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Abstract

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LOW-COMPLEXITY STAP VIA SUBSPACE TRACKING IN COMPOUND-GAUSSIAN ENVIRONMENT

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ABSTRACT

This paper considers the subspace tracking approach for low-complexity space-time adaptive processing (STAP) in the non-homogeneous compound-Gaussian environment. Specifically, a normalized subspace tracking (NST) and an instantaneously normalized subspace tracking (iNST) detectors are proposed to mitigate the effect of the time-varying texture (power) component on the detection performance and track the subspace of the stationary speckle component. On one hand, the two proposed detectors can be considered as a fast implementation of the normalized eigen-canceler by replacing the conventional eigen-decomposition with the subspace tracking techniques. On the other hand, they improve existing subspace tracking-based STAP detectors which mostly deal with homogeneous environment and ignore the power variation among rang bins. Extensive simulations confirm that the proposed detectors are able to provide performance gain over conventional subspace tracking-based STAP detectors in the compound-Gaussian environment.

Index Terms— Space-time adaptive processing, compound-Gaussian environment, subspace tracking, low computational complexity, reduced-rank detection.

1. INTRODUCTION

Space-time adaptive processing (STAP) methods have been shown to be effective in detecting air/ground moving objects [1–3]. Furthermore, STAP for the compound-Gaussian environments has been investigated over two decades [4–9] (and reference therein). A number of non-adaptive and adaptive STAP detectors are available for detecting moving objects in such non-Gaussian environments. Due to the additional time-correlated texture component, the optimum detection in the compound-Gaussian yields an implicit form in most cases. The solution to the optimum detector usually resorts to an expectation-maximization (EM) procedure. On the other hand, sub-optimal detectors in the compound-Gaussian case are usually expressed in simple and closed forms. Among these detectors are the normalized adaptive matched filter (NAMF) with the conventional sample covariance matrix (SCM) and the NAMF with the normalized SCM (NSCM).

Same as in the Gaussian case, the SCM in the compound-Gaussian environment imposes two major constraints: 1) the number of training data should be equal or larger than the dimension of the space-time covariance matrix; and 2) the computationally-demanding inversion of the space-time covariance matrix which is usually a large-dimension matrix. Recognizing that the speckle in the compound-Gaussian environment has a low-rank structure, reduced-rank detectors have been proposed in the literature, e.g., eigenvalue decomposition (EVD)-based eigencanceler [10]. The reduced-rank detector employs a projection of the received signal and steering vector into the null space of the clutter principle subspace. The EVD-based eigencanceler is able to reduce the training requirement to $\mathcal{O}(2r)$, where r is the rank of the clutter covariance matrix. However, the computational complexity of this method remains high as $\mathcal{O}(M^3N^3)$, where M is the number of spatial channels and N is the number of pulses. As the product MN becomes large, such high computational complexity makes the EVD-based eigencanceler prohibitive from real-time applications. To address this issue, the subspace tracking technique has been introduced to the STAP in [11, 12] to deal with homogeneous environments. It has been shown that the application of the subspace tracking technique greatly reduces the computational complexity involved in STAP, without significant performance sacrifice.

In this paper, we consider a low-complexity STAP strategy which is based on the subspace tracking and designed for the non-homogeneous compound-Gaussian environment. Existing subspace tracking based STAP detectors fail to account for the power variation across range bins. To address this issue, two modified subspace tracking STAP detectors, the normalized ST (NST) detector and the instantaneously normalized ST (iNST) detector, are proposed to mitigate the effect of the time-varying texture component and to track the subspace of the speckle component. Specifically, the NST detector introduces a normalization factor in the test statistic and handle the power variation in the test statistic level, while the iNST detector performs two normalization operations: first an instantaneous normalization on the received signal at each range bin and then the normalization at the test statistic. On one hand, the proposed detectors are training-efficient, due to its exploitation of the low-rank structure of the speckle component, as compared to the covariance matrix based approach, e.g., the NAMF detectors with the SCM and NSCM; on the other hand, they are also computationally more efficient than the EVD-based eigencanceler, due to the fast subspace tracking technique.

