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Abstract

In this paper, an energy optimizing extremum seeking controller is developed for vapor compression systems (VCS) that automatically discovers sets of inputs that minimizes the energy consumption while the machine is in operation. This controller optimizes an input-output map (from VCS inputs to electrical energy consumed) in realtime, and without relying on a model of the dynamics of a vapor compression system. A detailed algorithm and rules for tuning the controller gains will be described. Experiments are performed on an inverter-driven room air conditioner that demonstrate convergence of inputs to their optimal values, resulting in an improvement in COP of 10-20% for some operating points.

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Extremum Seeking Control for Energy Optimization of Vapor Compression Systems

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ABSTRACT

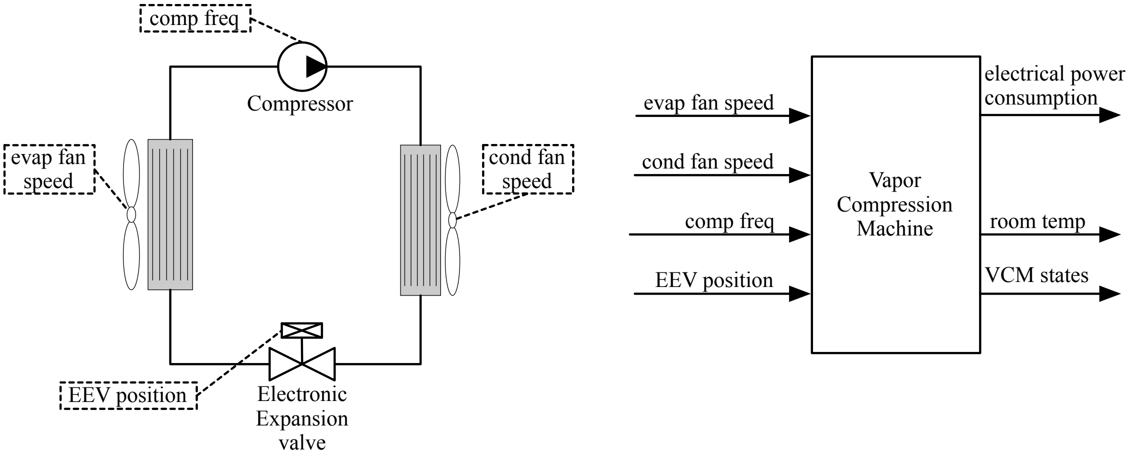
In this paper, an energy optimizing extremum seeking controller is developed for vapor compression systems (VCS) that automatically discovers sets of inputs that minimizes the energy consumption while the machine is in operation. This controller optimizes an input-output map (from VCS inputs to electrical energy consumed) in realtime, and without relying on a model of the dynamics of a vapor compression system. A detailed algorithm and rules for tuning the controller gains will be described. Experiments are performed on an inverter-driven room air conditioner that demonstrate convergence of inputs to their optimal values, resulting in an improvement in COP of 10-20% for some operating points.

1. INTRODUCTION

Vapor compression systems (VCS), such as heat pumps, refrigeration and air-conditioning systems, are widely used in industrial and residential applications. The introduction of variable speed compressors, electronically-positioned valves, and variable speed fans to the vapor compression cycle has greatly improved the flexibility of the operation of such systems. This increased flexibility allows the heat delivered by the machine to be directly matched to the load, and this design has proved to be more energy efficient than the duty cycling characteristic of vapor compression systems with fixed speed compressors.

Further, the combination of commanded inputs to the VCS that delivers a particular amount of heat is often not unique, and various combinations of inputs (compressor speed, fan speeds, etc.) constitute sets of inputs where each set will cause the VCS to consume different amounts of energy. As the operating conditions vary (heat load, outdoor air temperature), the inputs to the VCS necessarily must change to regulate various quantities to their setpoints. However, by operating the VCS with sets of inputs where each set has been determined to be energy optimal for a particular heat load, the coefficient of performance (COP) can be dramatically improved.

