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Learning-based Adaptive Control for Nonlinear Systems

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Learning-based Adaptive Control for Nonlinear Systems

Mouhacine Benosman

Abstract— **We present in this paper a preliminary result on learning-based adaptive trajectory tracking control for nonlinear systems. We propose, for the class of nonlinear systems with parametric uncertainties which can be rendered integral Input-to-State stable w.r.t. the parameter estimation errors input, that it is possible to merge together the integral Input-to-State stabilizing feedback controller and a model-free extremum seeking (ES) algorithm to realize a learning-based adaptive controller. We show the efficiency of this approach on a mechatronic example.**

I. Introduction

Extremum seeking (ES) is a well known approach by which one can search for the extremum of a cost function associated with a given process performance (under some conditions) without the need of detailed modelling of the process, e.g. [1], [2], [3]. Several ES algorithms with their stability analysis have been proposed, e.g. [4], [5], [2], [6], [3], [6], [1], [7], [8], and many applications of ES algorithms have been reported, e.g [9], [10], [11], [12], [13].

On the other hand, classical adaptive control deals with controlling partially unknown processes based on their uncertain model, i.e., controlling plants with parameters uncertainties. Classical adaptive methods can be classified as 'direct', where the controller is updated to adapt to the process, or 'indirect', where the model is updated to better reflect the actual process. Many adaptive methods have been proposed over the years for linear and nonlinear systems, we could not possibly cite here all the design and analysis results that have been reported, instead we refer the reader to e.g. [14], [15] and the references therein for more details. What we want to underline here is that these results in 'classical' adaptive control are mainly based on the structure of the model of the system, e.g. linear vs. nonlinear, with linear uncertainties parametrization vs. nonlinear parameterizations, etc.

Another adaptive control paradigm is the one which uses 'learning schemes' to estimate the uncertain part of the process. Indeed, in this paradigm the learning-based controller, based either on machine learning theory, neural networks, fuzzy systems, etc. is trying either to estimate the parameters of an uncertain model, or the structure of a deterministic or a stochastic function representing part or totality of the model. Several results have been proposed in this area as well, and we refer the reader to e.g. [16] and the references therein for more details.

We want to concentrate in this paper on the use of ES theory in the 'learning-based' adaptive control paradigm. Indeed, several results were recently developed in this direction, e.g. [17], [18], [19], [20], [9], [10], [12], [13]. For instance in [17], [18] the authors used a model-free ES, i.e., only based on a desired cost function, to estimate parameters of a linear state feedback to compensate for unknown parameters for linear systems. In [19], [20] an extremum seeking-based controller for nonlinear affine systems with linear parameters uncertainties was proposed. The controller drives the states of the system to unknown optimal states that optimize a desired objective function. The ES controller used in [19], [20] is not model-free in the sense that it is based on the known part of the model, i.e., it is designed based on the objective function and the nonlinear model structure. A similar approach is used in [9], [10] when dealing with more specific examples. In [12], the authors used for the case of electromagnetic actuators, a model-free ES, i.e., only based on the cost function without the use of the systems model, to learn the 'best' feedback gains of a passive robust state feedback. Similarly, in [12], a backstepping controller was merged with a model-free ES to estimate the uncertain parameters of a nonlinear model for electromagnetic actuators. Although, no stability analysis was presented for the full controller (i.e., backstepping plus ES estimator), very promising numerical results where reported.

In this work we propose to generalize the idea of [12], for the class of nonlinear system with parametric uncertainties which can be rendered iISS w.r.t. the parameter estimation errors. The idea is based on a modular design, where we first design a feedback controller which makes the closed-loop tracking error dynamic iISS w.r.t. the estimation errors and then complement this iISS-controller with a mode-free ES algorithm that can minimize a desired cost function, by tuning, i.e., estimating, the unknown parameters of the model. The modular design simplifies the analysis of the total controller, i.e., iISS-controller plus ES estimation algorithm. We first propose this formulation in the general case of nonlinear systems and then show a case-study on a mechatronic example.

