MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Further Developments and Applications of Network Reference Governor for Constrained Systems

Di Cairano, S.; Kolmanovsky, I.V.

TR2012-048 June 2012

Abstract

This paper develops further the network reference governor, which is a predictive algorithm for modifying commands sent to the remote system to satisfy state and control constraints. Due to the network communication, the governor must account for a delay that can be time-varying and unknown. The paper summarizes the results on network reference governor theory, and demonstrate its operation on a case study of a attitude control of a spacecraft with a very flexible appendage, where the commands are transmitted remotely over a network to the spacecraft, and hence delayed by a bounded, unknown delay. In this case study, the reference governor ensures that the elastic deflections of the appendage and the control signal satisfy the imposed limits while the spacecraft performs a reorientation maneuver. The paper then presents the novel theoretical construction of a less conservative network reference governor for the case when the delay is long but only slowly time-varying, with known bounds on the rate of change. A spacecraft relative motion control example with constraints on thrust and Line Of Sight (LoS) cone positioning is considered to illustrate these theoretical developments.

American Control Conference

Further Developments and Applications of Network Reference Governor for Constrained Systems

Stefano Di Cairano, Ilya V. Kolmanovsky

***Abstract*— This paper develops further the network reference governor, which is a predictive algorithm for modifying commands sent to the remote system to satisfy state and control constraints. Due to the network communication, the governor must account for a delay that can be time-varying and unknown. The paper summarizes the results on network reference governor theory, and demonstrate its operation on a case study of a attitude control of a spacecraft with a very flexible appendage, where the commands are transmitted remotely over a network to the spacecraft, and hence delayed by a bounded, unknown delay. In this case study, the reference governor ensures that the elastic deflections of the appendage and the control signal satisfy the imposed limits while the spacecraft performs a reorientation maneuver. The paper then presents the novel theoretical construction of a less conservative network reference governor for the case when the delay is long but only slowly time-varying, with known bounds on the rate of change. A spacecraft relative motion control example with constraints on thrust and Line Of Sight (LoS) cone positioning is considered to illustrate these theoretical developments.**

I. Introduction

In aerospace applications, remotely piloted and unmanned air and space vehicles rely on communication relay networks and links for exchanging commands and measurements with ground stations. Electronic control modules within these vehicles are in turn interconnected with internal communication networks exchanging subsystem input and output information. The new generation of spacecraft, based on Plug-n-Play standards [1], will incorporate modern networking protocols that facilitate rapid, flexible, modular satellite constructions. These network-based data bus protocols introduces communication delays that are network-traffic dependent, and thus time-varying. Hence, control systems need to robustly compensate the variable disturbances introduced by the network communication [2]–[4].

A particular class of control algorithms that have been extended to the networked control framework are the reference governors. Reference governors [5]–[8] are constrained control algorithms that modify the exogenous reference (or command) to a plant stabilized by an inner-loop controller, in order to enforce constraints on closed-loop system outputs. Since the reference governor relies on on-line optimization which may be computationally intensive for certain embedded applications, locating it remotely with respect to the system being controlled may be justified for these systems. Previous contributions in the area of reference governor for remote controller executions can be found in [9], [10], for network-induced delays that are multiple of the sampling period. More recently in [11], the authors have proposed a scheme that guarantees robustness to delays that are not necessarily multiple of the sampling period. The approach is based on reformulating the system dynamics so that the effect of the network-induced delay appears as an additive disturbance withe magnitude affinely dependent on the change in the reference governor output. The reference governor was modified to account for this disturbance thereby resulting in a scheme that effectively trades-offs the rate of convergence to the desired reference versus the magnitude of the induced disturbances due to the command variations. In this paper, in Section II we briefly review the recent results in [11] on our approach to network reference governor design. In Section III we present an application of these results to a challenging attitude control problem for a spacecraft with a very flexible appendage subject to control input magnitude constraints and constraints on maximum appendage deflection. In Section IV we present a new network reference governor construction with reduced conservativeness for the case of a slowly varying delay. In Section V we illustrate the results using a case study of a remotely piloted spacecraft relative motion control. Finally, concluding remarks are summarized in Section VI

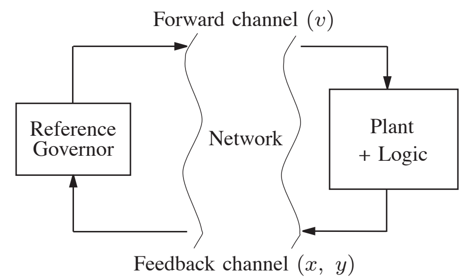

Fig. 1. Network reference governor schematics

Notation: The sets of real, nonnegative real, and nonnegative integer numbers are denoted by $\mathbb{R}$, $\mathbb{R}_{0+}$, and $\mathbb{Z}_{0+}$, respectively. For a signal $a(t)$, $t \in \mathbb{R}_{0+}$, sampled with sampling period T_s we denote by $a(k)$ the value at the k^{th} sampling instant, that is $a(k) = a(t_k)$, where $t_k = kT_s$.

S. Di Cairano was with Ford Research and Adv. Engineering, Dearborn, MI, dicairano@ieee.org. He is now with Mitsubishi Electric Research Laboratory, Cambridge, MA.

