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Comparison of Quaternary Block-Coding and Sphere-Cutting for High-Dimensional Modulation

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OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

As the capacity limits of the fiber networks are approached [1], optimization of modulation schemes represents a crucial trade-off between performance, cost, complexity and latency [2]. Recently, significant gains have been demonstrated by optimizing optical modulation format design in higher dimensions [3–5]. For low spectral efficiency (SE), high-dimensional modulation formats based on spherical cutting of an optimal lattice [6] and binary block coding were introduced and applied to ultra long-haul optical fiber systems [5, 7]. In this paper, we investigate high-dimensional modulation for intra-channel SE between 1 and 1.5 b/s/Hz/dim, corresponding to 4-6 b/s/Hz over a 4-D channel (e.g. single mode fiber). We introduce high-dimensional modulation based on quaternary block coding, which requires only 4 levels per quadrature at the transmitter. We analyze the performance of selected quaternary block coded modulation formats, and compare them with conventional, spherical lattice-cut and time-domain hybrid dual-polarization quadrature phase shift keying (DP-QPSK) and $8-ary$ quadrature amplitude modulation (DP-8QAM).

2. High-dimensional modulation format design and quaternary block coding

For p bits/symbol, a modulation format maps p-bit encoded binary words to 2^p points in an N dimensional vector field, yielding p/N bits/symbol/dimension SE. Thus, the design constitutes determination of (i) the locations of 2^p constellation points in the N dimensional vector field and (ii) the corresponding 2^p bit labelings. Here, we will describe three modulation design methodologies: time-domain hybrid QAM; sphere-cutting; and block coding.

Time-domain hybrid modulation takes the form of serially allocated multiple sub-formats inside the ‘supersymbol’ [8]. The proportion of the sub-formats is adjusted for a given p/N, N is determined by the size of the supersymbol, and constellation is formed by the sub-formats’ constellations with average power adjusted for the target bit error ratio (BER). In this paper, we will consider a hybrid modulation format obtained by DP-QPSK and DP-8QAM sub-formats yielding an overall SE of 1.25 bits/symbol/dimension.

The second high-dimensional modulation approach is sphere-cutting. 2^p symbol points are determined from the infinite, densest known lattice in the N dimensional space by minimizing the average constellation energy [6, 7]. For bit labelings of each constellation point, a random search method was employed. Inside an ordinary DP transmitter, spherical lattice-cut modulation is implemented by a look-up table of size 2^p by N (see Fig. 1(b)). For spherical lattice-cut modulation formats, we use the notation ‘Sp: δb - βD’ for $p = \delta$ and $N = \beta$.

Linear block codes constitute the third approach to high-dimensional modulation. Although this topic has been investigated previously for binary block codes [3, 5, 9], here we will investigate non-binary block codes. Assuming 2^L signal levels per dimension, the block code should have word length N and code rate of $p/(L \times N)$ for a resultant SE of p/N b/s/Hz/dim. In this paper, focusing on quaternary block codes [10], we restrict $L = 2$ yielding SE less than 2 bits/s/Hz/dim. Small N is desired since maximum-likelihood (ML) demodulation complexity scales with $O(N \times 2^p)$. It should be noted that the quaternary coded constellation in Fig. 1(c) requires only 4 levels per quadrature, which

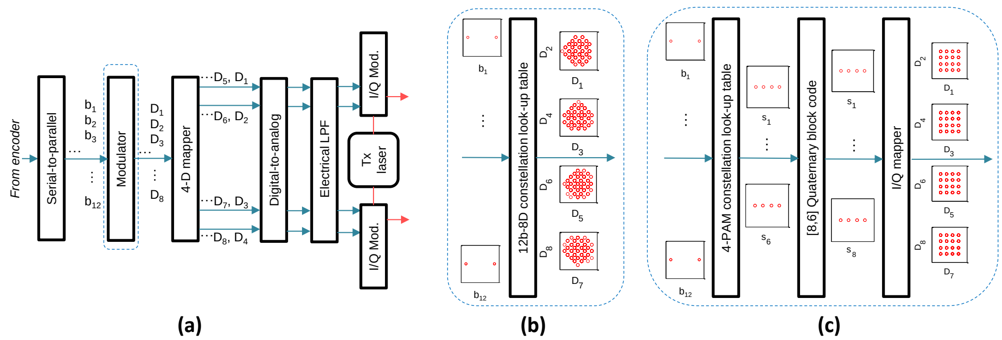

Fig. 1: (a) Coherent transmitter employing a 8-D modulation with $p = 12$. Implementation of modulator block for (b) sphere-cut lattice constellation, (c) quaternary block coded constellation.

is significantly less than that of the sphere-cut constellation in Fig. 1(b), resulting in a lowering of requirements on transmitter digital-to-analogue convertor (DAC) resolution.