This paper is organized as follows: Section II is used to

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recall some notations and definitions. In Section III we present the main result of this paper, namely the ES-based learning adaptive controller. Section IV is dedicated to an application example, and the paper ends with a Conclusion in Section V.

II. Preliminaries

Throughout the paper we will use $\|.\|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $\|x\| = \sqrt{x^T x}$. We will use $\dot{(.)}$ for the short notation of time derivative. We denote by C^k functions that are k times differentiable. A function is said analytic in a given set, if it admits a convergent Taylor series approximation in some neighborhood of every point of the set. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$. Let us now introduce the definition of Local Integral Input-to-State Stability (iISS).

Definition 1 (Local Integral Input-to-State Stability [21]): Consider the system

$$\dot{x} = f(t, x, u) \tag{1}$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, and $f : [0, \infty) \times \mathcal{D} \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The inputs are assumed to be measurable and locally bounded functions $u : \mathbb{R}_{\geq 0} \to \mathcal{D}_u \subseteq \mathbb{R}^m$. Given any control $u \in \mathcal{D}_u$ and any $\xi \in \mathcal{D}_0 \subseteq \mathcal{D}$, there is a unique maximal solution of the initial value problem $\dot{x} = f(t, x, u)$, $x(t_0) = \xi$. Without loss of generality, assume $t_0 = 0$. The unique solution is defined on some maximal open interval, and it is denoted by $x(\cdot, \xi, u)$. System (1) is locally integral input-to-state stable (LiISS) if there exist functions α, $\gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that, for all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$, the solution $x(t, \xi, u)$ is defined for all $t \geq 0$ and

$$\alpha(\|x(t, \xi, u)\|) \leq \beta(\|\xi\|, t) + \int_0^t \gamma(\|u(s)\|) ds \tag{2}$$

for all $t \geq 0$. Equivalently, system (1) is LiISS if and only if there exist functions $\beta \in \mathcal{KL}$ and γ_1, $\gamma_2 \in \mathcal{K}$ such that

$$\|x(t, \xi, u)\| \leq \beta(\|\xi\|, t) + \gamma_1 \left(\int_0^t \gamma_2(\|u(s)\|) ds \right) \tag{3}$$

for all $t \geq 0$, all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$.

III. Learning-based adaptive controller

Consider the system (1), with an additional argument representing parametric uncertainties $\Delta \in \mathbb{R}^p$

$$\dot{x} = f(t, x, \Delta, u) \tag{4}$$

We associate with (4), the output vector

$$y = h(x) \tag{5}$$

where $h : \mathbb{R}^n \to \mathbb{R}^h$.

The control objective here is for y to asymptotically track a desired smooth vector time-dependent trajectory $y_{ref} : [0, \infty) \to \mathbb{R}^h$.

Let us now define the output tracking error vector as $e_y(t) = y(t) - y_{ref}(t)$.

We then assume the following

Assumption 1: There exists a robust control feedback $u_{iss}(t, x, \hat{\Delta}) : \mathbb{R}^{\times}\mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^m$, with $\hat{\Delta}$ being the dynamic estimate of the uncertain vector Δ, such that, the closed-loop error dynamics

$$\dot{e}_y = f_{e_y}(t, e_y, e_\Delta) \tag{6}$$

is iISS from the input vector $e_\Delta = \Delta - \hat{\Delta}$ to the state vector e_y.

Remark 1: Assumption 1 might seem too general, however, several control approaches can be used to design a controller u_{iss} rendering an uncertain system iISS, for instance backstepping control approach has been shown to achieve such a property for parametric strict-feedback systems, e.g. [15]. This is a preliminary report, and we do not pretend here to present a detailed solution for all the cases. A more detailed study of how to achieve Assumption 1 for specific classes of systems and how to use it in the context of ES learning-based adaptive control, will be presented in our future reports.

