma(K)\sqrt{\mathcal{W}_1}} \sqrt{\frac{2\pi\gamma_{th}}{\Gamma_0}} \left(\frac{\Lambda_{0,1}}{\Lambda_{0,1} + \gamma_{th}}\right)^K$$

$$\times \sum_{j=0}^{K-1} (-\mathcal{W}_1)^{K-1-j} \binom{K-1}{j} \Gamma_{\mathsf{inc}}\left(\mathcal{W}_1, j + \frac{3}{2}\right). \tag{29}$$

Using (28) and (29) in (3), a closed-form expression for AOD can be obtained.

B. Single Interferer

With a single interferer we have $K = 1$, and (29) simplifies to

$$\mathrm{LCR}(\gamma_{th}) = f_{\max,0}\sqrt{2\pi} e^{-\frac{\gamma_{th}}{\Gamma_0}} \sqrt{\frac{\gamma_{th}}{\Gamma_0 \mathcal{W}_1}} \times \frac{\Lambda_{0,1} e^{\mathcal{W}_1} \Gamma_{\mathsf{inc}}\left(\mathcal{W}_1, \frac{3}{2}\right)}{\Lambda_{0,1} + \gamma_{th}}. \tag{30}$$

Using (7) and (30), a closed-form expression for AOD is given by

$$\mathrm{AOD}(\gamma_{th}) = \frac{(\Lambda_{0,1} + \gamma_{th}) e^{\frac{\gamma_{th}}{\Gamma_0}} - \Lambda_{0,1}}{f_{\max,0}\sqrt{2\pi \frac{\gamma_{th}}{\Gamma_0 \mathcal{W}_1}} \Lambda_{0,1} e^{\mathcal{W}_1} \Gamma_{\mathsf{inc}}\left(\mathcal{W}_1, \frac{3}{2}\right)}. \tag{31}$$

C. No Interference

In the absense of CCI (i.e., *noise-limited environment*), we have $\Omega_k = 0$, $\forall k = 1, 2, \ldots, K$. In other words, we set $\Lambda_{0,1} \to \infty$ in (30) and (31), to arrive at

$$\mathrm{LCR}(\gamma_{th}) = f_{\max,0}\sqrt{\frac{2\pi\gamma_{th}}{\Gamma_0}} \exp\left(-\frac{\gamma_{th}}{\Gamma_0}\right), \tag{32}$$

$$\text{and} \quad \mathrm{AOD}(\gamma_{th}) = \frac{\exp\left(\frac{\gamma_{th}}{\Gamma_0}\right) - 1}{f_{\max,0}\sqrt{2\pi \frac{\gamma_{th}}{\Gamma_0}}}, \tag{33}$$

respectively. Note that (32) and (33) coincide with the LCR and AOD of SNR on Rayleigh fading channels [1], [14].

D. Interference-Limited Environment

In an interference-limited regime, the average noise power N is negligible compared with the total average interference power. Letting $N \to 0$ in (7), $F_{\gamma(t)}(\gamma_{th})$ can be written as

$$F_{\gamma(t)}(\gamma_{th}) = 1 - \prod_{k=1}^{K} \frac{\Lambda_{0,k}}{\gamma_{th} + \Lambda_{0,k}}. \tag{34}$$

However, from (26) it is not obvious as to how the LCR expression reduces for interference-limited regime. By letting $N \to 0$ and moving Γ_0 into the $\sqrt{(\cdot)}$ expression, and noting that $\Gamma_k/\Gamma_0 = 1/\Lambda_{0,k}$, we can re-write the last step of (23) as

$$\mathrm{LCR}(\gamma_{th}) = \frac{\sigma_0\sqrt{2\gamma_{th}}}{\sqrt{\pi}} \int_{u_1=0}^{\infty} \cdots \int_{u_K=0}^{\infty} du_1 \ldots du_K$$

$$\times \sqrt{\sum_{k=1}^{K} u_k \left(\frac{1}{\Lambda_{0,k}} + \frac{\gamma_{th}\sigma_k^2}{\Lambda_{0,k}^2 \sigma_0^2}\right)} e^{-\sum_{k=1}^{K} u_k \left(1 + \frac{\gamma_{th}}{\Lambda_{0,k}}\right)}. \tag{35}$$

Similar to the derivation of (26), by defining

$$\mathcal{Z}_k = \frac{1 + \frac{\gamma_{th}}{\Lambda_{0,k}}}{\frac{1}{\Lambda_{0,k}} + \frac{\gamma_{th}\sigma_k^2}{\Lambda_{0,k}^2\sigma_0^2}} = \frac{\Lambda_{0,k}\sigma_0^2(\Lambda_{0,k} + \gamma_{th})}{\Lambda_{0,k}\sigma_0^2 + \gamma_{th}\sigma_k^2}, \tag{36}$$

$$\nu_k = u_k \left(1 + \frac{\gamma_{th}}{\Lambda_{0,k}}\right), \tag{37}$$

$$\text{and} \quad \omega_k = \prod_{i=1, i \neq k}^{K} \frac{\mathcal{Z}_i}{\mathcal{Z}_i - \mathcal{Z}_k}, \tag{38}$$