This research was not sponsored by MELCO or any of its subsidiaries.

I.V. Kolmanovsky is with Dept. Aerospace Engineering, the University of Michigan, Ann Arbor, MI, ilya@umich.edu. His research is supported in part by the National Science Foundation, Award Number 1130160 to the University of Michigan.

Where a is a vector, $\|a\|_p$ indicates the p-norm and $[a]_i$ is the i^{th} component, while where A is a matrix, $\|A\|_p$ indicates the induced p-norm. Where $\mathcal{X}$ is a set, $\mathrm{int}[\mathcal{X}]$ indicates the interior, and $\mathcal{X} \oplus \mathcal{Y}$ is the Minkowski sum of sets $\mathcal{X}$ and $\mathcal{Y}$.

II. Network Reference Governor for Systems with Unkwnown Variable Delay

We first introduce the system model, and review the main results on network reference governor in [11].

Consider the control architecture shown in Figure 1, which consists of a linear plant (2a) (possibly interconnected with a local controller) and a remotely located reference governor (2c). The commands and measurements are exchanged through a communication network and are subject to time delay. In [11] the authors have shown that when delays are present in both the command and the measurement channels, an equivalent representation where the delay is present only in the command channel can be obtained. Consequently, in this paper we focus on the dynamics where the delay affects only the command channel,

$$\dot{x}(t) = A_c x(t) + B_c r(t - \delta(t)), \quad (1a)$$
$$y(t) = C_c x(t), \quad (1b)$$

where $x \in \mathbb{R}^n$, $r \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $\delta(t) \in \mathbb{R}_{0+}$, for all $t \in \mathbb{R}_{0+}$, A_c is strictly Hurwitz, i.e., plant is asymptotically stable (possibly because of a local controller), and (A_c, C_c) is observable. The signal r is interpreted as a reference command specified to achieve the desired equilibrium.

Consider a delay free version of (1a) (i.e., for all $t \in \mathbb{R}_{0+}$, $\delta(t) = 0$) which is sampled with period T_s. We assume that observability is preserved, and we consider pointwise in-time output constraints $Hy(k) \leq K$, where the set $\mathcal{C} = \{y \in \mathbb{R}^p : Hy \leq K\}$ is compact and $0 \in \mathrm{int}[\mathcal{C}]$. Instead of r, a *virtual reference* v is applied to the system, as generated by a reference governor [7], [8],

$$x(k+1) = Ax(k) + Bv(k) \quad (2a)$$
$$y(k) = Cx(k) \quad (2b)$$
$$v(k) = g(x(k), r(k)) \quad (2c)$$
$$Hy(k) \leq K, \quad (2d)$$

where $v(t) = v(k)$ for all $t \in [t_k, t_{k+1})$, $A = e^{A_c T_s}$ is strictly Schur, $B = \int_0^{T_s} e^{A_c(T_s - \tau)} B_c d\tau$. In what follows, the value of the state x is assumed to be known only at the sampling instants. The purpose of the reference governor $g(\cdot)$ in (2c) is to modify $r(k)$ to enforce

$$y(k) \in \mathcal{C}, \ \forall k \in \mathbb{Z}_{0+}. \quad (3)$$

Let $v(\cdot) : \mathbb{R}_{0+} \to \mathbb{R}^m$ be the virtual reference signal generated by the governor, and $\tilde{v}(\cdot) : \mathbb{R}_{0+} \to \mathbb{R}^m$ be the virtual reference signal received by the plant, where $\tilde{v}(t + \delta(t)) = v(t)$, and $\delta(t) \in \mathbb{R}_{0+}$ is the time-varying delay. In the sampled-data framework, the command $v(k)$ is generated at time t_k, $k \in \mathbb{Z}_{0+}$, and it is applied at $t_k + \delta(k)$, $\delta(k) \in \mathbb{R}_{0+}$, due to the network induced delay. We assume that for every $k \in \mathbb{Z}_{0+}$, $\delta(k) \in [0, \bar{\delta}]$, $\bar{\delta} < T_s$, and $\delta(k)$ is unknown at both network sides, plant and controller. The system evolution from state $x(t_k)$ is given by

$$x(k+1) = e^{A_c T_s} x(t_k) + \int_0^{\delta(k)} e^{A_c(T_s - \tau)} B_c v(t_{k-1}) d\tau + \int_{\delta(k)}^{T_s} e^{A_c(T_s - \tau)} B_c v(t_k) d\tau. \quad (4)$$

By defining $W(\delta) = -\int_0^{\delta} e^{A_c(T_s - \tau)} B_c d\tau$, from (4) we obtain the uncertain system

$$x(k+1) = Ax(k) + Bv(k) + W(\delta(k))\Delta v(k). \quad (5)$$

where $\Delta v(k) = v(k) - v(k-1)$. From (5), the effect of the network induced delay is a disturbance proportional to the virtual reference variation, $\Delta v(k)$. The smaller is the virtual reference variation, the smaller is the induced disturbance. Since A_c is Hurwitz, it is invertible and $W(\delta) = (A_c)^{-1}(e^{A_c(T_s - \delta)} - e^{A_c T_s})B_c$. The complete model of the system subject to input delay is

$$x(k+1) = Ax(k) + Bv(k) + d(k) \quad (6a)$$
$$d(k) \in \mathcal{D}(\Delta v(k)) \quad (6b)$$
$$\mathcal{D}(\Delta v(k)) = \{d \in \mathbb{R}^n : d = W(\delta)\Delta v(k), \ \delta \in [0, \bar{\delta}]\}. \quad (6c)$$

Next, model (6) is used to design a reference governor that robustly enforces the constraints with respect to the delay.