Let us define now the following cost function

$$Q(\hat{\Delta}) = F(e_y(\hat{\Delta})) \tag{7}$$

where $F : \mathbb{R}^h \to \mathbb{R},\ F(0) = 0,\ F(e_y) > 0$ for $e_y \neq 0$. We need the following assumptions on Q.

Assumption 2: The cost function Q has a local minimum at $\hat{\Delta}^* = \Delta$.

Assumption 3: The initial error $e_\Delta(t_0)$ is sufficiently small, i.e., The original parameters estimates vector $\hat{\Delta}$ is close enough to the actual parameters vector Δ.

Assumption 4: The cost function is analytic and its variation with respect to the uncertain variables is bounded in the neighborhood of Δ^*, i.e., $\|\frac{\partial Q}{\partial \Delta}(\tilde{\Delta})\| \leq \xi_2,\ \xi_2 > 0,\ \tilde{\Delta} \in \mathcal{V}(\Delta^*)$, where $\mathcal{V}(\Delta^*)$ denotes a compact neighborhood of Δ^*.

Remark 2: Assumption 2 simply means that we can consider that Q has at least a local minimum at the true values of the uncertain parameters.

Remark 3: Assumption 3 indicates that our result will be of local nature, meaning that our analysis holds in a small neighborhood of the actual values of the parameters.

We can now present the following Lemma.

Lemma 1: Consider the system (4), (5), with the cost function (7), then under Assumptions 1, 2, 3, and 4, the controller u_{iss}, where $\hat{\Delta}$ is estimated with the multi-parameter extremum seeking algorithm

$$\begin{aligned} \dot{x}_i &= a_i sin(\omega_i t + \tfrac{\pi}{2}) Q(\hat{\Delta}) \\ \hat{\Delta}_i &= x_i + a_i sin(\omega_i t - \tfrac{\pi}{2}),\ i \in \{1, ..., p\} \end{aligned} \quad (8)$$

with $\omega_i \neq \omega_j,\ \omega_i + \omega_j \neq \omega_k,\ i, j, k \in \{1, ..., p\}$, and $\omega_i > \omega^*,\ \forall i \in \{1, ..., p\}$, with ω^* large enough, ensures that the norm of the error vector e_y admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|, t) + \alpha(\int_0^t \gamma(\tilde{\beta}(\|e_\Delta(0)\|, t) + \|e_\Delta\|_{max}))ds,$$

where $\|e_\Delta\|_{max} = \frac{\xi_1}{\omega_0} + \sqrt{\sum_{i=1}^{i=p} a_i^2}$, $\xi_1, \xi_2 > 0$, $e(0) \in \mathcal{D}_e$, $\omega_0 = \max_{i \in \{1,...,p\}} \omega_i$, $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}, \tilde{\beta} \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$.

Proof: Consider the system (4), (5), then under Assumption 1, the controller u_{iss} ensures that the tracking error dynamic (6) is iISS between the input e_Δ and the state vector e_y, which by Definition 1, implies that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $e_\Delta \in \mathcal{D}_{e_\Delta}$, the norm of the error vector e_Δ admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|, t) + \alpha(\int_0^t \gamma(\|e_\Delta\|))ds \quad (9)$$

for all $t \geq 0$.
Now, we need to evaluate the bound on the estimation vector $\tilde{\Delta}$, to do so we use the results presented in [22]. First, based on Assumption 4, the cost function is locally Lipschitz, i.e., $\exists \eta_1 > 0,\ s.t.\ |Q(\Delta_1) - Q(\Delta_2)| \leq \eta_1 \|\Delta_1 - \Delta_2\|,\ \forall \Delta_1, \Delta_2 \in \mathcal{V}(\Delta^*)$. Furthermore, since Q is analytic it can be approximated locally in $\mathcal{V}(\Delta^*)$ with a quadratic function, e.g. Taylor series up to second order. Based on this and on Assumptions 2 and 3 , we can write the following bound ([22], pages 436-437):

