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Observer Forms for Perspective Systems ⋆

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Abstract

Estimation of 3D position information from 2D images in computer vision systems can be formulated as a state estimation problem for a nonlinear perspective dynamic system. The multi-output state estimation problem has been treated by several authors using methods for nonlinear observer design. This paper shows that a perspective system can be transformed to two observer forms, and provides constructive methods for arriving at the transformations. These observer forms lead to straightforward observer designs. First, it is shown that using an output transformation, the system admits an observer form which leads to an observer with linear error dynamics. A second observer design is based on a time scaled block triangular form. Both designs assume a commonly used observability condition. The designs are demonstrated in simulation.

1 Introduction

The problem of estimating 3D structure and motion from 2D perspective observations can be solved with a nonlinear observer design. The perspective system dynamics is obtained by considering the relative motion between a perspective camera and an observed object. The estimation of both structure and motion can be achieved by an observer for states and parameters. Existing approaches have used the extended Kalman filter [17] or adaptive observers [3, 6]. The problem of estimating structure when the motion parameters are measured or otherwise assumed available, has been considered using observer-based approaches in [15, 10, 16, 2, 9, 7, 1, 11].

Based on preliminary results in [8], this paper presents structure estimation results by showing how a perspective system can be transformed into two observer forms. These forms naturally lead to observers with simple error dynamics systems. The simplicity of the error dynamics leads to a straightforward stability analysis. Relative to existing related work, the results here show that it is possible to achieve linear time-invariant error dynamics without any constraints on the type of motion when an Observer Form (OF) with output transformation is considered [13]. Previous work in [5] considered the OF without output transformation, and required a constraint on the type of motion which potentially limited the application of the approach. A second contribution of the paper is to demonstrate the application of a Time-Scaled Block Triangular Observer Form (TBTOF) which was first introduced in [21]. The TBTOF is a generalization of OF and can therefore be applied to a wider class of systems. The advantage of the TBTOF is a simpler observer structure since it leaves part of the dynamics untransformed.

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2 Background

2.1 Perspective dynamic systems

A perspective dynamic system with three states and two outputs, derived assuming a calibrated pinhole camera and observations of feature points on a rigid object, can be written as

$$\dot{\zeta} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \zeta + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \; y = \begin{bmatrix} \zeta_1/\zeta_3 & \zeta_2/\zeta_3 \end{bmatrix}^T, \quad (1)$$

where we assume $a_{ij}, b_i, 1 \leq i, j \leq 3$ are constant and $\zeta_3 > 0$ [14].

As in e.g. [2], it is convenient to work in state coordinates defined by $\xi = [\xi_1, \xi_2, \xi_3]^T = [\zeta_1/\zeta_3, \zeta_2/\zeta_3, 1/\zeta_3]^T$ which transforms (1) into

$$\begin{aligned} \dot{\xi}_1 &= (a_{11} - a_{33})\xi_1 + a_{12}\xi_2 + a_{13} + (b_1 - b_3\xi_1)\xi_3 \\ &\quad - a_{31}{\xi_1}^2 - a_{32}\xi_1\xi_2 \\ \dot{\xi}_2 &= a_{21}\xi_1 + (a_{22} - a_{33})\xi_2 + a_{23} + (b_2 - b_3\xi_2)\xi_3 \\ &\quad - a_{31}\xi_2\xi_1 - a_{32}{\xi_2}^2 \\ \dot{\xi}_3 &= -\xi_3\left(a_{31}\xi_1 + a_{32}\xi_2 + a_{33} + b_3\xi_3\right) \\ y &= \begin{bmatrix}\xi_1 & \xi_2\end{bmatrix}^T. \end{aligned} \tag{2}$$

In the ξ coordinates the dynamics are nonlinear and the output function is linear.

In the following we use the notation $L_f h$ for the Lie derivative of a function $h : \mathbb{R}^n \to \mathbb{R}$ along a vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ and $L_f^k h(x)$ for the k times repeated Lie derivative. The notation dh is used for the gradient of a function $h : \mathbb{R}^n \to \mathbb{R}$. Given vector fields $f, g : \mathbb{R}^n \to \mathbb{R}^n$, $ad_f g$ denotes the Lie bracket $[f, g]$ and $ad_f^i g$ is the repeated Lie bracket $ad_f^i g = [f, ad_f^{i-1} g]$ and $ad_f^0 g = g$.