A. Network Reference Governor

The standard reference governor (2c) [8] generates the virtual reference $v(k)$ in (2a) at time $k \in \mathbb{Z}_{0+}$ as the minimum distance projection of $r(k)$ onto the set of commands that enforce (3) for all $h \geq k$, $h \in \mathbb{Z}_{0+}$,

$$v(k) = \arg\min_{v} \quad \|r(k) - v\|_2^2 \quad (7a)$$
$$\text{subject to} \quad (x(k), v) \in \mathcal{O}_\infty, \quad (7b)$$

where $\mathcal{O}_\infty$ is the *maximum output admissible set* [7]. If $(x(k), v(k)) \in \mathcal{O}_\infty$ and $v(h) = v(k)$ for all $h \in \mathbb{Z}_{0+}, h \geq k$, then $y(h) \in \mathcal{C}$ for all $h \geq k$. For stable linear systems under the previous assumptions, $\mathcal{O}_\infty$ is positive invariant, bounded, convex, and has an arbitrary close inner approximation described by a finite number of linear inequalities [7].

However, due to the network delay, the reference governor must ensure that a set of states belongs to $\mathcal{O}_\infty$,

$$v(k) = \arg\min_{v} \quad \|r(k) - v\|_2^2 \quad (8a)$$
$$\text{subject to} \quad (x(k+1), v) \in \mathcal{O}_\infty, \quad (8b)$$
$$\forall d(k) \in \mathcal{D}(\Delta v(k)). \quad (8c)$$

In order to make (8) computationally tractable, we approximate the disturbance set (8c). Constraints (8b), (8c) enforce

$$(\tilde{x}(k+1) \oplus \mathcal{D}(\Delta v(k)), v(k)) \subseteq \mathcal{O}_\infty,$$

where $\tilde{x}(k+1) = Ax(k) + Bv(k)$ is the delay free next state computed from (5) for $\delta(k) = 0$. The set $\mathcal{D}(\Delta v)$ can be bounded [11] by any polyhedral set affinely dependent on Δv. A simple approach is to use the infinity norm

$$\bar{W} = \max_{\delta \in [0, \bar{\delta}]} \|W(\delta)\|_\infty, \quad (9)$$

which can be computed by solving a nonlinear scalar optimization problem. Note that $\min_{\delta\in[0,\bar{\delta}]}\|W(\delta)\|_\infty = 0$. By (9), we define $\tilde{\mathcal{D}}(\Delta v) \supset \mathcal{D}(\Delta v)$ as

$$d \in \tilde{\mathcal{D}}(\Delta v) \quad \triangleq \quad \{d \in \mathbb{R}^n : \|d\|_\infty \leq \bar{W}\|\Delta v\|_\infty\}. \quad (10)$$

By using $\tilde{\mathcal{D}}$, (8) is formulated as the quadratic program (QP),

$$\begin{aligned}
v(k) = \arg\min_{v,\xi} \quad & \|r(k) - v\|_2^2 && (11a)\\
\text{s.t.} \quad & H_x(Ax(k) + Bv + \eta_i \bar{W}\xi) \\
& \qquad + H_v v \leq h, && (11b)\\
& \xi \geq [v - v(k-1)]_j, && (11c)\\
& \xi \geq -[v - v(k-1)]_j, && (11d)\\
& i = 1, \ldots, n_v,\ j = 1, \ldots, m,
\end{aligned}$$

where $\mathcal{O}_\infty = \{(x, v) : H_x x + H_v v \leq h\}$ and $\{\eta_i\}_{i=1}^{n_v}$ is the set of vertices of the unitary ∞-norm ball in $\mathbb{R}^n$.

Theorem 1 ([11]): Let $x_e(v)$ be the equilibrium associated with command v, and $r(k) = r$ for all $k \geq 0$. Let $\mathcal{O}_\infty$ be compact and convex with $(x_e(v), v) \in \text{int}[\mathcal{O}_\infty]$, for all $v, r \in \Gamma$, where Γ is a compact set of strictly steady state admissible references. For the network reference governor based on (8) and (11), there exists a finite index $\bar{k} \in \mathbb{Z}_{0+}$ such that $v(\bar{k}) = r$. □

B. Unbounded delays and command overtaking

If the delay is larger than the sampling period, $\bar{\delta} \geq T_s$, the problem becomes significantly more complex. In particular, it is possible that the delay variation causes command overtaking ($v(k)$ is received after $v(k+j)$, $j \in \mathbb{Z}_+$). The approach in [11] avoids the conservativeness of previous approaches [10] by embedding some logic at the plant. The reference governor generates commands that are robust with respect to a certain set of disturbances, and the logic at the plant verifies the admissibility of the command upon reception.