Conventionally, methods for maximizing the energy efficiency rely on the use of mathematical models of the physics of vapor compression systems. These model-based approaches attempt to describe the influence of commanded inputs on the thermodynamic behavior of the system and the consumed energy, and they are used to predict the combination of inputs that both meets the heat load requirements and minimizes energy. However, these models of vapor compression systems rely on simplifying assumptions in order to produce a mathematically tractable representation. These assumptions often ignore important effects or do not consider installation-specific characteristics such as room size, causing the model of the system to deviate from actual behavior of the system. Additionally, the variation in these systems during the manufacturing process are often so large as to produce vapor compression systems of the same type that exhibit different input-output characteristics, and therefore a single model cannot accurately describe the variations among copies produced as the outcome of a manufacturing process.



A. B.

Figure 1: (A) The vapor compression system consists of a compressor, condensing heat exchanger, electronically controlled expansion valve, and evaporating heating exchanger. The inputs to the VCS that are manipulated by the control system include (i) the compressor frequency, (ii) the condenser fan speed, (iii) the EEV position, and (iv) the evaporator fan speed. **(B)** The block diagram representation of the VCS outputs electrical power consumption and other outputs used for regulation.

Finally, model-based optimization methods are difficult to derive and calibrate, and often do not describe variations over long time scales, such as those due to refrigerant losses or accumulation of debris on the heat exchangers.

Recently, a class of optimizing controllers called “extremum seeking” has received increased attention in the controls literature since a rigorous proof of stability was published by Krstić (2000). Its ability to automatically discover inputs that optimize a metric of performance without requiring a model of the system distinguish it from other nonlinear control techniques such as feedback linearization, sliding mode control or model reference adaptive control. Recently, extremum seeking has enabled feedback control of challenging nonlinear systems such as those that exhibit changes in the structure of the input-output relationship, and it continues to facilitate the optimization of internal combustion engine (Killingsworth 2009, Popović 2006), process control (Hudon 2008, Tyagi 2006), and mobile robots (Mayhew 2008, Zhang 2007). Although extremum seeking control has previously been applied to the broad area of HVAC systems and building optimization (Pengfei 2009, Sane 2006), the authors believe this work represents the first application of extremum seeking to vapor compression systems.

In this paper, we present a model-free optimizing controller that automatically discovers inputs to the vapor compression system that minimize energy consumption while still regulating zone temperature and various internal thermodynamic properties. This optimizing controller perturbs one input while monitoring the response of the electrical power consumed, and drives that input so that the overall energy consumption is minimized. Meanwhile, a conventional feedback controller regulates zone temperature and any internal vapor compression system states. The end result is that for a given thermodynamic condition, the full set of VCM inputs is selected that both meets comfort requirements and minimizes electrical power consumption.

The rest of the paper is organized as follows: Section 2 describes the vapor compression system under study and presents necessary conditions for extremum seeking control. Section 3 presents the extremum seeking algorithm and how it is employed in this application. Experimental results are shown in Section 4, and concluding remarks are offered in Section 5.

2. VAPOR COMPRESSION MACHINE

A simplified hydraulic diagram for a single indoor unit vapor compression machine is shown in Figure 1-A. Recent advancements in microcontroller technology and actuator design have provided a higher degree of controllability for