2.2 Observability

In order to apply the definition of observability in [13] we compute

$$\Omega = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_f h_1 \\ dL_f h_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ * & * & b_1 - b_3\xi_1 \\ * & * & b_2 - b_3\xi_2 \end{bmatrix}.$$

We note the codistribution Ω has dimension 3 and system (2) is observable with observability indices $k_1 = 2, k_2 = 1$ provided $(b_1 - b_3\xi_1)^2 + (b_2 - b_3\xi_2)^2 \neq 0$. The value of the output $y = [b_1/b_3, b_2/b_3]^T$ where the system loses observability is called the *focus of expansion* [10]. In this paper, we assume $b_1 - b_3\xi_1 \neq 0$ so that $h_1, h_2, L_f h_1$ are coordinates. The constraint $b_1 - b_3\xi_1 \neq 0$ in the approach proposed here can be generalized to $c_1(b_1 - b_3\xi_1) + c_2(b_2 - b_3\xi_2) \neq 0$ for some $c_1, c_2 \in \mathbb{R}$.

3 Observer forms for perspective systems

Without output transformation it was shown in [5] that (1) admits an OF under the constraint $b_2 = b_3 = 0$ given the observability assumption $b_1 - b_3\xi_1 \neq 0$. In this paper we provide two results which extend [5]. The first result shows the existence of an output transformation $\bar{y} = \Psi(y)$ and state transformation $z = \Phi(\xi)$ such that (1) is transformable to OF with weaker motion constraints relative to [5]. The second result demonstrates the existence of a TBTOF which provides coordinates allowing for a straightforward observer design, albeit with the same constraint on the motion which appeared in [5] for dynamic error linearization. The results were derived with the help of a Maple library for observer error linearization [4].

3.1 Observer Form with output transformation

We have the following theorem which provides the necessary and sufficient conditions for transformation to OF without output transformation.

Theorem 1 ([22]) *The dynamic system*

$$\dot{\zeta} = f(\zeta), \quad y = h(\zeta),$$

where $\zeta \in \mathbb{R}^n, y \in \mathbb{R}^2$, is locally transformable to OF with the change of state coordinates $z = \Phi(\zeta)$ if and only if the following three conditions are locally satisfied:

(*i*) *The codistributions*

$$\begin{aligned} R_j^l &= \operatorname{span}\{dL_f^k h_i : 0 \le k \le k_j - 1, i \neq j, \\ &\qquad 1 \le i \le 2, dL_f^k h_j : 0 \le k \le k_j - 2)\} \\ R_j^r &= \operatorname{span}\{dL_f^k h_i : 0 \le k \le \min(k_i, k_j) - 1, i \neq j, \\ &\qquad 1 \le i \le 2, dL_f^k h_j : 0 \le k \le k_j - 2\} \end{aligned}$$

satisfy $\dim R_j^l = \dim R_j^r$, $1 \le j \le 2$.

(*ii*) *There exist vector fields r_i, $1 \le i \le 2$, such that*

$$\begin{aligned} &L_{r_i} L_f^{k-1} h_j = \delta_{i,j} \cdot \delta_{k,k_j}, \\ &1 \le i \le 2, \quad 1 \le k \le k_i, \quad 1 \le j \le 2, \end{aligned}$$

where $\delta_{i,j} = 1$ when $i = j$ and zero otherwise.

(*iii*) $[ad_{-f}^k r_i, ad_{-f}^l r_j] = 0, 1 \le i, j \le 2; 0 \le k \le k_i - 1; 0 \le l \le k_j - 1$.