(35) can be simplified to

$$\mathrm{LCR}(\gamma_{th}) = \frac{\sigma_0\sqrt{2\gamma_{th}}}{\sqrt{\pi}} \left\{\prod_{k=1}^{K} \frac{\Lambda_{0,k}}{\Lambda_{0,k} + \gamma_{th}}\right\} \int_{\nu_1=0}^{\infty} \cdots \int_{\nu_K=0}^{\infty}$$

$$\times \sqrt{\sum_{k=1}^{K} \frac{\nu_k}{\mathcal{Z}_k}} \times e^{-\sum_{k=1}^{K} \nu_k} d\nu_1 \cdots d\nu_K \tag{39}$$

whose closed-form solution is readily obtained from (55) in Appendix-B as

$$\mathrm{LCR}(\gamma_{th}) = f_{\max,0}\pi \sqrt{\frac{\gamma_{th}}{2}} \left\{\prod_{k=1}^{K} \frac{\Lambda_{0,k}}{\Lambda_{0,k} + \gamma_{th}}\right\} \sum_{k=1}^{K} \frac{\omega_k}{\mathcal{Z}_k}. \tag{40}$$

IV. Conclusion

The duration of outage is more important than the probability of outage in analyzing the performance of a communication system with multiple co-channel interferers. With this motivation, in this paper, we presented the analytical LCR and AOD of the received SINR on Rayleigh fading channels with CCI. A closed-form expression for the LCR and the AOD is obtained for the general case of unequal power multiple co-channel interferers traveling at different speeds, and with additive white Gaussian noise.

Appendix A
On the Equivalence of ZCR and the SINR LCR

Let $X(t) = S(t) - \gamma_{th} I(t)$ denote the difference between the signal power $S(t)$ and the interference-plus-noise power $I(t)$ that is scaled by the threshold γ_{th}. The time derivative of $X(t)$ is denoted by $\dot{X}(t)$ and is equal to $\dot{S}(t) - \gamma_{th}\dot{I}(t)$. Denote the auxiliary random processes $A(t) = \dot{S}(t)$ and $B(t) = I(t)$. With these, the joint pdf of $X(t), \dot{X}(t), A(t)$ and $B(t)$ is written as

$$f_{X,\dot{X},A,B}(x,y,a,b) = \left(\frac{\partial\left(X,\dot{X},A,B\right)}{\partial\left(S,I,\dot{S},\dot{I}\right)}\right)^{-1} \times f_{S,I,\dot{S},\dot{I}}(u,v,z,w)\Big|_{u=x+\gamma_{th}b,v=b,z=a,w=(a-y)/\gamma_{th}}, \quad (41)$$

where, for simplicity, we have dropped the time index and $\frac{\partial\left(X,\dot{X},A,B\right)}{\partial\left(S,I,\dot{S},\dot{I}\right)}$ is the Jacobian which is computed as

$$\frac{\partial\left(X,\dot{X},A,B\right)}{\partial\left(S,I,\dot{S},\dot{I}\right)} = \begin{vmatrix} 1 & -\gamma_{th} & 0 & 0 \\ 0 & 0 & 1 & -\gamma_{th} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \gamma_{th}. \quad (42)$$

Upon using (42) in (41), the joint pdf of $X, \dot{X}, A$ and B simplifies to

$$f_{X,\dot{X},A,B}(x,y,a,b) = \frac{1}{\gamma_{th}} f_{S,I,\dot{S},\dot{I}}\left(x+\gamma_{th}b, b, a, \frac{a-y}{\gamma_{th}}\right). \quad (43)$$

The joint pdf of X and $\dot{X}$ is obtained from (43) by integrating over A and B as

$$f_{X,\dot{X}}(x,y) = \int_{a=-\infty}^{\infty}\int_{b=0}^{\infty} \frac{1}{\gamma_{th}} f_{S,I,\dot{S},\dot{I}}\left(x+\gamma_{th}b, b, a, \frac{a-y}{\gamma_{th}}\right) da\, db. \quad (44)$$

The zero crossing rate of $\dot{X}$ at the level $X = 0$ is

$$\mathrm{ZCR}(0) = \int_{y=0}^{\infty} y f_{X,\dot{X}}(0,y) dy = \int_{a=-\infty}^{\infty}\int_{b=0}^{\infty}\int_{y=0}^{\infty} dy\, da\, db \times \frac{y}{\gamma_{th}} f_{S,I,\dot{S},\dot{I}}\left(\gamma_{th}b, b, a, \frac{a-y}{\gamma_{th}}\right). \quad (45)$$