$$\begin{aligned} \|e_\Delta(t)\| - \|d(t)\| \leq \|e_\Delta(t) - d(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|, t) + \tfrac{\xi_1}{\omega_0}, \\ \Rightarrow \|e_\Delta(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|, t) + \tfrac{\xi_1}{\omega_0} + \|d(t)\| \\ \Rightarrow \|e_\Delta(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|, t) + \tfrac{\xi_1}{\omega_0} + \sqrt{\textstyle\sum_{i=1}^{i=p} a_i^2}, \end{aligned}$$

with $\tilde{\beta} \in \mathcal{KL},\ \xi_1 > 0,\ t \geq 0$, $\omega_0 = \max_{i \in \{1,...p\}} \omega_i$, $d(t) = [a_1 sin(\omega_1 t + \frac{\pi}{2}), ..., a_p sin(\omega_p t + \frac{\pi}{2})]^T$. which together with the bound (9) completes the proof. ∎

Remark 4: The estimated parameters upper bounds used in Lemma 1 are correlated to the choice of the first order multi-variable extremum seeking (MES) (7) and (8), however, these bounds can be easily changed by using other MES algorithms, e.g. [23], [4], which is due to the modular design of the controller, that uses the iISS robust part to ensure boundedness of the error dynamics and the learning part to improve the tracking performance.

Remark 5: We point out here that ISS can be substituted for iISS if we are dealing with time-invariant systems and solving a regulation problem instead of a time-varying trajectory tracking.

IV. Case study

A. Controller design

We study here the example of electromagnetic actuators modelled by the nonlinear equations [24]

$$\begin{aligned} m\tfrac{d^2x}{dt^2} &= k(x_0 - x) - \eta\tfrac{dx}{dt} - \tfrac{ai^2}{2(b+x)^2} + f_d \\ u &= Ri + \tfrac{a}{b+x}\tfrac{di}{dt} - \tfrac{ai}{(b+x)^2}\tfrac{dx}{dt},\ 0 \leq x \leq x_f \end{aligned} \quad (10)$$

where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 the initial spring length, η the damping coefficient, $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b being constant parameters of the coil, f_d a constant term modelling unknown disturbance force, e.g. static friction, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance (assumed to be dependent on the position of the armature), $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil.
We consider there the control problem of the electromagnetic system assuming uncertainties on the spring constant k , the damping coefficient η, and the additive disturbance f_d.
Let us define the state vector $\mathbf{z} := [z_1\ z_2\ z_3]^T = [x\ \dot{x}\ i]^T$. The objective of the control is to make the variables (z_1, z_2) robustly track a sufficiently smooth (i.e. C^2) time-varying position and velocity trajectories $z_1^{ref}(t),\ z_2^{ref}(t) = \frac{dz_1^{ref}(t)}{dt}$ that satisfy the following constraints: $z_1^{ref}(t_0) = z_{1_{int}},\ z_1^{ref}(t_f) = z_{1_f},\ \dot{z}_1^{ref}(t_0) = \dot{z}_1^{ref}(t_f) = 0,\ \ddot{z}_1^{ref}(t_0) = \ddot{z}_1^{ref}(t_f) = 0$, where t_0 is the starting time of the trajectory, t_f is the final time, $z_{1_{int}}$ is the initial position and z_{1_f} is the final position.