If the conditions of Theorem 1 hold for all $\zeta \in \mathbb{R}^n$ and the n vector fields in Condition (iii) are complete, the OF coordinates are globally defined. One approach to deriving the OF state coordinates which include an output transformation is to use Theorem 1 with a general output function $\Psi(y)$ instead of the original function h. Without an output transformation, Condition (i) is not satisfied for $b_2 - b_3\xi_2 \neq 0$. This can be seen from R_1^l, R_1^r:

$$R_1^l = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_f h_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ * & * & b_2 - b_3\xi_2 \end{bmatrix}$$

$$R_1^r = \begin{bmatrix} dh_1 \\ dh_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

which have different dimension unless $b_2 - b_3\xi_2 = 0$. The dimensions of R_1^l, R_1^r can be made equal if we transform the second output: $\bar{y} = [\bar{y}_1, \bar{y}_2]^T = [\xi_1, \Psi_2(\xi_1, \xi_2)]^T$. Relative to this output we have

$$R_1^l = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\partial \Psi_2}{\partial \xi_1} & \frac{\partial \Psi_2}{\partial \xi_2} & 0 \\ * & * & \frac{\partial \Psi_2}{\partial \xi_1}(b_1 - b_3\xi_1) + \frac{\partial \Psi_2}{\partial \xi_2}(b_2 - b_3\xi_2) \end{bmatrix}.$$

The condition $\dim R_1^l = \dim R_1^r$ yields a PDE

$$\frac{\partial \Psi_2}{\partial \xi_1}(b_1 - b_3\xi_1) + \frac{\partial \Psi_2}{\partial \xi_2}(b_2 - b_3\xi_2) = 0 \qquad (3)$$

whose general solution is

$$\Psi_2(\xi_1, \xi_2) = F\left(\frac{b_3\xi_2 - b_2}{b_3(b_3\xi_1 - b_1)}\right).$$

We choose F as the identity function:

$$\Psi_2(\xi_1, \xi_2) = \frac{b_3\xi_2 - b_2}{b_3(b_3\xi_1 - b_1)}.$$

We remark that this choice restricts $b_3 \neq 0$ for Ψ_2 to be well-defined. Other choices for Ψ_2 were investigated in order to eliminate this condition on the motion parameters. In all cases, varying Ψ_2 led to points of singularity at certain values of the motion parameters.

Next, we introduce the first component for the output transformation which for simplicity is taken to be a function of ξ_1 alone: $\bar{y} = [\bar{y}_1, \bar{y}_2]^T = [\Psi_1(\xi_1), \Psi_2(\xi_1, \xi_2)]^T$. Using Condition (ii) we obtain a non-unique solution for r_2 and an r_1 dependent on $\frac{d\Psi_1}{dy_1}$:

$$r_1 = \frac{1}{(b_1 - b_3\xi_1)\frac{d\Psi_1}{d\xi_1}} \frac{\partial}{\partial \xi_3}, \; r_2 = (b_3\xi_1 - b_1)\frac{\partial}{\partial \xi_2} + \rho(\xi_1)\frac{\partial}{\partial \xi_3},$$

where we have assumed $\rho(\xi_1)$ is some function of ξ_1 to be determined. We remark that for systems with all observability indices equal, then no such degree of freedom results from Condition (ii).

Next, we compute $ad_{-f}r_1$ and verify the Lie bracket Condition (iii). The condition $[r_1, r_2] = 0$ holds for all $\xi \in \mathbb{R}^3$ except $b_3\xi_1 - b_1 = 0$. The condition $[r_1, ad_{-f}r_1] = 0$ if

$$\frac{d^2\Psi_1}{d\xi_1^2}(b_1 - b_3\xi_1) - 2b_3\frac{d\Psi_1}{d\xi_1} = 0. \qquad (4)$$

Solving this ODE gives

$$\Psi_1(\xi_1) = C_1 + \frac{C_2}{b_3\xi_1 - b_1},$$

where we choose $C_1 = 0$ and $C_2 = 1$. Hence,

$$\bar{y}_1 = \frac{1}{b_3\xi_1 - b_1}, \quad \bar{y}_2 = \frac{b_3\xi_2 - b_2}{b_3(b_3\xi_1 - b_1)}. \qquad (5)$$

Using this Ψ_1 the condition $[r_2, ad_{-f}r_1] = 0$ requires

$$\frac{d\rho}{d\xi_1}(b_3\xi_1 - b_1) + a_{32}b_1 - b_3a_{12} - \rho(\xi_1)b_3 = 0.$$