Let the delay $\delta(k)$ be possibly unbounded, i.e., $\delta(k) \in \mathbb{R}_{0+}$, for all $k \in \mathbb{Z}_{0+}$ and assume an IID probability density function of δ is known, $\phi(\delta) : \mathbb{R}_{0+} \to \mathbb{R}_{0+}$. Select $T_s > 0$ and $0 < \alpha < 1$ such that $\int_0^{\alpha T_s} \phi(\delta) d\delta = \varrho$, for some $\varrho > 0$, finite, and set $\bar{\delta} = \alpha T_s$. At every control cycle $k \in \mathbb{Z}_{0+}$, the plant logic sends to the controller the state $x(k)$ and $v(t_k^-)$, the command that was applied just before the beginning of the current sampling instant. The controller sends to the plant $v(k) = \bar{v}$ computed from (8), together with $r_{v(k)} = r_v \in \mathbb{R}^m$, the corresponding value of the desired reference r. The plant logic checks whether the command can be practically applied, since robustness only up to a delay length $\bar{\delta}$ is guaranteed, and whether it is better than the one currently applied. Such a checks are performed by using $\mathcal{O}_\infty$, the state at the beginning of the current sampling period, $x(k)$, and the worst case command variation $\overline{\Delta v}$ during the current sampling period.

Thus, the command $\bar{v}$ is actuated if and only if: (i), $\|r_v - \bar{v}\|_2^2 < \|r_v - v_{act}\|_2^2$, where v_{act} is the command currently applied to the plant, and (ii), $((Ax(k) + B\bar{v}) \oplus \mathcal{D}(\overline{\Delta v}), \bar{v}) \subseteq \mathcal{O}_\infty$, where $\mathcal{D}(\overline{\Delta v}) = \{d \in \mathbb{R}^n : \|d\|_\infty \leq \bar{W}\overline{\Delta v}\}$. Condition (i) is a *liveness* condition that requires that the newly received command improves the reference tracking performance, while assuming that r_v is the current value of the reference. Condition (ii) is a *safety* condition that requires that the new command maintains output admissibility. The following result was proven.

Corollary 1 ([11]): Let the assumptions of Theorem 1 hold, and a uniform bound $\bar{k} \in \mathbb{Z}_{0+}$ on the time required for the virtual reference to converge to the reference exists. Let $\varrho > 0$, $\alpha > 0$, $\bar{\delta} = \alpha T_s$. For the case of the $\delta > T_s$ the command converges asymptotically to the reference r in probability, i.e., $\lim_{r\to\infty} \mathbf{P}[\|v(k) - r\|_2^2 = 0] = 1$

III. Orientation control of a spacecraft with a flexible appendage

We illustrate the network reference governor based control using simulations of a single degree of freedom reorientation maneuver for a spacecraft with a flexible appendage. The command r is the requested orientation angle. This command is transmitted through a communication network with a time-varying delay. The objective is to keep the deflections of the flexible appendage in the specified range and enforce the control constraints. With x_1[rad] denoting the orientation angle of the spacecraft bus and x_2[m] denoting the deflection of the flexible appendage, the physical model of the spacecraft has the following form,

$$(J + ml^2)\ddot{x}_1 + ml\ddot{x}_2 = u,$$

$$m\ddot{x}_2 + kx_2 + ml\ddot{x}_1 = 0,$$

where $m = 1$kg, $J = 50$kgm^2, $k = 0.1$N/m, and $l = 40$m. The nominal controller for the control torque u[Nm] is of Linear Quadratic type, $u = -Kx + H_r r$, and is designed so that x_1 tracks the command r in steady-state. The constraints are prescribed as

$$-0.2 \leq x_2(t) \leq 0.2, \quad -0.8 \leq u(t) \leq 0.8.$$

The update period of the reference governor is 1s. In constructing $\mathcal{O}_\infty$, we used a discrete-time model obtained for 0.25s update period. It can be shown that for a reorientation of -1.2rad, the constraints are severely violated. We report the simulation results for two scenarios: short time delay which can randomly vary in $[0, 1]$s, and long time delay which can randomly vary in $[0, 10]$s, i.e., up to 10 times the sampling period. For the second case, the network reference governor implementation was based on Section II-B.

The simulation results for the short time delay case are shown in Figures 2,3. Unlike the conventional reference governor, the networked reference governor strictly enforces the appendage deflection constraint despite the time-varying delay. The orientation change maneuver is only slightly slowed down with the networked reference governor versus the conventional reference governor.

The responses for the long time delay case are shown in Figures 4,5. The constraints are enforced despite the delay up to 10 times the update period of the reference governor.

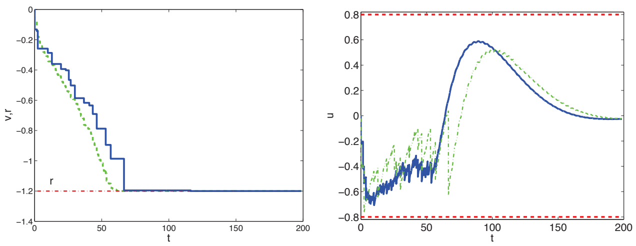

Fig. 2. Time histories of virtual reference, v, (left) and control signal, u, (right) with conventional (dashed) and networked reference governor (solid) for the short time delay case.