Returning to the SINR process $\gamma(t) = S(t)/I(t)$, we have $\dot{\gamma}(t) = -S(t)\dot{I}(t)/I^2(t) + \dot{S}(t)/I(t)$. With the auxiliary random processes $A(t) = \dot{S}(t)$ and $B(t) = I(t)$, the Jacobian $\frac{\partial(\gamma,\dot{\gamma},A,B)}{\partial(S,I,\dot{S},\dot{I})}$ is

$$\begin{aligned} \frac{\partial\left(\gamma,\dot{\gamma},A,B\right)}{\partial\left(S,I,\dot{S},\dot{I}\right)} &= \begin{vmatrix} \frac{1}{I} & -\frac{S}{I^2} & 0 & 0 \\ -\frac{\dot{I}}{I^2} & \frac{2S\dot{I}}{I^3} - \frac{\dot{S}}{I^2} & \frac{1}{I} & -\frac{S}{I^2} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} \\ &= \begin{vmatrix} \frac{1}{I} & 0 & 0 \\ -\frac{\dot{I}}{I^2} & \frac{1}{I} & -\frac{S}{I^2} \\ 0 & 1 & 0 \end{vmatrix} \\ &= \frac{1}{I}\begin{vmatrix} \frac{1}{I} & -\frac{S}{I^2} \\ 1 & 0 \end{vmatrix} \\ &= \frac{S}{I^3}. \end{aligned} \quad (46)$$

Using (46), the joint pdf of $\gamma, \dot{\gamma}, A$ and B is obtained as

$$\begin{aligned} f_{\gamma,\dot{\gamma},A,B}(x,y,a,b) &= \left(\frac{\partial\left(\gamma,\dot{\gamma},A,B\right)}{\partial\left(S,I,\dot{S},\dot{I}\right)}\right)^{-1} \times f_{S,I,\dot{S},\dot{I}}(u,v,z,w)\Big|_{u=xb,v=b,z=a,w=(a-by)/x} \\ &= \frac{b^2}{\gamma} f_{S,I,\dot{S},\dot{I}}\left(xb, b, a, \frac{a-by}{x}\right). \end{aligned} \quad (47)$$

Similar to (44) and (45), the level crossing rate of $\dot{\gamma}$ at the level $\gamma = \gamma_{th}$ is

$$\begin{aligned} \mathrm{LCR}(\gamma_{th}) &= \int_{y=0}^{\infty} y f_{\gamma,\dot{\gamma}}(\gamma_{th}, y) dy \\ &= \int_{a=-\infty}^{\infty}\int_{b=0}^{\infty}\int_{y=0}^{\infty} \frac{yb^2}{\gamma_{th}} f_{S,I,\dot{S},\dot{I}}\left(\gamma_{th}b, b, a, \frac{a-by}{\gamma_{th}}\right) dy\, da\, db \\ &= \int_{a=-\infty}^{\infty}\int_{b=0}^{\infty}\int_{t=0}^{\infty} \frac{t\, dt\, da\, db}{\gamma_{th}} f_{S,I,\dot{S},\dot{I}}\left(\gamma_{th}b, b, a, \frac{a-t}{\gamma_{th}}\right). \end{aligned} \quad (48)$$

Upon comparing (48) with (45), we conclude that the ZCR of $X(t) = S(t) - \gamma_{th}I(t)$ at a level $X(t) = 0$ is equal to the LCR of $\gamma(t) = S(t)/I(t)$ at a level $\gamma(t) = \gamma_{th}$. When the signal process is independent of the interference process, (45) and (48) are simplified as

$$\begin{aligned} \mathrm{ZCR}(0) &= \int_{t=0}^{\infty} \frac{t\, dt}{\gamma_{th}} \times \int_{a=-\infty}^{\infty}\int_{b=0}^{\infty} f_{S,\dot{S}}\left(\gamma_{th}b, a\right) f_{I,\dot{I}}\left(b, \frac{a-t}{\gamma_{th}}\right) da\, db \\ &= \mathrm{LCR}(\gamma_{th}). \end{aligned} \quad (49)$$