Solving this ODE gives

$$\rho(\xi_1) = (b_3\xi_1 - b_1)C_3 + (a_{32}b_1 - b_3a_{12})/b_3,$$

and choosing $C_3 = 0$ gives $\rho = (a_{32}b_1 - b_3a_{12})/b_3$. The state transformation is $z = \Phi(\xi) = [\Phi_1(\xi), \Psi_1(\xi), \Psi_2(\xi)]^T$ with Φ_1 given by

$$\begin{aligned}
\Phi_1(\xi) &= \frac{c_1(\xi_1)\xi_3 + c_2\xi_2 + c_3\xi_1 + c_4}{2b_3(b_3\xi_1 - b_1)^2} \\
c_1(\xi_1) &= 2b_3^2(b_3\xi_1 - b_1) \\
c_2 &= 2b_3(a_{12}b_3 - a_{32}b_1) \\
c_3 &= 2b_3(a_{11}b_3 - a_{31}b_1) \\
c_4 &= b_3(b_3a_{13} - (a_{33} + a_{11})b_1 - b_2a_{12}) + a_{31}b_1^2 + b_2a_{32}b_1.
\end{aligned}$$

Applying this state transformation and the output transformation (5) gives the OF

$$\dot{z} = \begin{bmatrix} \eta_1(\bar{y}) \\ z_1 + \eta_2(\bar{y}) \\ \eta_3(\bar{y}) \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} z_2 \\ z_3 \end{bmatrix}. \qquad (6)$$

The functions η_i, $i = 1, 2, 3$ are polynomials:

$$\begin{aligned}
\eta_1(\bar{y}) &= d_{11}\bar{y}_2^3 + d_{12}\bar{y}_2\bar{y}_1^2 + d_{13}\bar{y}_1\bar{y}_2^2 + d_{14}\bar{y}_1^2 + d_{15}\bar{y}_2^2 \\
&\quad + d_{16}\bar{y}_1\bar{y}_2 + d_{17}\bar{y}_1 + d_{18}\bar{y}_2 + d_{19} \\
\eta_2(\bar{y}) &= d_{21}\bar{y}_1^2 + d_{22}\bar{y}_1\bar{y}_2 + d_{23}\bar{y}_1 + d_{24}\bar{y}_2 + d_{25} \\
\eta_3(\bar{y}) &= d_{31}\bar{y}_2^2 + d_{32}\bar{y}_1\bar{y}_2 + d_{33}\bar{y}_1 + d_{34}\bar{y}_2 + d_{35},
\end{aligned}$$

where d_{ij} are constants depending on $b_i, a_{ij}, 1 \leq i, j \leq 3$.

An observer for (2) can now be constructed as

$$\dot{\hat{\zeta}} = f(\hat{\zeta}) + \frac{\partial \zeta}{\partial z}(\hat{\zeta})(\eta(\bar{y}) - \eta(\Psi(h(\hat{\zeta})))) + K(\bar{y} - \Psi(h(\hat{\zeta}))) \qquad (7)$$

and we introduce parameters ω_p, ζ_p, and α_p and $K = [\omega_p^2, 2\zeta_p\omega_p, \alpha_p]^T$ so that the characteristic equation for the error dynamics of (7) in the z-coordinates is

$$(s + \alpha_p)(s^2 + 2\zeta_p\omega_p s + \omega_p^2) = 0. \qquad (8)$$

This allows us to directly control the rate of convergence of the estimate error and represents an important design feature of the proposed method.

The state transformation $z = \Phi(\xi)$ and the output transformation $\bar{y} = \Psi(y)$ require $b_3 \neq 0$. However, the case $b_3 = 0$ can be handled using the same procedure using a linear output transformation $\bar{y}_1 = \xi_1, \bar{y}_2 = b_1\xi_2 - b_2\xi_1$, which is valid when $b_1 \neq 0$. For the case $b_1 = b_3 = 0$, no output transformation is required [5].

The above approach can also be applied to the planar perspective system

$$\dot{\zeta} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \zeta + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad y = \frac{\zeta_1}{\zeta_2},$$

which does not admit an OF without output transformation [18]. The details of the procedure are straightforward and not provided. It is also interesting to note that since the system is two dimensional, it is transformable to OF using a time scale transformation.