The rest of this paper is organized as follows. Section 2 contains the signal model and briefly reviews prior solutions. Section 3 introduces the two proposed STAP detectors for the compound-Gaussian environment. Simulation results are provided for performance comparison in Section 4. Conclusions are finally drawn in Section 5.

2. SIGNAL MODEL AND PRIOR SOLUTIONS

2.1. Signal Model

The compound-Gaussian clutter is a product of a positive scalar λ_k (texture) and a multi-dimensional complex Gaussian vector $\mathbf{z}_k$

P. Wang was with Mitsubishi Electric Research Laboratories as a summer intern of 2010 when this work was conducted.

(speckle) with mean zero and covariance matrix $\mathbf{R}$ as

$$\mathbf{x}_k = \lambda_k \mathbf{z}_k \in \mathbb{C}^{N\times 1}, k = 0, 1, \cdots, K \tag{1}$$

where $\mathbf{z}_k \sim CN(\mathbf{0}, \mathbf{R})$ and k is the index of range bins. The conditional distribution of $\mathbf{x}_k$ is

$$\mathbf{x}_k \,|\lambda_k \sim CN\left(0, \lambda_k^2 \mathbf{R}\right), \tag{2}$$

which implies power variations over different range bins. It can be shown that the compound-Gaussian model can be interpreted by the well known spherically invariant random process (SIRP) and has been verified extensively in the literature with measured data.

The complete distribution of $\mathbf{x}_k$ can be obtained as

$$f(\mathbf{x}_k) = \frac{1}{\pi^N |\mathbf{R}|} h_N\left(\mathbf{x}_k^H \mathbf{R}^{-1} \mathbf{x}_k\right), \tag{3}$$

where the function $f_N(\cdot)$ is defined as

$$h_N(x) = \int_0^\infty s^{-2N} \exp(-x/s^2) f_\lambda(s)\, ds, \tag{4}$$

with $f_\lambda(s)$ specifying the distribution of the texture component λ. The most common probability density functions (pdf) include the Weibull and the *K*-distribution. Specifically, the Weibull pdf is

$$f_\lambda(s) = abs^{b-1} \exp(-as^b), s \geq 0, a, b > 0 \tag{5}$$

with the shape parameter $0 < b \leq 2$, while the *K*-distribution pdf is

$$f_\lambda(s) = \frac{a^{v+1} s^v}{2^{v-1}\Gamma(v)} K_{v-1}(as), s \geq 0, a, v > 0 \tag{6}$$

where $\Gamma(\cdot)$ is the gamma function and $K_v(\cdot)$ is the modified Bessel function of the second-kind. Due to the independence between λ and $\mathbf{z}_k$, the clutter covariance matrix is given by

$$\mathbf{R}_x = E\left\{\mathbf{x}_k \mathbf{x}_k^H\right\} = E\left\{\lambda_k^2\right\} \mathbf{R} = P_\lambda \mathbf{R}, \tag{7}$$

where $\mathbf{R} = E\left\{\mathbf{z}_k \mathbf{z}_k^H\right\}$ and $P_\lambda = E\left\{\lambda_k^2\right\}$ specifies the texture power. Since $P_\lambda \mathbf{R} = (P_\lambda/q)(q\mathbf{R})$, we constrain the speckle covariance matrix to $\mathrm{tr}\left\{\mathbf{R}\right\} = N$.