To start, we first write the system (10) in the following form

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \tfrac{k}{m}(x_0 - z_1) - \tfrac{\eta}{m}z_2 - \tfrac{a}{2m(b+z_1)^2}z_3^2 \\ \dot{z}_3 &= -\tfrac{R}{\frac{a}{b+z_1}}z_3 + \tfrac{z_3}{b+z_1}z_2 + \tfrac{u}{\frac{a}{b+z_1}} \end{aligned} \quad (11)$$

The authors in [25] have shown that under the following feedback controller

$$
\begin{aligned}
u=&\tfrac{a}{b+z_1}\Bigg(\tfrac{R(b+z_1)}{a}z_3-\tfrac{z_2 z_3}{(b+z_1)}+\tfrac{1}{2z_3}\Big(\tfrac{a}{2m(b+z_1)^2}(z_2-z_2^{ref})-c_2(z_3^2-\tilde{u})\Big)\Bigg)\\
&+\tfrac{2mz_2}{z_3}\Bigg(\tfrac{\hat{k}}{m}(x_0-z_1)-\tfrac{\hat{\eta}}{m}z_2+\tfrac{\hat{f}_d}{m}+c_3(z_1-z_1^{ref})+c_1(z_2-z_2^{ref})-\dot{z}_2^{ref}\\
&+\kappa_1(z_2-z_2^{ref})\|\psi\|_2^2\Bigg)+\tfrac{m(b+z_1)}{z_3}\Bigg(\Big(\tfrac{\hat{k}}{m}(x_0-z_1)-\tfrac{\hat{\eta}}{m}z_2+\tfrac{\hat{f}_d}{m}-\tfrac{a}{2m(b+z_1)^2}z_3^2\\
&-\dot{z}_2^{ref}\Big)\Big(c_1+\kappa_1\|\psi\|_2^2-\tfrac{\hat{\eta}}{m}\Big)-\tfrac{\hat{\eta}}{m}\dot{z}_2^{ref}\Bigg)+\tfrac{m(b+z_1)}{z_3}\Big(2\kappa_1(z_2-z_2^{ref})\\
&\left(\tfrac{(x_0-z_1)(-z_2)}{m^2}+\tfrac{z_2\left(\tfrac{\hat{k}}{m}(x_0-z_1)-\tfrac{\hat{\eta}}{m}z_2+\tfrac{\hat{f}_d}{m}-\tfrac{az_3^2}{2m(b+z_1)^2}\right)}{m^2}\right)\\
&-\kappa_2(z_3^2-\tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2\left[\left|c_1+\kappa_1\|\psi\|_2^2-\tfrac{\hat{\eta}}{m}\right|^2+\left|2\kappa_1(z_2-z_2^{ref})\right|^2\left|\tfrac{z_2}{m^2}\right|^2\right]\|\psi\|_2^2\\
&-\kappa_3(z_3^2-\tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2\|\psi\|_2^2+\tfrac{m(b+z_1)}{z_3}\Big(-\tfrac{\hat{k}}{m}z_2-\ddot{z}_2^{ref}+c_3(z_2-z_2^{ref})\Big)\Big),
\end{aligned}
\tag{12}
$$

with

$$
\begin{aligned}
\tilde{u}=&\tfrac{2m(b+z_1)^2}{a}\left(\tfrac{\hat{k}}{m}(x_0-z_1)-\tfrac{\hat{\eta}}{m}z_2+\tfrac{\hat{f}_d}{m}+c_3(z_1-z_1^{ref})\right.\\
&\left.+c_1(z_2-z_2^{ref})-\dot{z}_2^{ref}\right)+\tfrac{2m(b+z_1)^2}{a}\left(\kappa_1(z_2-z_2^{ref})\|\psi\|_2^2\right),
\end{aligned}
\tag{13}
$$

where $\hat{k},\ \hat{\eta},\ \hat{f}_d$ are the system parameter estimates, and $\psi \triangleq (\frac{x_0-z_1}{m},\ \frac{z_2}{m},\ \frac{1}{m})^T$; the system (10), (12) and (13), is locally iISS- (LiISS).