The method described above to compute the OF coordinates uses Theorem 1. Alternatively, one can use a method based on a so-called Generalized Characteristic Equation (GCE) [12]. For a two output system with observability indices $(2, 1)$, the GCEs are

$$L_f^2\Psi_1(y) = L_f\eta_2(\Psi(y)) + \eta_1(\Psi(y)),\ L_f\Psi_2(y) = \eta_3(\Psi(y)).$$

Expanding the GCEs and performing coefficient matching leads to necessary and sufficient conditions on the transformability to OF. In particular the so-called polynomial condition results: $\partial^2 L_f^2\Psi_1(y)/\partial\dot{y}_1^2 = 0$ and $\partial L_f\Psi_2(y)/\partial\dot{y}_1 = 0$. We assume the output transformation for the first subsystem to only depend on y_1, i.e. $\bar{y}_1 = \Psi_1(y_1)$. We are able to solve for Ψ_1 s.t. the system satisfies a polynomial condition. That is, $L_f^2\bar{y}_1$ is linear in $\dot{y}_1$ with coefficients depending on y:

$$\ddot{\bar{y}}_1 = \frac{d^2\Psi_1}{dy_1^2}\dot{y}_1^2 + \frac{d\Psi_1}{dy_1}\left(\alpha_2(y)\dot{y}_1^2 + \alpha_3(y)\dot{y}_1 + \alpha_4(y)\right). \quad (9)$$

In order to remove the dependence on $\dot{y}_1^2$ on the RHS of (9) we have the ODE

$$\frac{d^2\Psi_1}{dy_1^2} + \alpha_2(y)\frac{d\Psi_1}{dy_1} = 0, \quad \alpha_2(y) = \frac{2b_3}{b_3y_1 - b_1}. \quad (10)$$

We notice that ODE (10) is the same as (4), hence the GCE approach leads to the same Ψ_1 as computed above. For the second subsystem we assume a more general dependence for the output transformation: $\Psi_2(y)$. Following the similar procedure as that used for the first subsystem we have

$$\dot{\bar{y}}_2 = \frac{\partial\Psi_2}{\partial y_1}\dot{y}_1 + \frac{\partial\Psi_2}{\partial y_2}\dot{y}_2 = \frac{\partial\Psi_2}{\partial y_1}\dot{y}_1 + \frac{\partial\Psi_2}{\partial y_2}\left(\alpha_5(y)\dot{y}_1 + \alpha_6(y_1)\right),$$

and the PDE

$$\frac{\partial\Psi_2}{\partial y_1} + \frac{\partial\Psi_2}{\partial y_2}\alpha_5(y) = 0, \quad \alpha_5(y) = \frac{b_3y_2 - b_2}{b_3y_1 - b_1}. \quad (11)$$

One can see that PDE (11) is the same as (3). Hence, the GCE approach yields the same output transformation $\Psi_2(y)$ as computed above.

3.2 Time-scaled block triangular observer form

The system (2) in observable form is already in BTF [20]. We attempt to transform the first subsystem to BTOF [19]. Defining the observable coordinates as $x = [x_1^T, x_{21}]^T = [x_{11}, x_{12}, x_{21}]^T = [h_1(\xi), L_fh_1(\xi), h_2(\xi)]^T$, one can compute the starting vector $g_1 = \partial/\partial x_{12}$ according to [21, Eq. (6)] and verify that the Lie bracket [21, Eq. (7)] is not satisfied. We introduce the time scaling transformation for the first subsystem

$$\frac{d\tau_1}{dt} = s_1(y) > 0,$$

where $s_1(y)$ is the time scaling function (TSF) to be determined. We apply [21, Prop. 3.1]

$$dL_{g_1}L_{F^1}^{k_1}h_1 = l_{k_1}\frac{1}{s_1}\frac{\partial s_1}{\partial y_1}dL_{F^1}h_1 \mod \{dz_1^1\},$$

with $k_1 = 2$, $F^1 = f_1 = x_{12}\partial/\partial x_{11} + (L_f^2h_1(x))\partial/\partial x_{12}$, and $l_2 = 2$. This yields the PDE for s_1