2.2. Prior Solutions

The optimum solution to the binary hypothesis testing problem in the compound-Gaussian clutter needs to integrate the pdf over the distribution of the texture, which results in no close-form solutions. However, suboptimal solutions are available and include the NAMF with standard sample covariance matrix and the NAMF with the normalized sample covariance matrix, given by

$$T_{\text{S-NAMF}} = \frac{\left|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{x}_0\right|^2}{\left(\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{s}\right)\left(\mathbf{x}_0^H \hat{\mathbf{R}}^{-1} \mathbf{x}_0\right)}, \tag{8}$$

with $\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^H$, and

$$T_{\text{N-NAMF}} = \frac{\left|\mathbf{s}^H \tilde{\mathbf{R}}^{-1} \mathbf{x}_0\right|^2}{\left(\mathbf{s}^H \tilde{\mathbf{R}}^{-1} \mathbf{s}\right)\left(\mathbf{x}_0^H \tilde{\mathbf{R}}^{-1} \mathbf{x}_0\right)}, \tag{9}$$

with $\tilde{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^K \tilde{\mathbf{x}}_k \tilde{\mathbf{x}}_k^H$ and $\tilde{\mathbf{x}}_k = \frac{\mathbf{x}_k}{\sqrt{\mathbf{x}_k^H \mathbf{x}_k / K}}$ is the instantaneously normalized training signal. In space-time adaptive processing, the space-time covariance matrix usually exhibits a low rank structure. According to the Brennan's rule, the rank of the space-time covariance matrix is much smaller than the overall space-time dimension. By exploiting this structure, the normalized eigencanceler was proposed as [7,10]

$$T_{\text{N-EVD}} = \frac{\left|\mathbf{s}^H\left(\mathbf{I} - \mathbf{U}\mathbf{U}^H\right)\mathbf{x}_0\right|^2}{\left[\mathbf{s}^H\left(\mathbf{I} - \mathbf{U}\mathbf{U}^H\right)\mathbf{s}\right]\left[\mathbf{x}_0^H\left(\mathbf{I} - \mathbf{U}\mathbf{U}^H\right)\mathbf{x}_0\right]} \tag{10}$$

where $\mathbf{U}$ is obtained by perform the EVD of either the sample covariance matrix or the normalized sample covariance matrix.

3. SUBSPACE TRACKING-BASED DETECTORS FOR THE COMPOUND-GAUSSIAN ENVIRONMENT

Direct application of the subspace tracking technique to the compound-Gaussian environment fails to take into account the power variation over range bins and hence the subspace tracking technique cannot accurately track the subspace of the speckle component. In this section, two subspace tracking-based STAP detectors are proposed to mitigate the effect of the power variation when the subspace tracking is applied to the compound-Gaussian environment.

3.1. Subspace Tracking for STAP

The subspace tracking first reduces the signal dimension of $\mathbf{x}_k$ from MN to r, where $\mathrm{rank}(\mathbf{R}) < r \ll MN$. Denote by $\mathbf{W}$ of dimension $MN \times r$ the subspace concentration matrix. The compressed signal after subspace concentration process can be mathematically expressed as:

$$\mathbf{y}_k = \mathbf{W}^H \mathbf{x}_k, \tag{11}$$

with $\mathbf{W}$ given by the following optimization function

$$\mathbf{W} = \arg\min_{\hat{\mathbf{W}}} \left\|\mathbf{y}_k - \hat{\mathbf{W}}\hat{\mathbf{W}}^H \mathbf{x}_k\right\|^2. \tag{12}$$

The optimization problem in (12) can be numerically solved with subspace tracking algorithms such as the the conventional projection approximation subspace tracking (PAST) [13] and fast approximated power iteration (FAPI) [11, 14].

3.2. NST STAP Detector

The NST detector is proposed to replace the EVD of the normalized eigencanceler of (10) by some subspace tracking techniques. The NST detector, on one hand, improves existing subspace tracking-based STAP detector in [11] for the compound-Gaussian environment; on the other hand, it reduces the computational complexity of the normalized eigencanceler from $\mathcal{O}(M^3N^3)$ to about $\mathcal{O}(3MNr + 3r^2)$.