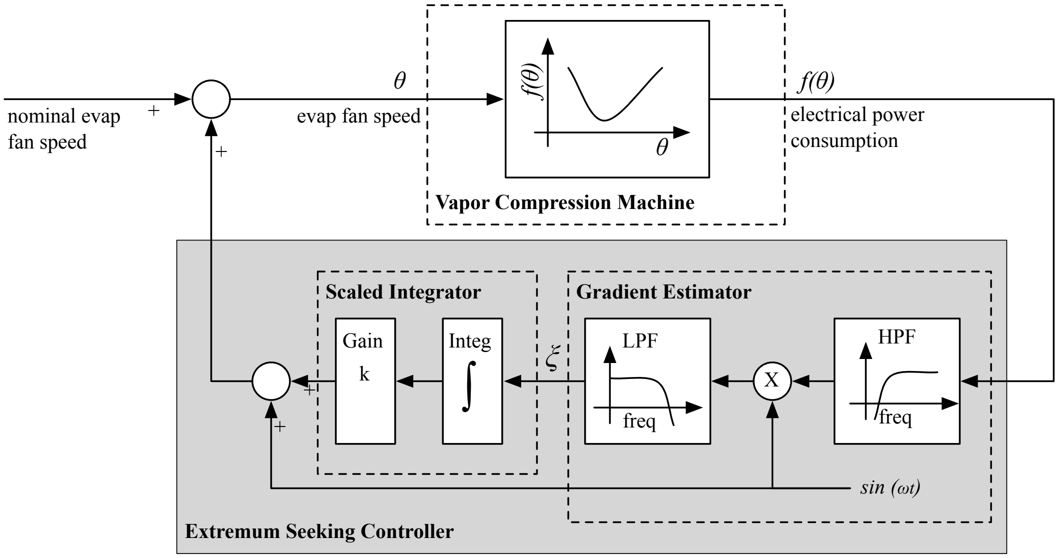


Figure 2: The extremum seeking algorithm consists of a gradient estimator and a scaled integrator. This controller perturbs one input to the vapor compression system, and uses the response to that perturbation to drive the inputs in the direction that minimizes electrical power consumption.

this class of systems. For example, compressors are now commonly driven by electrical inverters, allowing variable speed operation, the expansion valve has advanced from thermostatic control to electronic control, and the two heat exchanger fans are also now variable speed devices. This shift to electronic actuator control under the direction of an onboard microcontroller has resulted in increased flexibility and presents an opportunity for multi-objective optimization. In this paper, we exploit increased actuator capability to (1) meet customer comfort requirements (by regulating zone temperature) and (2) minimize electrical power consumption with an extremum seeking controller.

The inputs to the vapor compression system are represented in a block diagram in Figure 1-B. The system inputs include compressor frequency, electronic expansion valve (EEV) opening position, and the fan speeds for both the indoor unit and outdoor unit. (When the VCS is providing cooling, the indoor unit heat exchanger acts as an evaporator, and the outdoor unit heat exchanger acts as a condenser; the opposite pairing occurs when the VCS is a heat pump.) The outputs include the zone temperature (room dynamics are lumped into this representation), internal states of the vapor compression machine that may be used by the feedback controller for equipment regulation, and electrical power consumption that will be used by the extremum seeking controller.[1]

A necessary requirement for extremum seeking control is that the relationship between a system input and the output to be extremized must be convex. In the context of this application, we require convexity in the input-output map between one of the vapor compression system inputs (we have selected evaporator fan speed, although other inputs would work provided convexity holds) and the electrical power consumption. We have demonstrated the existence of convexity in this relationship through dynamic models and the experimental results shown in Section 4. The next section presents an extremum seeking algorithm and describes how it can be designed and tuned for a vapor compression system.

3. EXTREMUM SEEKING ALGORITHM

Extremum seeking control is an adaptive algorithm where the goal is to optimize an objective function in realtime by driving inputs to values that minimize the cost function. Although the goal is to minimize some cost function, it is assumed that the cost function itself, as well as plant dynamics are unknown.

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[1] Practically speaking, electrical power consumption is not typically a signal directly measured by most vapor compression systems, however, an estimated version of this signal is usually a straightforward relation of actuator values.

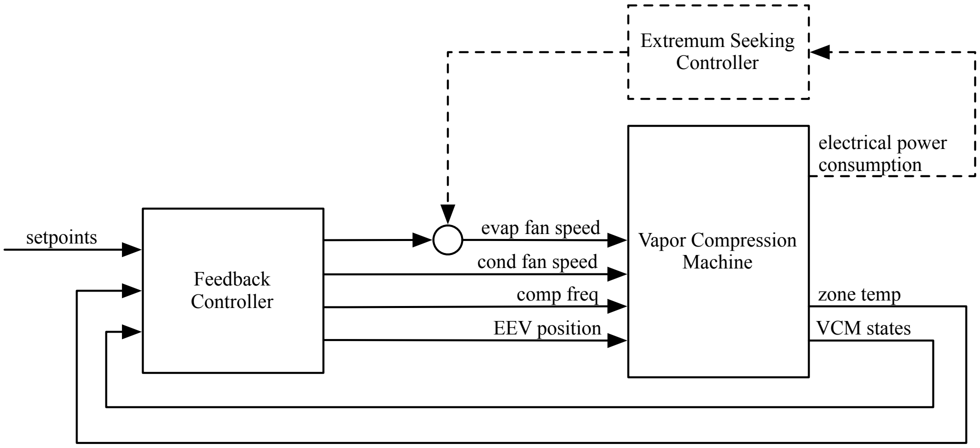


Figure 3: The extremum seeking controller is configured to work with the closed loop vapor compression system. The existing feedback controller generates commands for all four VCS inputs, but the command to the evaporator fan is modified by the extremum seeking controller. The result is that all four inputs are driven to values that minimizes power consumption.