Appendix B
A Useful Integral with a Closed-form Solution

Consider the following integral:

$$\mathcal{I} = \int_{u_1=0}^{\infty}\cdots\int_{u_K=0}^{\infty} \sqrt{a_0 + \sum_{k=1}^{K}\frac{u_k}{\lambda_k}} \times e^{-\sum_{k=1}^{K} u_k} du_1 \ldots du_K, \quad (50)$$

here $a_0, \lambda_1, \ldots, \lambda_K$ are positive real numbers. The above integral can be interpreted as the expected value of $\sqrt{a_0 + Z}$, where $Z = \sum_{k=1}^{K}\frac{U_k}{\lambda_k}$. Here, $U_1, \ldots, U_K$ are i.i.d exponential r.vs each with unity-mean. Since the Laplace transform (LT) of the PDF of U_k, $\mathcal{L}_{U_k}(s) = E[\exp(-sU_k)]$, is given by $\frac{1}{1+s}$, and the LT of the PDF of a sum of independent r.vs is simply the product of individual LTs, the LT of the PDF of Z, $\mathcal{L}_Z(s)$, can be written as

$$\mathcal{L}_Z(s) = \prod_{i=1}^{L}\frac{\lambda_i}{\lambda_i + s} = \sum_{k=1}^{K} c_k\lambda_k\frac{1}{\lambda_k + s}, \quad (51)$$

where the second equality in the above is due to partial-fractions expansion, and c_ks can be found easily as

$$c_k = \frac{1}{\lambda_k}\lim_{s\to-\lambda_k}\left\{\mathcal{L}_Z(s)\times(s+\lambda_k)\right\} = \prod_{i=1,i\neq k}^{K}\frac{\lambda_i}{\lambda_i - \lambda_k}. \quad (52)$$

Upon inverting (51), the PDF, $f_Z(z)$, of Z is

$$f_Z(z) = \sum_{k=1}^{K} c_k\lambda_k\exp(-\lambda_k z), \quad z \geq 0. \quad (53)$$

With the help of (53), (50) can be evaluated as

$$\begin{aligned}\mathcal{I} &= E\left[\sqrt{a_0+Z}\right] \\ &= \sum_{k=1}^{K} c_k\lambda_k \int_{x=0}^{\infty} \sqrt{a_0+x}\exp(-\lambda_k x)dx \\ &= \sum_{k=1}^{K} c_k \frac{\exp(a_0\lambda_k)}{\sqrt{\lambda_k}} \int_{z=a_0\lambda_k}^{\infty} \exp(-z)z^{\frac{1}{2}}dz \\ &= \sum_{k=1}^{K} c_k \frac{\exp(a_0\lambda_k)}{\sqrt{\lambda_k}} \Gamma_{\mathsf{inc}}\left(a_0\lambda_k, \frac{3}{2}\right), \end{aligned} \tag{54}$$

where the third equality in the above equation is due to the change of integration variable from x to z by $z = (a_0 + \lambda_k)x$, and $\Gamma_{\mathsf{inc}}(u, n) = \int_{x=u}^{\infty} \exp(-x)x^{n-1}dx$ is the complimentary incomplete Gamma function [16].

When $a_0 = 0$, using $\Gamma_{\mathsf{inc}}\left(0, \frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$ [16], (54) reduces to

$$E\left[\sqrt{\sum_{k=1}^{K} \frac{U_k}{\lambda_k}}\right] = \frac{\sqrt{\pi}}{2} \sum_{k=1}^{K} \frac{c_k}{\sqrt{\lambda_k}}. \tag{55}$$

We are also interested in the case of $\lambda_1 = \lambda_2 = \cdots = \lambda_K$. In this case, the PDF of Z can be conveniently expressed as

$$f_Z(z) = \frac{\lambda_1^K \exp(-\lambda_1 z)z^{K-1}}{\Gamma(K)},\ z \geq 0. \tag{56}$$

Now, upon re-evaluating the integral of (54), we obtain

$$\begin{aligned}\mathcal{I} &= \frac{\lambda_1^K}{\Gamma(K)} \int_{y=0}^{\infty} \sqrt{a_0+y}e^{-\lambda_1 y}y^{K-1}dy \\ &= \frac{e^{a_0\lambda_1}}{\Gamma(K)\sqrt{\lambda_1}} \int_{t=a_0\lambda_1}^{\infty} \sqrt{t}e^{-t}(t-a_0\lambda_1)^{K-1}dt \\ &= \frac{e^{a_0\lambda_1}}{\Gamma(K)\sqrt{\lambda_1}} \sum_{j=0}^{K-1}(-1)^{K-1-j}(a_0\lambda_1)^{K-1-j}\binom{K-1}{j} \\ &\quad \times \int_{t=a_0\lambda_1}^{\infty} e^{-t}t^{j+\frac{1}{2}}dt \\ &= \frac{e^{a_0\lambda_1}}{\Gamma(K)\sqrt{\lambda_1}} \sum_{j=0}^{K-1}(-1)^{K-1-j}(a_0\lambda_1)^{K-1-j}\binom{K-1}{j} \\ &\quad \times \Gamma_{\mathsf{inc}}\left(a_0\lambda_1, j+\frac{3}{2}\right), \end{aligned} \tag{57}$$

where the second step of the above equation is due to the change of integration variable y to t by $t = (a_0 + y)\lambda_1$, and the last equality is due to the definition of complimentary incomplete Gamma function [16].

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