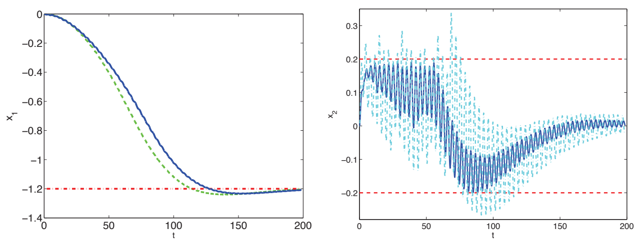

Fig. 3. The response of spacecraft bus orientation x_1 (left) and of the appendage deflection, x_2 (right) with conventional (dashed) and networked reference governor (solid) for the short time delay case.

The response is slower and clearly more conservative to compensate for the potential effects of longer delay.

IV. Network Reference Governor for Systems with Slowly Varying Delays

The conservativeness of the networked reference governor algorithm can be reduced if additional information is available on the time delay dynamics. In several applications, delay depends on the network loading [12], which changes slowly over time. Hence, the delay is also slowly time-varying. Motivated by this, we consider the case when the time delay is long, but *slowly* varying, where "slowly" will be mathematically defined later. In the case of long delays, that are common in space applications, the delay variation cannot be neglected, because the slow variation is integrated over long periods (the time for the command to reach the plant and to generate effects observable by the controller), hence resulting in significative changes.

Consider system (1a), and the delay-free constrained discrete-time model of the plant and reference governor (2c) under the previous assumptions of closed-loop stability and observability, compactness of $\mathcal{C}$, and $0 \in \mathrm{int}[\mathcal{C}]$.

Let the time delay be bounded, $\delta \in [0, \bar{\delta}]$, and slowly varying according to the dynamics

$$\dot{\delta}(t) = \sigma(t) \quad (12a)$$
$$\underline{\sigma} \leq \sigma(t) \leq \overline{\sigma}, \quad (12b)$$

where the delay rate of change with $\sigma(t)$ is unknown and time-varying. In this context, slowly varying means that $\overline{\sigma} + |\underline{\sigma}| < 1$ (preferably, $\ll 1$). This condition also avoids the command overtaking discussed in Section II-B.

At $t_k > 0$, let $\delta(k)$ be the current delay duration, which is supposed to be known to the controller (either measured

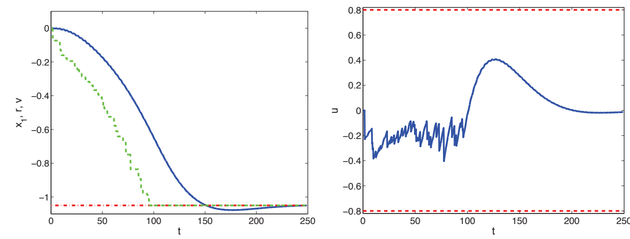

Fig. 4. The responses of v and x_1 (left) and of u (right) with networked reference governor for the long time delay case.

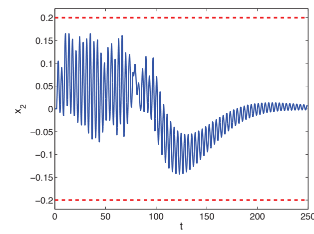

Fig. 5. The response of the appendage deflection x_2 for the long time delay case.

at the plant or estimated by the controller itself) together with the state measurement, $x(k)$. For the moment, assume no uncertainty on previously issued commands (i.e., $\tilde{v}(t_i + \delta(i)) = v(i)$, for all $i \leq k$). This assumption will be relaxed later. The command issued at t_k is expected to reach the plant at time $t_k + \delta(k)$ when the state takes value

$$x(t_k + \delta(k)) = e^{A_c \delta(k)} x(k) + \int_0^{\delta(k)} e^{A_c(\delta(k) - \tau)} B_c \tilde{v}(t_k + \tau - \delta(k)) d\tau. \quad (13)$$

If the delay were constant, the predicted state $x(t_k + \delta(k))$ could be used in the controller without accounting for uncertainty. However, since the delay is changing, the reference governor command reaches the plant at $t + \delta(t) + \varsigma(t)$, where $\varsigma(t)$ is the cumulated delay change from the time the new reference is commanded, to the time it is received,

$$\varsigma(t) = \int_t^{t+\delta(t)+\varsigma(t)} \sigma(t) dt.$$

The bounds on ς can be computed by considering a constant value for $\sigma(t)$, so that $\varsigma(t) = \sigma(\delta(t) + \varsigma(t))$, and

$$\varsigma(t) = \frac{\sigma(t)}{1 - \sigma(t)} \varsigma(t). \quad (14)$$

For $\sigma(t) < 1$, (14) is monotonic, hence due to the slow variation assumption, we obtain

$$\frac{\underline{\sigma}}{1 - \underline{\sigma}} \delta(t) \leq \varsigma(t) \leq \frac{\overline{\sigma}}{1 - \overline{\sigma}} \delta(t).$$