Let the the electrical power consumed by the vapor compression system $f(\theta)$ be a convex function of the evaporator fan speed θ. The goal of the extremum seeking controller is then to find θ^* such that $f(\theta^*)$ is minimal. As shown in Figure 2, the extremum seeking algorithm can be viewed as a realtime gradient descent controller consisting of two parts: (1) a gradient estimator and (2) a scaled integrator that drives the input in response to the estimated gradient.

The gradient estimator adds a sinusoidal perturbation to the system input, and extracts the part of the system response due to that perturbation. A high pass filter passes the system's response to the sinusoidal perturbation, and this response is then multiplied by a sinusoid in order to obtain a signal that contains terms proportional to the gradient. A low pass filter removes unnecessary components, and the resulting signal ξ represents the estimated gradient. Conventionally, the perturbation frequency ω is chosen to be slower than the unknown dominant plant dynamics in order to avoid plant excitation, and the high pass filter cutoff frequency is selected to pass the sinusoidal perturbation frequency ($\omega_{hp} > \omega$).

The scaled integrator is then used to drive the actuator command in the direction of the gradient. A scaling factor k is used to control the rate of actuator adaptation and tailor the algorithm to problems where the objective is either a minimization ($k<0$) or maximization ($k>0$) of the objective function. In this manner, the extremum seeking controller aims to drive the gradient to zero, which ensures that the input θ is driven to its unknown optimal value θ^* and the resulting objective function $f(\theta)$ itself is driven to its unknown optimal value f^*.

The extremum seeking algorithm makes no assumption about model structure (other than the existence of convexity), and therefore this class of adaptive controllers tend to be robust to time-varying plants and many types of nonlinearities. For more information on the extremum seeking algorithm, see (Ariyur 2003).

3.1 Interfacing with Regulating Controllers

Whereas the extremum seeking controller performs optimization, a conventional feedback controller is still required to perform the regulation that is characteristic of all vapor compression systems. In particular, the feedback controller must be designed to regulate zone temperature, reject disturbances originating from variable heat loads and outdoor air temperature fluctuations, and stabilize the internal states of the VCS.

Figure 3 shows the typical feedback controller regulating the four inputs to the VCS with the addition of an extremum seeking controller. The extremum seeking controller modifies the command sent to the evaporator fan. This modification, consists of two components (1) a sinusoidal perturbation intended to estimate the gradient of the power consumption at the current operating point, and (2) a correction signal that is intended to drive the evaporator command signal to a value that minimizes power. This correction signal is added to the nominal signal output from the feedback controller.

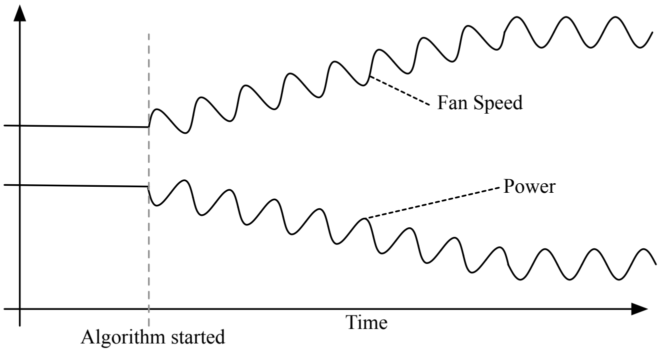


Figure 4: When the extremum seeking algorithm is started in a siutation where the evaporator fan is slower than optimal, the sinusoidal perturbation to the fan speed command will cause a response in the measured power. Meanwhile the feedback controller continues to regulate zone temperature while commanding all four inputs. The result is that all four inputs are driven to values that minimizes power consumption.