$$\frac{4b_3}{b_3y_1 - b_1} = \frac{2}{s_1}\frac{\partial s_1}{\partial y_1}.$$

Solving this PDE yields the time scaling transformation

$$\frac{d\tau_1}{dt} = (b_3y_1 - b_1)^2 = s_1(y) > 0.$$

Defining $\bar{f}_1 = f_1/s_1$ and calculating the vector fields $\bar{g}_1 = s_1g_1, ad_{-\bar{f}_1}\bar{g}_1$, we can verify the Lie bracket condition $[\bar{g}_1, ad_{-\bar{f}_1}\bar{g}_1] = 0$. However, [21, Eq. (8)] requires $\frac{\partial}{\partial y_2}ad_{-f_1}\bar{g}_1 = 0$ which is satisfied if and only if

$$a_{12}b_3 - a_{32}b_1 = 0. \quad (12)$$

This constraint also appears in the dynamic error linearization in [5]. Given (12), the transformation of state can be solved from $\frac{\partial \Phi_1(x_1)}{\partial x_1}[ad_{-\bar{f}_1}\bar{g}_1, \bar{g}_1] = I_2$, where I_2 is an identity matrix and

$$\begin{aligned} ad_{-\bar{f}_1}\bar{g}_1 &= \begin{bmatrix} 1 \\ \frac{-2b_3x_{12}+c_2x_{11}+c_3}{b_1-b_3x_{11}} \end{bmatrix} \\ c_3 &= -2\,a_{33}b_1 + 3\,b_3a_{13} + a_{11}b_1 + a_{12}b_2 \\ c_2 &= 2\,b_3a_{11} - 3\,a_{31}b_1 - a_{32}b_2 - a_{33}b_3. \end{aligned}$$

This gives the transformation to TBTOF:

$$z = \begin{bmatrix} z_{11} \\ z_{12} \\ z_{21} \end{bmatrix} = \begin{bmatrix} \Phi_1(x) \\ \Phi_2(x) \\ \Phi_3(x) \end{bmatrix} = \begin{bmatrix} x_{11} \\ \frac{2x_{12}b_3^2+c_2b_1-c_3b_3-2c_2x_{11}b_3}{2b_3^2(b_1-b_3x_{11})^2} \\ x_{21} \end{bmatrix},$$

where we have reused the notation for z and Φ. In z coordinates the system is

$$\begin{bmatrix} \frac{dz_{11}}{d\tau_1} \\ \frac{dz_{12}}{d\tau_1} \\ \frac{dz_{21}}{dt} \end{bmatrix} = \begin{bmatrix} z_{12} + \beta_{11}(z_{11}, y_2) \\ \beta_{12}(z_{11}, y_2) \\ \beta_{21}(z_1, z_{21}) \end{bmatrix}, \quad y = \begin{bmatrix} z_{11} \\ z_{21} \end{bmatrix}.$$

The TBTOF allows for a straightforward observer design

$$\begin{bmatrix} \frac{d\hat{z}_1}{d\tau_1} \\ \frac{d\hat{z}_{21}}{dt} \end{bmatrix} = \begin{bmatrix} A_1\hat{z}_1 + \beta_1 + L_1C_1(z_1 - \hat{z}_1) \\ \hat{\beta}_{21} + L_2(z_{21} - \hat{z}_{21}) \end{bmatrix},$$

where $\hat{z}_1 = [\hat{z}_{11}, \hat{z}_{12}]^T, A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C_1 = [1, 0]^T, \beta_1 = [\beta_{11}, \beta_{12}]^T, \hat{\beta}_{21} = \beta_{21}(\hat{z}_{12}, y)$, and L_1 is chosen so that $A_1 - L_1C_1$ is Hurwitz, and $L_2 > 0$. The corresponding error dynamics in the new time scale is

$$\begin{bmatrix} \frac{d\tilde{z}_1}{d\tau_1} \\ \frac{d\tilde{z}_{21}}{dt} \end{bmatrix} = \begin{bmatrix} A_1 - L_1C_1 & 0 \\ 0 & -L_2 \end{bmatrix} \tilde{z} + \begin{bmatrix} 0 \\ \beta_{21} - \hat{\beta}_{21} \end{bmatrix},$$

whose zero solution is globally exponentially stable (GES) assuming $b_3x_{11} - b_1 \neq 0$ and β_{21} globally Lipschitz in z_{12}. Assuming there exist positive constants T_0, ε such that