Next, we define the cost function

$$Q(\hat{\Delta})=\int_0^{t_f} q_1(z_1(s)-z_1(s)^{ref})^2 ds+\int_0^{t_f} q_2(z_2(s)-z_2^{ref}(s))^2 ds, \tag{14}$$

where $q_1, q_2 > 0$, and $\hat{\Delta}=(\hat{\Delta}_k,\ \hat{\Delta}_\eta,\ \hat{\Delta}_{f_d})^T$ represents the vector of the learned parameters, defined such that

$$
\begin{aligned}
\hat{k}(t) &= k_{nominal}+\hat{\Delta}_k(t)\\
\hat{\eta}(t) &= \eta_{nominal}+\hat{\Delta}_\eta(t)\\
\hat{f}_d(t) &= f_{d-nominal}+\hat{\Delta}_{f_d}(t)
\end{aligned}
\tag{15}
$$

where $k_{nominal},\ \eta_{nominal},\ f_{d-nominal}$ are the nominal values of the parameters, and the $\hat{\Delta}_i$s are computed using a discrete version of (8), given by

$$
\begin{aligned}
&x_k(k'+1)=x_k(k')+a_k t_f sin(\omega_k k' t_f+\tfrac{\pi}{2})Q\\
&\hat{\Delta}_k(k'+1)=x_k(k'+1)+a_k sin(\omega_k k' t_f-\tfrac{\pi}{2}),\\
&x_\eta(k'+1)=x_\eta(k')+a_\eta t_f sin(\omega_\eta k' t_f+\tfrac{\pi}{2})Q\\
&\hat{\Delta}_\eta(k'+1)=x_\eta(k'+1)+a_\eta sin(\omega_\eta k' t_f-\tfrac{\pi}{2}),\\
&x_{f_d}(k'+1)=x_{f_d}(k')+a_{f_d} t_f sin(\omega_{f_d} k' t_f+\tfrac{\pi}{2})Q\\
&\hat{\Delta}_{f_d}(k'+1)=x_{f_d}(k'+1)+a_{f_d} sin(\omega_{f_d} k' t_f-\tfrac{\pi}{2}),
\end{aligned}
\tag{16}
$$

eventually, we conclude based on Lemma 1, that the controller (12), (13), (15) and (16), ensures that the tracking error norm is bounded with a decreasing function of the estimation error.