Thus, under the assumption of no uncertainty on previously issued commands, the state at $t_k + \delta(k) + T_s$ is

$$x(t_{k+1} + \delta(t)) = e^{A_c T_s} x(t_k + \delta(k)) + \int_0^{T_s} e^{A_c(T_s - \tau)} B_c v(k) d\tau - \int_{a(\varsigma(k))}^{b(\varsigma(k))} e^{A_c(T_s - \varsigma(k))} B_c \Delta v(k), \quad (15)$$

where if $\varsigma(k) < 0$, $a(\varsigma(k)) = \varsigma(k)$ and $b(\varsigma(k)) = 0$, while if $\varsigma(k) > 0$, $a(\varsigma(k)) = 0$ and $b(\varsigma(k)) = \varsigma(k)$.

As a result, we obtain again a linear system with input multiplicative additive uncertainty,

$$x(t_{k+1} + \delta(k)) = Ax(t_k + \delta(k)) + Bv(k) + \Omega(\varsigma(k))\Delta v(k), \tag{16}$$

where $\Omega(\varsigma) = -\int_{\min\{0,\varsigma\}}^{\max\{0,\varsigma\}} e^{A_c(T_s-\tau)} B_c d\tau$. The value of $\varsigma(k)$ is unknown, yet it depends on $\delta(k)$ and on the bounds of the delay rate of change. Thus, we can bound Ω and over-approximate the uncertainty set for a given input variation Δv, for instance by using the infinity norm,

$$\bar{\Omega}(\delta) = \max_{\substack{\|\Delta v\|_\infty = 1, \\ \frac{\underline{\sigma}}{1-\underline{\sigma}}\delta \leq \varsigma \leq \frac{\overline{\sigma}}{1-\overline{\sigma}}\delta}} \|\Omega(\varsigma)\Delta v\|_\infty. \tag{17}$$

Hence, the relations

$$\begin{aligned} \Omega(\varsigma)\Delta v &\in \mathcal{D}(\delta, \Delta V) \\ \mathcal{D}(\delta, \Delta V) &= \{d \in \mathbb{R}^n : \|d\|_\infty \leq \bar{\Omega}(\delta)\|\Delta v\|_\infty\}, \end{aligned} \tag{18}$$

overbound the uncertainty induced by the variation of the time delay. By ensuring that

$$(x(t_{k+1} + \delta(k)) \oplus \mathcal{D}(\delta(k), \Delta v(k)), v(k)) \subseteq \mathcal{O}_\infty, \tag{19}$$

the system state is in the maximum output admissible set for every feasible delay-induced uncertainty.

From the reasoning in Section II-A and from (18), at every step $k \in \mathbb{Z}_{0+}$ the reference governor solves the following optimization problem

$$\begin{aligned} v(k) = \arg\min_{v,\xi} \quad & \|r(k) - v\|_2^2 && \text{(20a)} \\ \text{s.t.} \quad & H_x(Ax(t_k + \delta(k)) + Bv && \\ & \quad + \eta_i \bar{\Omega}(\delta(k))\xi) + H_v v \leq h, && \text{(20b)} \\ & \xi \geq [v - v(k-1)]_j, && \text{(20c)} \\ & \xi \geq -[v - v(k-1)]_j, && \text{(20d)} \\ & i = 1, \ldots, n_v, \; j = 1, \ldots, m, && \end{aligned}$$

and, if the problem is feasible, it sends $v(k)$ as new reference to the plant, otherwise it sends $v(k) = v(k-1)$.

A. Uncertainty in prediction

In (13), it was assumed that the previous values of the applied reference were perfectly known. However, due to the variations in the delay between when the command is sent from the controller and when it is applied to the plant, this is not the case. For every reference change $\Delta v(i) \neq 0$ at time $t_i \in \mathbb{R}_{0+}$ we can compute the expected arrival time range $\mathcal{T}_a(i) = \left[t_i + \delta(i) - \frac{\underline{\sigma}}{1-\underline{\sigma}}\delta(i), t_i + \delta(i) + \frac{\overline{\sigma}}{1-\overline{\sigma}}\delta(i)\right]$, and the over-approximation (18) of the uncertainty induced on the state, $\mathcal{D}(\delta(i), \Delta v(i))$. At any time $t_k \in \mathbb{R}_{0+}$ if

$$[t_k, t_k + \delta(k)] \cap \mathcal{T}_a(t_i) \neq 0, \tag{21}$$

for some $i < k$, $i \in \mathbb{Z}_{0+}$, (i.e., for some $t_i < t_k$), during the prediction interval $[t_k, t_k + \delta(k)]$ the reference change commanded at t_i may take place. Thus, the associated uncertainty will need to be accounted for in prediction. Let $\Delta\mathcal{V}(k)$ be the set of all reference changes expected to add uncertainties on the prediction performed at time t_k, i.e.,

$$\Delta\mathcal{V}(k) = \{\Delta v(i) \neq 0 : [t_k, t_k + \delta(k)] \cap \mathcal{T}_a(i) \neq \emptyset\}.$$