Since we are particularly interested in a low-complexity STAP in the compound-Gaussian environment, we choose the modified FAPI algorithm [12] to track the clutter principle subspace. The modified FAPI algorithm uses two consecutive approximations within the conventional FAPI algorithm and further reduces the computational complexity. It is noted that the FAPI was derived from the approximated power iteration (API) algorithm [15]. In API, the auxiliary matrix $\mathbf{Z}$ is updated as

$$\mathbf{Z}_k = \frac{1}{\beta}\mathbf{\Theta}_k^H\left[\mathbf{I}_r - \mathbf{g}_k \mathbf{y}_k^H\right]\mathbf{Z}_{k-1}\mathbf{\Theta}_k^{-H}, \tag{13}$$

Algorithm 1 NST-based STAP Procedure

Initialization: $\mathbf{W}_0 = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0}_{(n-r)\times r} \end{bmatrix}$, $\mathbf{Z}_0 = \mathbf{I}_r$.

Step 1 (Subspace Tracking Procedure):

for $k = 1, 2, \cdots, K$ **do**
- $\mathbf{y}_k = \mathbf{W}_{k-1}^H \mathbf{x}_k$
- $\mathbf{h}_k = \mathbf{Z}_{k-1}\mathbf{y}_k$
- $\mathbf{g}_k = \frac{\mathbf{h}_k}{\beta + \mathbf{y}_k^H \mathbf{h}_k}$
- $\epsilon_k^2 = \|\mathbf{x}_k\|^2 - \|\mathbf{y}_k\|^2$
- $\tau_k = \frac{\epsilon_k^2}{1+\epsilon_k^2\|\mathbf{g}_k\|^2 + \sqrt{1+\epsilon_k^2\|\mathbf{g}_k\|^2}}$
- $\eta_k = 1 - \tau_k \|\mathbf{g}_k\|^2$
- $\mathbf{y}_k' = \eta_k \mathbf{y}_k + \tau_k \mathbf{g}_k$
- $\mathbf{h}_k' = \mathbf{Z}_{k-1}^H \mathbf{y}_k'$
- $\boldsymbol{\epsilon}_k = \frac{\tau_k}{\eta_k}\left(\mathbf{Z}_{k-1}\mathbf{g}_k - \left((\mathbf{h}_k')^H \mathbf{g}_k\right)\mathbf{g}_k\right)$ and (18): $\mathbf{Z}_k = \frac{1}{\beta}\left(\mathbf{Z}_{k-1} - \mathbf{g}_k \mathbf{h}_k'\right)$
- $\mathbf{e}_k' = \eta_k \mathbf{x}_k - \mathbf{W}_{k-1}\mathbf{y}_k'$
- $\mathbf{W}_k = \mathbf{W}_{k-1} + \mathbf{e}_k' \mathbf{g}_k^H$

end for

Step 2 (STAP Detection):
- $\mathbf{W} = \mathbf{W}_K$
- $T_{\text{NST}} = \frac{\left|\mathbf{s}^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{x}_0\right|^2}{\left[\mathbf{s}^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{s}\right]\left[\mathbf{x}_0^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{x}_0\right]} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{NST}}$

where

$$\boldsymbol{\Theta}_k = \mathbf{W}_{k-1}^H \mathbf{W}_k, \tag{14}$$

and $\mathbf{g}_k$ of length r has the same definition as that in [14]. It is important to observe that the last term in (13), $\boldsymbol{\Theta}_k^{-H}$, not only incurs $\mathcal{O}(r^3)$ operation but also may enhance noise if $\boldsymbol{\Theta}_k$ is noisy. Motivated by this observation, two consecutive approximations of (13) are made. First, recalling that $\boldsymbol{\Theta}$ is nearly orthonormal [14], it is reasonable to approximate $\boldsymbol{\Theta}_k^{-H}$ as

$$\boldsymbol{\Theta}_k^{-H} \approx \boldsymbol{\Theta}_k. \tag{15}$$

Then, observing that $\mathbf{W}$ comprises of orthonormal column vectors. the second approximation

$$\boldsymbol{\Theta}_k \approx \mathbf{I}_r, \tag{16}$$

holds. With both approximations, $\mathbf{Z}_k$ in (13) now takes the following form:

$$\mathbf{Z}_k = \frac{1}{\beta}\boldsymbol{\Theta}_k^H \left[\mathbf{I}_r - \mathbf{g}_k \mathbf{y}_k^H\right]\mathbf{Z}_{k-1}. \tag{17}$$