$$\int_t^{t+T_0} s_1(\xi)\mathrm{d}\xi \geq \varepsilon, \quad \forall t \geq t_0$$

we conclude the zero solution of the error dynamics is GES in the original time. The observer in x-coordinates and t time is

$$\begin{aligned} \dot{\hat{x}} = &\begin{bmatrix} \frac{s_1(y)}{s_1(\hat{y})} f_1(\hat{x}) \\ f_2(\hat{x}) \end{bmatrix} \\ &+ \left(\frac{\partial \hat{z}}{\partial \hat{x}}\right)^{-1} \begin{bmatrix} s_1(y)\left(\beta_1 - \hat{\beta}_1^* + L_1(y_1 - C_1\hat{z}_1)\right) \\ \hat{\beta}_{21} - \hat{\beta}_{21}^* + L_2(y_2 - \hat{z}_{21}) \end{bmatrix}, \end{aligned}$$

where f_2 is the third component of the dynamics in observable form, $\hat{\beta}_1^* = \beta_1(\hat{z}_{11}, \hat{y}_2), \hat{\beta}_{21}^* = \beta_{21}(\hat{z}_1, \hat{y}_2)$. The TBTOF coordinates involve the transformation of a 2-dimensional subsystem of the perspective dynamics. It does not transform the 2nd subsystem dynamics, i.e., the dynamics of $y_2 = x_{21} = z_{21} = \zeta_2/\zeta_3$. Hence, the advantage this design has over the OF-based design is a simpler means of construction and a simpler expression for the change of coordinates and observer. This simplicity has the practical benefit of reduced sensitivity to motion parameter error.

4 Simulations

We simulate the OF and TBTOF observers for the system

$$\dot{\zeta} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \zeta + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

The eigenvalues of error dynamics are placed at -4 (OF) and -2.5 (TBTOF), and the initial conditions (ICs) in the format of $[\zeta_1, \zeta_2, \zeta_3, \hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3]^T$ are

$$\begin{aligned} IC1 :&[-1, 2, 2, -1/6, 1/3, 1/3]^T \\ IC2 :&[-1, 2, 1, -0.03, 0.12, 0.30]^T \\ IC3 :&[-2, 3, 4, -0.4, 2.4, 0.4]^T. \end{aligned} \tag{13}$$

For the OF-based observer, a plot of the norm of the error in the original coordinates $\left\|\tilde{\zeta}\right\| = \left\|\zeta - \hat{\zeta}\right\|$ is presented in Figure 1, using solid (IC1), dashed (IC2), and dashed-dot (IC3) lines for the initial conditions in (13). For the TBTOF-based observer, the corresponding simulation result is given in Figure 2. Uniformly distributed noise with an amplitude of .005 was added to the output to demonstrate the designs' robustness to measurement noise. The TBTOF-based design exhibits more noise in the estimate error which can be due to the faster error convergence.

To compare the OF-based design with an existing ap-

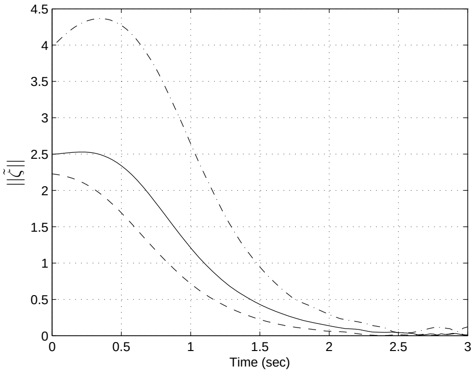

Fig. 1. Norm of state estimate error in ζ-coordinates using the OF-based design.

proach in [11], we consider the example in [2, 9, 11]:

$$\dot{\zeta} = \begin{bmatrix} -0.2 & 0.4 & -0.6 \\ 0.1 & -0.2 & 0.3 \\ 0.3 & -0.4 & 0.4 \end{bmatrix} \zeta + \begin{bmatrix} 0.5 \\ 0.25 \\ 0.3 \end{bmatrix},$$