In order to guarantee robustness, there are two approaches. The first approach is not to command a reference change whenever $|\Delta\mathcal{V}(t)| \neq 0$. This does not require further complications in the calculations, yet reduces the control bandwidth, and as a consequence the closed-loop performance. The second approach is to consider in prediction the additional uncertainty caused by the expected reference change, which, at time t_k, is over-approximated by

$$\tilde{\mathcal{D}}(\Delta\mathcal{V}(k)) = \bigoplus_{\Delta v(i) \in \Delta\mathcal{V}(k)} \mathcal{D}(\delta(i), \Delta v(i)). \tag{22}$$

Thus, the state at $t_k + \delta(k)$ is such that

$$x(t_k + \delta(k)) \in \left\{\tilde{x}(t + \delta(t)) \oplus \tilde{\mathcal{D}}(\Delta\mathcal{V}(k))\right\}, \tag{23}$$

where $\tilde{x}(t_k + \delta_k)$ is the state prediction neglecting the uncertainty induced by the previous commands, i.e., the right hand side of (13). As a consequence, the reference governor problem with uncertain initial state is solved by imposing that all the vertices of the set at the right hand side of (23) belongs to the maximum output admissible set. Let $\{\varphi_h\}_{h=1}^{n_d}$, be the vertices of $\tilde{\mathcal{D}}(\Delta\mathcal{V}(t_k))$, then the reference governor problem becomes

$$\begin{aligned} v(k) = \arg\min_{v,\xi} \quad & \|r(k) - v\|_2^2 && \text{(24a)} \\ \text{s.t.} \quad & H_x(A(\tilde{x}(t_k + \delta(k)) + \varphi_h) + Bv && \\ & \quad + \eta_i \bar{W}\xi) + H_v v \leq h, && \text{(24b)} \\ & \xi \geq [v - v(k-1)]_j, && \text{(24c)} \\ & \xi \geq -[v - v(k-1)]_j, && \text{(24d)} \\ & i = 1, \ldots, n_v, \; j = 1, \ldots, m, && \\ & h = 1, \ldots, n_d. && \end{aligned}$$

The uncertainty in (24) due to (23) decreases when the command is maintained constant. Thus, if (24) becomes unfeasible, by not issuing new commands the feasibility will be recovered, while constraint satisfaction is still guaranteed.

The algorithm for the reference governor in the presence of slowly varying time delay is summarized in Algorithm IV.1, where the buffer $\mathbb{E}$ contains relevant previous arrival time ranges and command changes, to compute (22).

V. Spacecraft Relative Motion Control

To illustrate the approach of Section IV, we consider a relative spacecraft motion control problem. The spacecraft is to be maneuvered to a desired position in a circular orbit which is assumed to be the origin of the Hill's frame. The waypoints are transmitted remotely to the spacecraft through a relay network and the communications are delayed. We assume that the delay is in the forward channel (transmitted commands are delayed but spacecraft states are acquired without delay), and is time-varying with bounded rate, $\overline{\sigma} = -\underline{\sigma} = 0.4$. The transmitted command is the target in-track position, v. The spacecraft controller responds to this

1. At t_k receive $(x(k), \delta(k))$
2. Compute $\bar{\Omega}(\delta(k))$, $\mathcal{T}_a(k)$
3. Compute $\tilde{x}(t+\delta(t))$, $\Delta\mathcal{V}(k)$, $\mathcal{D}(\Delta\mathcal{V}(k))$.
4. find $\bar{\imath} = \min\{i \in \mathbb{Z}_{0+} : \Delta v(i) \neq 0,\ \Delta v(i) \in \Delta\mathcal{V}(k)\}$.
   For all $j < \bar{\imath}$, remove $e(j) = (\mathcal{T}_a(j), \Delta v(j))$ from buffer $\mathbb{E}$.
5. Solve problem (24)
6. if (24) is feasible
   send $v(k)$ to the plant
   store $e(k) = (\mathcal{T}_a(k), \Delta v(k))$ in buffer $\mathbb{E}$
   else
   send $v(k) = v(k-1)$ to the plant
   endif

Algorithm IV.1: Networked reference governor algorithm in the presence of slowly varying delays.

command with the thrust vector $U = [u_x\ u_y]^{\mathrm{T}} = -K \cdot (X - X_e(\tilde{v}))$, to maneuver the spacecraft to the equilibrium $X_e(\tilde{v}) = [0\ \tilde{v}\ 0\ 0]^{\mathrm{T}}$, where $X = [x\ y\ \dot{x}\ \dot{y}]$ is the vector of relative positions and velocities of the spacecraft in Hill's frame, K is the Linear Quadratic Regulator gain and $\tilde{v}(t) = v(t - \delta(t))$. The dynamic equations of motion are the Hill-Clohessy-Wiltshire (HCW) equations. The mass of the spacecraft is 100kg and the nominal orbit is 850km. The constraints are imposed on the thrust (kN) as $-0.0004 \leq u_x(t) \leq 0.0004$, $-0.0004 \leq u_y(t) \leq 0.0004$ and $x(t)$, $y(t)$ must adhere to Line Of Sight (LOS) cone constraints shown by dashed lines in Figure 6-left. The LQ gain is obtained for high control weighting to reduce the maneuver fuel consumption.