It should be pointed out that the total computational complexity of the modified FAPI is $\mathcal{O}\left(3NMr + 3r^2\right)$, which is about $\mathcal{O}(2r^3)$ less operation as compared to the conventional FAPI. Incorporating (17) into the procedure of the FAPI [14], we can find the update function for $\mathbf{Z}_k$ using (17) is given as follows.

$$\mathbf{Z}_k = \frac{1}{\beta}\left(\mathbf{Z}_{k-1} - \mathbf{g}_k \mathbf{h}_k'\right), \tag{18}$$

where the definitions of $\mathbf{h}_k'$ and $\boldsymbol{\epsilon}_k$ are the same as those in [14] and given in Algorithm 1.

With the identified clutter principal subspace using the modified FAPI technique, the NST detector is shown as

$$T_{\text{NST}} = \frac{\left|\mathbf{s}^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{x}_0\right|^2}{\left[\mathbf{s}^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{s}\right]\left[\mathbf{x}_0^H\left(\mathbf{I}-\mathbf{W}\mathbf{W}^H\right)\mathbf{x}_0\right]} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{NST}}, \tag{19}$$

Algorithm 2 iNST-based STAP Procedure

Initialization: $\mathbf{W}_0 = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0}_{(n-r)\times r} \end{bmatrix}$, $\mathbf{Z}_0 = \mathbf{I}_r$.

Step 1 (Subspace Tracking Procedure):

for $k = 1, 2, \cdots, K$ **do**
- $\tilde{\mathbf{x}}_k = \frac{\mathbf{x}_k}{\sqrt{\mathbf{x}_k^H \mathbf{x}_k / K}}$
- $\mathbf{y}_k = \mathbf{W}_{k-1}^H \tilde{\mathbf{x}}_k$
- $\mathbf{h}_k = \mathbf{Z}_{k-1}\mathbf{y}_k$
- $\mathbf{g}_k = \frac{\mathbf{h}_k}{\beta + \mathbf{y}_k^H \mathbf{h}_k}$
- $\epsilon_k^2 = \|\tilde{\mathbf{x}}_k\|^2 - \|\mathbf{y}_k\|^2$
- $\tau_k = \frac{\epsilon_k^2}{1+\epsilon_k^2\|\mathbf{g}_k\|^2 + \sqrt{1+\epsilon_k^2\|\mathbf{g}_k\|^2}}$
- $\eta_k = 1 - \tau_k \|\mathbf{g}_k\|^2$
- $\mathbf{y}_k' = \eta_k \mathbf{y}_k + \tau_k \mathbf{g}_k$
- $\mathbf{h}_k' = \mathbf{Z}_{k-1}^H \mathbf{y}_k'$
- $\boldsymbol{\epsilon}_k = \frac{\tau_k}{\eta_k}\left(\mathbf{Z}_{k-1}\mathbf{g}_k - \left((\mathbf{h}_k')^H \mathbf{g}_k\right)\mathbf{g}_k\right)$ and (18): $\mathbf{Z}_k = \frac{1}{\beta}\left(\mathbf{Z}_{k-1} - \mathbf{g}_k \mathbf{h}_k'\right)$
- $\mathbf{e}_k' = \eta_k \tilde{\mathbf{x}}_k - \mathbf{W}_{k-1}\mathbf{y}_k'$
- $\mathbf{W}_k = \mathbf{W}_{k-1} + \mathbf{e}_k' \mathbf{g}_k^H$