Quaternary block coded high-dimensional modulation consists three stages as shown in Fig. 1(c):

1) Quaternary symbol mapping: Encoded bits $b_t \in \{0,1\}$ are mapped to quaternary symbols $s_t \in \{0,1,2,3\}$ by the 4-PAM Gray mapper function $G : (b_{2m}, b_{2m+1}) \to s_m$, for $m = 1, ..., p/2$.

2) Quaternary block coding: Quaternary symbols are mapped to codewords according to: $\begin{bmatrix} s_1 & \cdots & s_N \end{bmatrix}^T = \mathbf{Q} \begin{bmatrix} s_1 & \cdots & s_p/2 \end{bmatrix}^T$ (mod 4), where $\mathbf{Q}$ is the generator matrix of the $[N, p/2, h]$ quaternary code (codeword length N, input symbol length $p/2$ and minimum Lee distance h). For quaternary coded modulation formats, we use the notation 'Qu: pb - ND $[N, p/2, h]$'. Generator matrices of the quaternary codes used in this work may be found in [11].

3) Signal mapping: Zero-mean quaternary encoded blocks $\begin{bmatrix} D_1 & \cdots & D_N \end{bmatrix}^T = \begin{bmatrix} s_1 & \cdots & s_N \end{bmatrix}^T - 1.5$ are mapped to N signal dimensions as constellation points.

3. Noise sensitivity

We first evaluate additive white Gaussian noise (AWGN) channel performances of high-dimensional modulation formats for signal-to-noise ratio (SNR) regime concerning uncoded BER thresholds of modern forward error correction (FEC) codes [12]. SNR is defined as E_b/N_0 where E_b is the energy per bit and N_0 is the unilateral power spectral density of the noise per dimension. BER vs. SNR curves obtained by Monte-Carlo simulations are shown in Fig. 2. We note from Fig. 2(a) that for 1 b/s/Hz/dim, quaternary coded modulation outperforms DP-QPSK for BER lower than 3×10^{-3}, while sphere-cutting offers the best performance for BERs up to 10^{-2}. Fig. 2(b) demonstrates that while both quaternary coding and sphere-cutting offer very similar performance over the range of SNRs considered, they both outperform hybrid modulation for BERs less than approximately 10^{-2}. Fig. 2(c) shows that for spectral efficiency equal to that of DP-8QAM, quaternary coding offers a benefit of up to 1 dB for a BER less than 10^{-2}, while sphere packing offers superior performance at BERs below 4×10^{-3}.

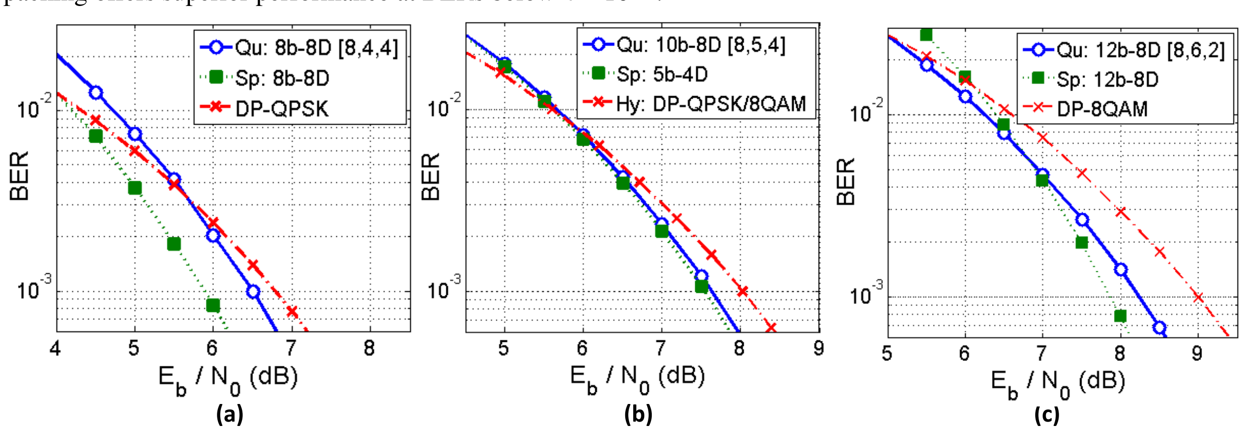

Fig. 2: BER vs E_b/N_0 for intra-channel spectral efficiencies of: (a) 1 b/s/Hz/dim, (b) 1.25 b/s/Hz/dim, (c) 1.5 b/s/Hz/dim.