The spacecraft is initially at $x(0) = -0.25$km and $y(0) = 4$km. The desired position is at the origin of the Hill's frame, hence $r(t) = 0$ for $t \geq 0$. Figures 6 and 7 illustrate the response governed by the network reference governor. The LoS and control input constraints are enforced during the maneuver despite the delay varies almost up to 100% during the maneuver.

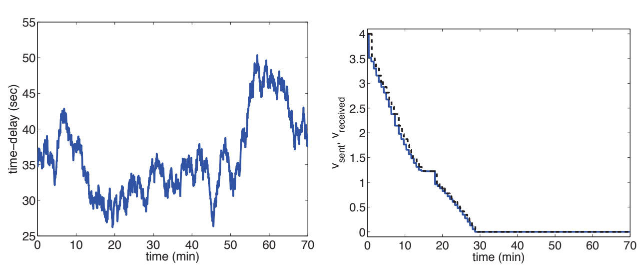

Fig. 6. The random, slowly varying time delay $\delta(t)$ (left) and the time histories (right) of v sent (solid) and $\tilde{v}$ received (right).

VI. Conclusions

In the paper, the network reference governor [11] has been extended to provide less conservative handling of the case when the time delay is long but slowly time-varying with known bounds on its time rate of change. The network reference governor uses state prediction forward in time by

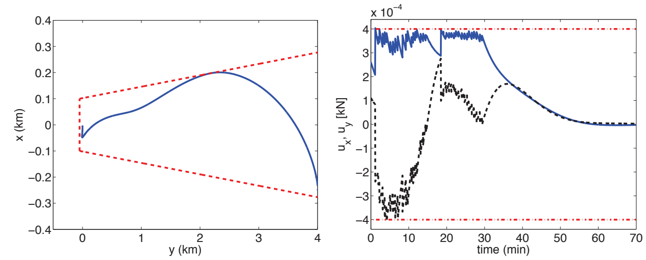

Fig. 7. The spacecraft relative motion trajectory in orbital track y versus radial x direction (left) and the time history of thrust forces in x and y directions (right).

the current delay value and accounts for the disturbance induced by the unknown time variation of the delay. As in [11], the delay-induced disturbance depends affinely on the change in the reference governor output and hence is under complete reference governor control. Two simulation examples based on challenging spacecraft attitude and orbital control problems were presented for the cases of randomly-varying and slowly-varying time delay, respectively. In both cases the network reference governor was shown to successfully handle the constraints despite the disturbances induced by the delay.

References

[1] F. Slane and A. Hooke, "Space plug and play avionics standards:progress, problems and a view of the future," in *AIAA Infotect@Aerospace*, 2007.

[2] P. Antsaklis and J. Baillieul, "Special issue on networked control systems," *IEEE Trans. Aut. Control*, vol. 49, no. 9, pp. 1421–1423, Sep. 2004.

[3] J. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proc. of IEEE* Special Issue on Technology of Networked Control Systems, vol. 95, no. 1, pp. 138–162, Jan. 2007.

[4] A. Bemporad, S. Di Cairano, E. Henriksson, and K. Johansson, "Hybrid model predictive control based on wireless sensor feedback: an experimental study," *Int. J. Robust Nonlinear Control*, vol. 20, no. 2, 2010, special issue "Industrial applications of wireless control".

[5] E. Gilbert and K. Tan, "Linear systems with state and control constraints: the theory and applications of maximal output admissible sets," *IEEE Trans. Aut. Control*, vol. 36, no. 9, pp. 1008–1020, 1991.

[6] A. Bemporad, A. Casavola, and E. Mosca, "Nonlinear control of constrained linear systems via predictive reference management," *IEEE Trans. Aut. Control*, vol. 42, no. 3, pp. 340–349, Mar 1997.

[7] E. Gilbert, I. Kolmanovsky, and K. Tan, "Discrete-time reference governors and the nonlinear control of systems with state and control constraints," *Int. J. Robust Nonlinear Control*, vol. 5, no. 5, pp. 487–504, 1995.

[8] E. Gilbert and I. Kolmanovsky, "Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor," *Automatica*, vol. 38, no. 12, pp. 2063–2073, 2002.

[9] A. Bemporad, "Predictive control of teleoperated constrained systems with unbounded communication delays," in *Proc. 37th IEEE Conf. on Decision and Control*, Tampa, FL, 1998, pp. 2133–2138.

[10] A. Casavola, M. Papini, and G. Franze, "Supervision of networked dynamical systems under coordination constraints," *IEEE Trans. Aut. Control*, vol. 51, no. 3, pp. 421–437, March 2006.

[11] S. Di Cairano and I. Kolmanovsky, "Rate limited reference governor for networked controlled systems," in *Proc. American Contr. Conf.*, Baltimore, MD, 2010, pp. 3704–3709.

[12] S. Di Cairano, K. Johansson, A. Bemporad, and R. Murray, "Discrete and hybrid stochastic state estimation algorithms for networked control systems," in *Hybrid Systems: Computation and Control*, ser. Lec. Notes in Computer Science. Springer-Verlag, 2008.