end for

Step 2 (STAP Detection):
- $\tilde{\mathbf{W}} = \mathbf{W}_K$
- $T_{\text{iNST}} = \frac{\left|\mathbf{s}^H\left(\mathbf{I}-\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\right)\mathbf{x}_0\right|^2}{\left[\mathbf{s}^H\left(\mathbf{I}-\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\right)\mathbf{s}\right]\left[\mathbf{x}_0^H\left(\mathbf{I}-\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\right)\mathbf{x}_0\right]} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{iNST}}$

where $\mathbf{W} = \mathbf{W}_K$ is the estimated principle clutter subspace based on K training signals and γ_{NST} is a threshold subject to the probability of false alarm. Compared with the normalized eigencanceler, the NST detector shares the same test statistic but replaces the EVD with the subspace tracking algorithm to track the clutter principle subspace with less computational complexity. Compared with the conventional subspace tracking based STAP detector [11], the additional normalization factor in the denominator of the NST test statistic is critical to handle power variation effect in the compound-Gaussian environment, especially when the number of training signals is small (see Section 4 for the discussion).

3.3. iNST STAP Detector

In fact, the above modified FAPI algorithm works well when the training signals $\mathbf{x}_k$ are homogeneous, i.e., they are all complex Gaussian random vectors with zero mean and covariance matrix $\mathbf{R}$. In the compound-Gaussian scenario, this homogeneous assumption is no longer valid and the normalization at the test statistic level maybe not sufficient to suppress the power variation. As such, one more normalization at the signal level is required, especially in the case of strong power variation cross range bins. A simple solution for this purpose is an instantaneous normalization at the signal level, i.e.,

$$\tilde{\mathbf{x}}_k = \frac{\mathbf{x}_k}{\sqrt{\frac{\mathbf{x}_k^H \mathbf{x}_k}{K}}}, \tag{20}$$

which produces training signals with approximately equal power. Then, the modified FAPI algorithm is applied to the normalized training signals $\tilde{\mathbf{x}}_k$ to track the clutter principle subspace of the speckle component. Denote $\tilde{\mathbf{W}}$ as the estimated clutter principle subspace by applying the subspace tracking algorithm to the normalized training signals $\tilde{\mathbf{x}}_k$. The iNST detector can be expressed

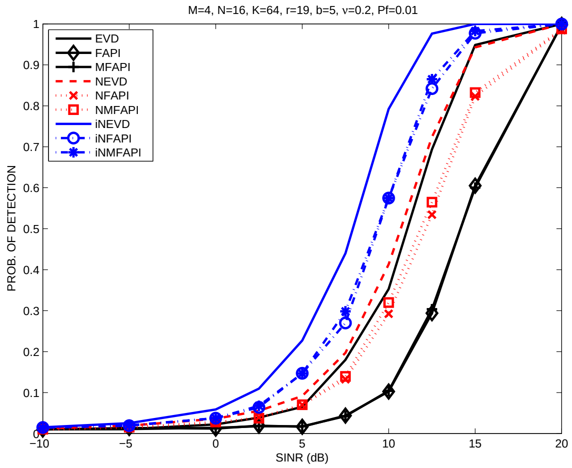

Fig. 1. Probability of detection versus SINR with moderate power variation ($b = 5$ and $v = 0.2$) and limited training signals ($K = MN$).

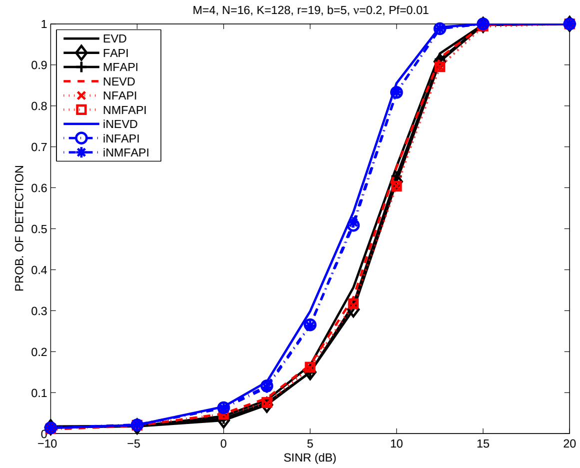

Fig. 2. Probability of detection versus SINR with moderate power variation ($b = 5$ and $v = 0.2$) and sufficient training signals ($K = 2MN$).