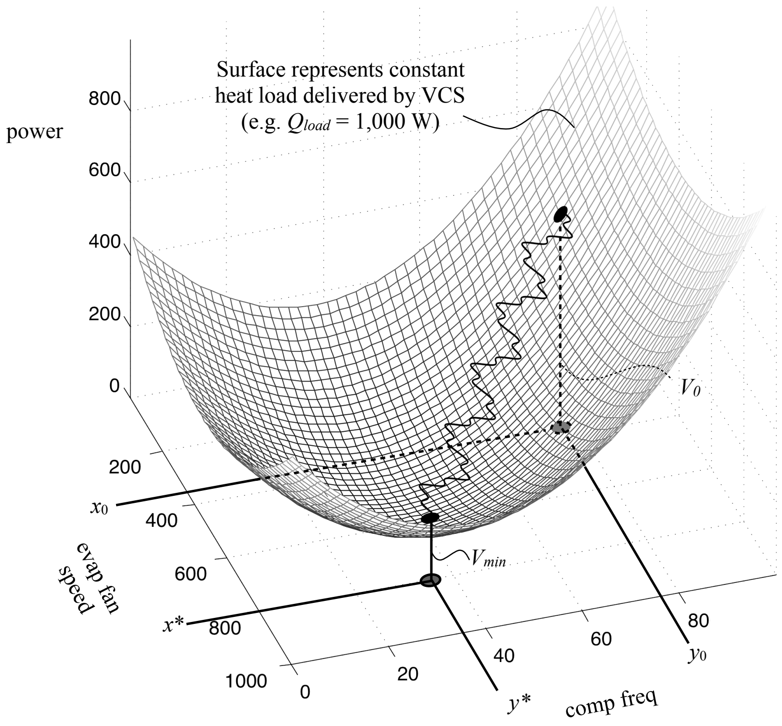
In order to keep the two controllers from competing over control of the evaporator fan, the extremum seeking controller is designed so that the adaptation is slow compared to the dominant dynamics of the VCS and its regulating controller. Therefore, any transient in zone temperature or internal VCS state due to the feedback controller part of the evaporator command will die out before the extremum seeking controller can misinterpret the transient as due to the sinusoidal perturbation.

Separating the time scales of the VCS and regulating controller from the extremum seeking controller allows all four inputs to be optimized in the following way: Consider a VCS that is controlled only by the regulating feedback controller in equilibrium with a room. The feedback controller nominally selects commands to the four actuators such that the room temperature is near its setpoint, and the VCS states are regulated. Further, assume the feedback controller has selected an evaporator fan speed that is slower than is optimal. When the extremum seeking algorithm is started (see Figure 4) a sinusoidal perturbation is added to the evaporator fan speed. As this perturbation causes an increased fan speed compared to the initial fan speed, more heat transfer occurs in the evaporator and the room temperature decreases. If the time scales are separated, the feedback controller will react to this decreased room temperature first, and select a slower compressor speed. After the transients resulting from the new feedback controller commands have died out, the resulting overall power consumption will be reduced because the compressor is the largest consumer of power. The extremum seeking controller will interpret this as “an increase in evaporator fan speed results in reduced power consumption” and will continue to drive the fan speed so that power is decreased. Meanwhile, the feedback controller continues to regulate zone temperature and internal VCS states and react to the disturbance introduced by the extremum seeking controller. In this way, the compressor (which is the largest consumer of power in the system) is influenced by the extremum seeking controller so that the *overall* system power is minimized.

This behavior is perhaps more easily understood on a surface plot. When viewed as a function of multiple VCS inputs, the overall power consumption has the form shown in Figure 5. The surface represents a constant amount of heat delivered by the VCS. We note that many combinations of system inputs will meet the heat load, but only a particular combination will do so while consuming minimum energy. By perturbing one input, the extremum seeking controller will drive the system from the initial operating point that consumes energy V_0, to the minimum energy V_{min}. With the feedback controller operating continuously, a perturbation from the extremum seeking controller will cause a response in all of the actuator commands. And because the input to the extremum seeking controller is overall power consumption of the entire machine, all four of the actuators will be driven to values that minimizes overall power consumption and thereby maximizes COP. This is experimentally demonstrated in the following section.

4. EXPERIMENTAL RESULTS

In order to validate the extremum seeking controller, an inverter-driven room air conditioner (Mitsubishi Electric, MUZ A09-NA) is instrumented and the indoor unit is installed in an adiabatic test chamber. Electric heaters



Surface represents constant heat load delivered by VCS (e.g. Q_{load} = 1,000 W)

Figure 5: Many combinations of inputs result in the same amount of heat delivered by a vapor compression system. However, one combination consumes a minimal amount of energy. By perturbing one input and measuring the overall power consumption, the extremum seeking controller will drive the system so that all inputs are steered to values that minimize power.