with the initial conditions as in [11]: $[\zeta_1(0), \zeta_2(0), \zeta_3(0)]^T = [1, 1.5, 2.5]^T$, $[\hat{\zeta}_1(0), \hat{\zeta}_2(0), \hat{\zeta}_3(0)]^T = [0, 0, 1]^T$. Simulations for three different sets of eigenvalues demonstrate the control of the rate of convergence of the estimate error: we take the parameters $\alpha_p = \omega_p = 4, 8, 12$ and $\zeta_p = 1$ in (8) which roughly compare with convergence corresponding to $\lambda = 10, 20, 30$ in [11]. The simulation results for $\tilde{\zeta}_3$ for both observers for no measurement noise are in Figure 3. To investigate the proposed design's ability to trade off robustness to noise with speed of convergence, uniformly distributed noise with an amplitude of .005 was added to the output of both designs. The simulation results for $\tilde{\zeta}_3$ are shown in Figure 4. We remark the OF-based design has favorable noise rejection properties relative to the design in [11]. This is because the term $\beta(y)$ in the expression for $\hat{\zeta}_3$ which can directly magnify the noise. Similarly, a large value of λ can amplify noise. The design in [11] is also reduced-order which implies estimates for ζ_1 and ζ_2 will depend directly on measurement noise. Further, the parameter λ in [11] requires knowledge of a bound on the size of system's state which makes the design of the rate of error convergence less direct relative to the OF-based design.

5 Conclusions

This paper has shown that a perspective system admits two observer forms. These observer forms naturally lead to observer designs with error dynamics which are easy to stabilize. The first observer form is the OF with output transformation which provides error convergence without motion constraints (assuming constant motion

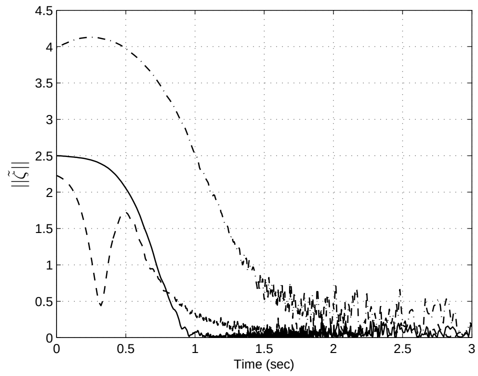

Fig. 2. Norm of state estimate error in ζ-coordinates using the TBTOF-based design.

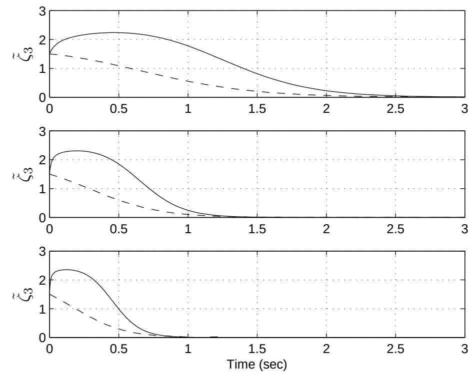

Fig. 3. Solid curves are state estimate error $\tilde{\zeta}_3$ using an OF design for three choices of eigenvalues: -4 (top), -8 (middle), and -12 (bottom). Dashed curves are observer in [11] for $\lambda = 10(top), 20(middle), 30(bottom)$.

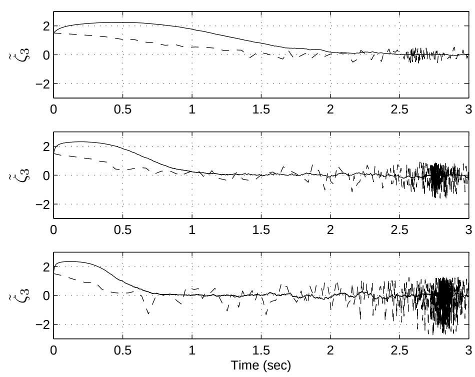

Fig. 4. Solid curves are state estimate error $\tilde{\zeta}_3$ using an OF design for three choices of eigenvalues: -4 (top), -8 (middle), and -12 (bottom). Dashed curves are observer in [11] for $\lambda = 10(top), 20(middle), 30(bottom)$. Noise was added to the output measurements.

parameters). The second observer form is a TBTOF which requires the same motion constraint as in previous work [5] on dynamic error linearization but has the advantage of a relatively simple observer structure. Future work involves generalizing the normal form-based approach to allow for time-varying and/or unknown motion parameters.

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