as

$$T_{\text{iNST}} = \frac{\left| \mathbf{s}^H \left(\mathbf{I} - \tilde{\mathbf{W}}\tilde{\mathbf{W}}^H \right) \mathbf{x}_0 \right|^2}{\left[\mathbf{s}^H \left(\mathbf{I} - \tilde{\mathbf{W}}\tilde{\mathbf{W}}^H \right) \mathbf{s} \right] \left[\mathbf{x}_0^H \left(\mathbf{I} - \tilde{\mathbf{W}}\tilde{\mathbf{W}}^H \right) \mathbf{x}_0 \right]} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{iNST}}, \tag{21}$$

where γ_{iNST} is a threshold subject to the probability of false alarm. In general, the iNST detector is similar to the NST except that the instantaneous normalization step is performed before applying the subspace tracking technique. The implementation of the iNST detector is summarized in Algorithm 2.

4. NUMERICAL EVALUATION

In this section, we evaluate three classes of detectors: 1) the conventional subspace tracking detectors designed for the homogeneous environment [11] (denoted as black curves); 2) the NST detectors with the FAPI and the introduced modified FAPI algorithms (denoted as red curves); and 3) the iNST detectors with the FAPI and the introduced modified FAPI algorithms (denoted as blue curves) in the K-distributed compound-Gaussian backgrounds. Within each class of detectors, the computationally extensive EVD-based eigencanceler is also implemented and its performance provides a reference on performance loss due to application of the subspace tracking technique. Several scenarios with different levels of power variation (i.e., changing parameters b and v in the K-distribution of clutter) and different sets of training signals (i.e., different K) are considered as follows.

We first consider a scenario with *moderate* power variation and *limited* training signals. More specifically, the K-distributed clutter is generated with parameters $b = 5$ and $v = 0.2$ and the number of training signals is $K = MN = 64$. Simulation results for three classes of detectors are shown in Fig. 1. It is seen that the NST detectors with the FAPI and modified FAPI algorithms (i.e., NFAPI and NMFAPI in the figure) improve detection performance of the conventional FAPI and modified FAPI approaches (i.e., FAPI and MFAPI in the figure) by about 3dB. while the iNST detectors with the FAPI and modified FAPI algorithms (i.e., iNFAPI and iNMFAPI in the figure) further improves the detection performance by about 6 dB with respect to the FAPI and MFAPI detectors. It is also seen that, in each class of detectors, the EVD-based detector provides an upper bound for the two subspace tracking based detectors. The performance losses between the EVD-based detector and the subspace tracking-based approach are about 2-3 dB in the NST and iNST categories, while the performance loss increases to about 4-5 dB in the conventional category without any normalization.

Then, we increase the number of training signals to $K = 2MN = 128$ while keeping the same level of power variation among training signals. Simulation results are shown in Fig. 2. When the training signals are sufficient, the performance of subspace tracking-based detectors both converge to the upper bound of the EVD-based detectors in all three classes of detection. In other words, with sufficient training signals, the performance loss due to the subspace tracking algorithm is negligible. It is also noted that the NST detectors, i.e., the NFAPI and NMFAPI, give the same performance as the conventional FAPI and MFAPI. In other words, there is no performance gain of the NST detectors by introducing the normalization only at the test statistic level. In contrast, the iNST detectors, i.e., the iNFAPI and iNMFAPI, show better detection performance than the conventional subspace tracking and the NST detectors with about 2-3 dB performance gain.

Next, we consider a scenario with *strong* power variation and *limited* training signals, where the K-distributed clutter is generated with parameters $b = 8$ and $v = 0.125$ and the number of training signals is $K = MN = 64$. Simulation results are shown in Fig. 3. Again, the subspace tracking detectors give detection performance with a certain gap with respect to their upper bounds provided by the EVD-based detector in all three classes of detection. The performance improvement from the conventional detectors to the NST detectors and from the NST detectors to the iNST detectors can be clearly observed. From Figs. 1 and 3, it can be concluded that the normalization at the test statistic level of the NST detector is more critical to improve the detection performance in the case of limited training signals.