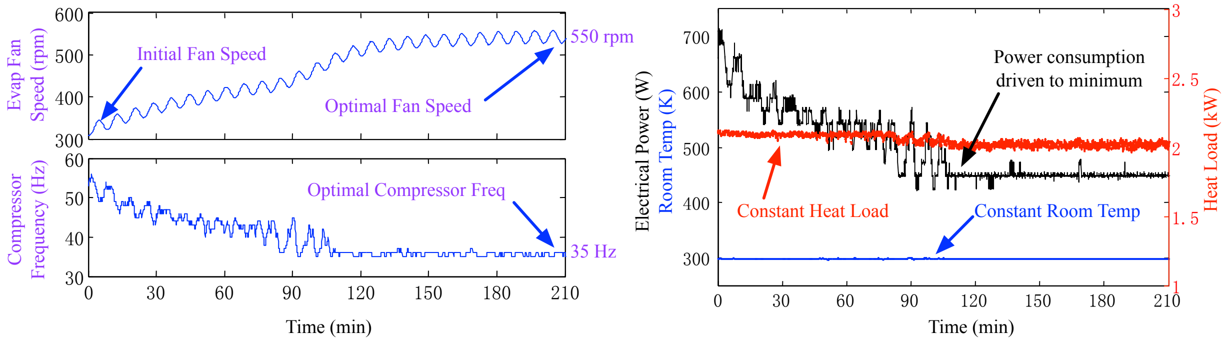
(McMaster-Carr, 3625K64) are used for the load, and an external custom controller is interfaced to the evaporator fan. Data logging equipment (National Instruments, NI cDAQ-9178) is configured to record temperatures and various VCS states. Power consumption is measured at the VCS connection to the external power supply (mains) using an electric power meter (Continental Control Systems, WNB-3Y-208-P).

Initially, a 2,000 W heat load is applied in the test chamber, the room temperature setpoint is set to 27°C, and the indoor fan is set to 300 rpm. The system is allowed to reach equilibrium under the direction of the feedback controller which forced the compressor to run at 56 Hz in order to maintain room temperature. The electric power consumption of the system in this state is 700 W, which yields a COP of 2.9.

At this point the extremum seeking controller is switched on, and the results are shown in Figure 6. The evaporator fan speed (Figure 6-A, top) shows the sinusoidal perturbation from the extremum seeking controller. As the extremum seeking controller increases the average evaporator fan speed, the compressor frequency decreases (Figure 6-A, bottom), which causes a decrease in electric power consumption (Figure 6-B). Additionally, the feedback controller maintains the room temperature during this constant-load experiment (Figure 6-B). The end result is that the compressor evaporator fan is driven from 300 rpm to 550 rpm, the compressor frequency decreases from 56 Hz to 35 Hz, and the overall power consumption drops from 700 W to 450 W. During this experiment, thermodynamic conditions in the test chamber do not change, implying that occupants in that environment would not notice changes in comfort levels. The COP at the end of the experiment is 4.4, or an increase of 52% from the beginning of the experiment.

5. CONCLUSIONS

In this paper, an extremum seeking controller is applied to a vapor compression system and is shown to automatically discover the inputs that optimize power consumption for a given heat load and outdoor air



A. B.

Figure 6: (A) The evaporator fan (top) is perturbed with an extremum seeking controller starting at $t=0$. This perturbation causes a response in the compressor frequency (bottom). (B) The overall power consumption (black line) is driven from about 700 W to 450 W while the heat load and room temperature remain constant, which results in an increase in COP from 2.9 to 4.4 for this operating point.

temperature. The extremum seeking algorithm is described with particular modifications for use with the existing feedback controllers characteristic to VCS that must regulate zone temperature and additional internal states of the vapor compression machine. The end result is that occupant comfort is maintained while the VCS adjusts its inputs so that overall power consumption is minimized, thereby achieving realtime optimization of performance. Experiments are performed that demonstrate the minimization of power consumption while zone temperature is regulated. Future work includes tuning of the extremum seeking parameters for increased convergence rates and extension of the algorithm to multiple indoor unit configurations.

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