4. Optical transmission performance

We simulated transmission performance over a 6000 km standard single-mode fiber (SSMF) link at a rate of 112 Gb/s per wavelength. Modulated symbols are mapped to the four dimensions by partially serializing the high-dimensional symbols as in Fig. 1(a) [7]. At the transmitter, DP-I/Q modulators were driven by rectangular pulses, filtered by a 5^{th} order Bessel filter with -3 dB bandwidth of 0.7 times the symbol rate. 5 wavelength channels with were simulated with 50 GHz spacing and no optical filtering. The link comprises 75 spans of 80 km SSMF with loss compensated by Erbium doped fiber amplifiers with 4.5 dB noise figure. In order to quantify performance over a single link for a wide variety of modulation formats, span loss budget was used as a performance metric [9]. Fiber parameters were as follows: $\gamma = 1.2$ /W/km; $D = 17$ ps/nm/km; $\alpha = 0.2$ dB/km. Other fiber effects such as dispersion slope and polarization mode dispersion were not simulated. An ideal homodyne coherent receiver was used, with a transfer function described by a 5^{th} order Bessel filter with -3 dB bandwidth 0.7 times the symbol rate, followed by sampling at twice the symbol rate. Following this, ideal chromatic dispersion equalization and data-aided least mean square equalization were employed. High-dimensional symbols were formed as in Fig. 1(a) prior to ML demodulation. We assumed a BER threshold of 4.6×10^{-3} for a 7% hard-decision FEC [12]. The plots for span loss budget vs. launch power for the considered modulation formats are given in Fig. 3.

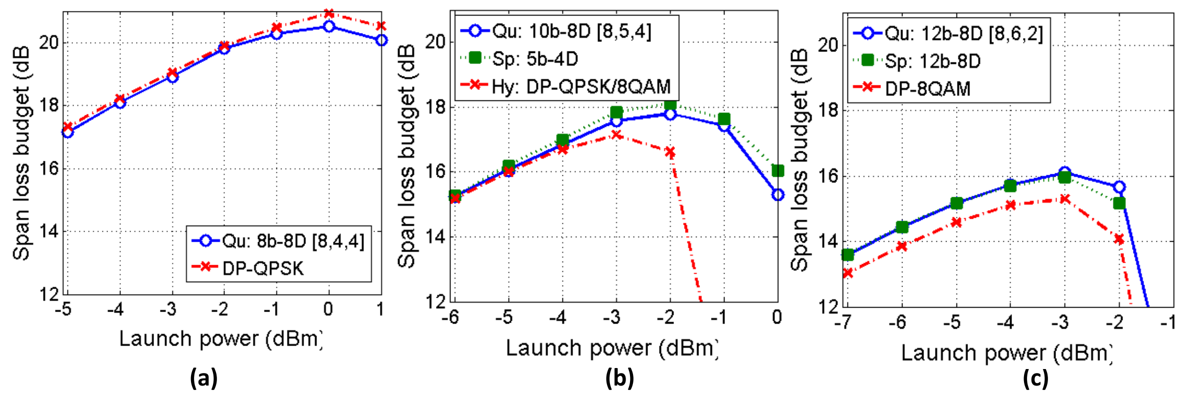

Fig. 3: Span loss budget vs launch power for intra-channel spectral efficiencies of: (a) 4; (b) 5; (c) 6 b/s/Hz

In the low launch power regime where linear propagation effects are dominant, quaternary block-coded and spherical lattice-cut formats provide gains up to 0.5 dB with respect to hybrid modulation formats, as expected from AWGN channel results (see Fig. 2). For higher launch powers where nonlinearity is dominant, performance improvements become more significant. DP-QPSK shown in Fig. 3(a) demonstrates robustness to nonlinearity because of its low peak-to-average power ratio, compared with quaternary coding. From Fig. 3(b), we note that hybrid modulation yields significantly worse performance for high launch powers, despite a small difference compared with quaternary coding and sphere-cutting for an AWGN channel; indicating a lower tolerance to nonlinearity. Fig 3(c) shows that the significant improvement of sphere-cutting and quaternary coding relative to DP-8QAM is maintained both in the linear and nonlinear transmission regimes.

5. Conclusions

We have proposed quaternary block coded high-dimensional modulation as an alternative to sphere-cutting with reduced requirements on transceiver DAC resolution. Performance was compared for both the AWGN channel, and transmission over a 6000 km SSMF link. For SE of 1.25 b/s/Hz/dim, or more, performance is comparable to sphere-cutting formats, although the gain over hybrid modulation is small. At SE of 1.5 b/s/Hz/dim, the difference is more significant with over 0.5 dB of gain compared with DP-8QAM at a BER of 4.6×10^{-3}.

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