Finally, we increase the number of training signal to $K = 2MN = 128$ and maintain the level of power variation. Simulation results are shown in Fig. 4. The result verifies that, with

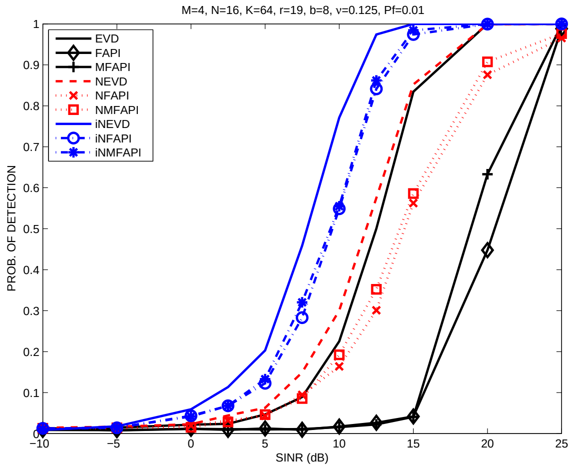

Fig. 3. Probability of detection versus SINR with strong power variation ($b = 8$ and $v = 0.125$) and limited training signals ($K = MN$).

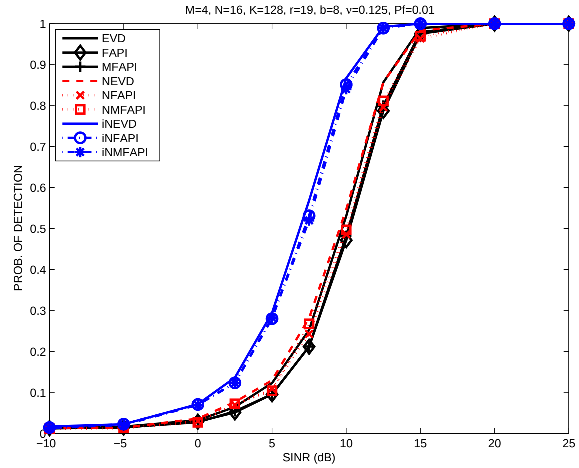

Fig. 4. Probability of detection versus SINR with strong power variation ($b = 8$ and $v = 0.125$) and sufficient training signals ($K = 2MN$).

sufficient training signals, the performance loss due to application of the subspace tracking algorithm is quite small by comparing the performance of the subspace tracking detectors and that of the EVD-based detector in all three classes of detectors. Moreover, in this case, the normalization at the test statistic level is not enough to suppress the power variation in the compound-Gaussian environment by comparing performance of the NST detectors (the NFAPI and NMFAPI) and that of the conventional detectors (FAPI and MFAPI). The performance improvement is possible by using the iNST detectors with the additional instantaneous normalization at the signal level. Comparison between Fig. 4 and Fig. 2 shows that the performance gain from the NST detectors to the iNST detectors increases from about 2-3 dB to about 3-4 dB, which implies that the instantaneous normalization is more critical in the case of strong power variation.

5. CONCLUSION

In this paper, motivated by the progress in the subspace tracking, we propose a low-complexity STAP strategy via subspace tracking in the nonhomogeneous compound-Gaussian environment. Specifically, the normalized and instantaneously normalized subspace tracking-based detectors are proposed to track the subspace of the speckle component and mitigate the effect of the time-varying texture component. On one hand, the proposed subspace tracking detectors are training efficient due to its exploit of the low-rank structure of the speckle component, as compared to the conventional covariance matrix based detectors, e.g., the NAMF; on the other hand, they are computationally more efficient than the EVD-based eigencanceler, thanks to the fast subspace tracking algorithms.

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