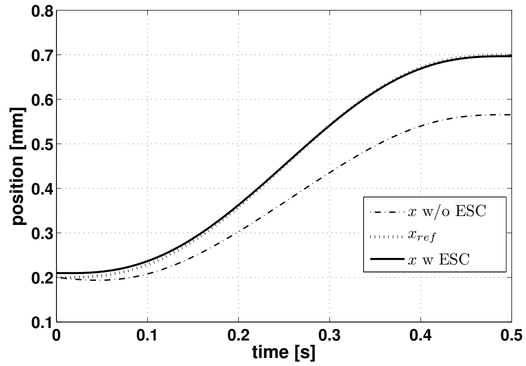
B. Numerical results

In this section, we illustrate our approach for the nonlinear electromagnetic actuator modelled by (10), using the system parameters given in Table I [26]. The reference trajectory is designed to be a 5^{th} order polynomial, $x^{ref}(t) = \sum_{i=0}^{5} a_i(\frac{t}{t_f})^i$ where the coefficients a_i are selected such that the following conditions are satisfied: $x^{ref}(0) = 0.2\ mm,\ x^{ref}(0.5) = 0.7\ mm,\ \dot{x}^{ref}(0) = 0,\ \dot{x}^{ref}(0.5) = 0, \ddot{x}^{ref}(0) = 0,\ \ddot{x}^{ref}(0.5) = 0$. We consider the uncertainties given by $\Delta k = -4.5$, $\Delta\eta = -0.7$ and $\Delta f_d = -7.5$. To make the simulation case more challenging we also introduced an initial error $x(0) = 0.01\ mm$ on the armature position. We implemented the controller (12) and (13) with the coefficients $c_1 = 100,\ c_2 = 100,\ c_3 = 2500,\ \kappa_1 = \kappa_2 = \kappa_3 = 0.25$, together with the learning algorithm (14), (15) and (16) with the coefficients $a_k = 0.5,\ \omega_k = 7.5,\ a_\eta = 0.2,\ \omega_\eta = 7.4,\ a_{f_d} = 1,\ \omega_{f_d} = 7.3,\ q_1 = q_2 = 100$. For more details about the tuning of the MES coefficients we refer the reader to [2], [22], [1], however, we underline here that the frequencies $\omega_i, \text{ı} = 1, 2, 3$ have been selected high enough to ensure efficient exploration on the search space and ensure convergence and that the amplitudes $a_i,\ i = 1, 2, 3$ of the dither signals, have been chosen such that the search is fast enough for this application. Here due to the cyclic nature of the problem, i.e., cyclic motion of the armature between 0 and x_f, the uncertain parameters estimate vector $(\hat{k},\ \ \hat{\eta},\ \ \hat{f}_d)^T$ is updated for each cycle, i.e., at the end of each cycle at $t = t_f$, the cost function Q is updated, and the new estimate of the parameters is computed for the next cycle. The purpose of using MES scheme along with iISS-backstepping controller is to improve the performance of the iISS-backstepping controller by better estimating the system parameters over many cycles, hence decreasing the error in the parameters over time to provide better trajectory following for the actuator. As can be seen in Figures 1(a) and 1(b), the robustification of the backstepping control via extremum seeking greatly improves the tracking performance. Figure 2(b) shows that the cost function decreases below 1 within 20 iterations. It can be seen in Figure 2(a) that the cost starts at an initial value around 9, and decreases rapidly afterwards. Moreover, the estimated parametric uncertainties $\hat{\Delta}k$, $\hat{\Delta}\eta$ and $\hat{\Delta}f_d$ converge to regions around the actual parameter values, as shown on Figure 3. The number of iterations for the estimate to reach the actual value of the parameters may appear to be high. The reason behind that is that the allowed uncertainties in the parameters are large, hence the extremum seeking

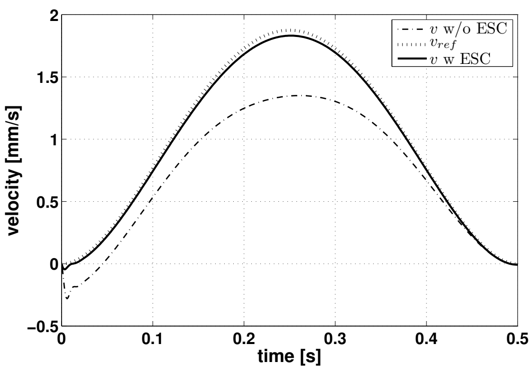
Parameter	Value
m	0.27 [kg]
R	6 [Ω]
η	7.53 [$\frac{kg}{s}$]
x_0	8 [mm]
k	158 [$\frac{N}{mm}$]
a	$14.96 \times 16\ \ ^{6}$ [$\frac{Nm^2}{A^2}$]
b	$4 \times 10\ \ ^{5}$ [m]

TABLE I

SYSTEM PARAMETER VALUES

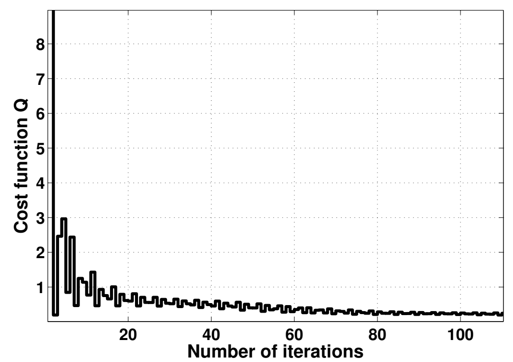


(a) Obtained Armature Position vs. Reference Trajectory

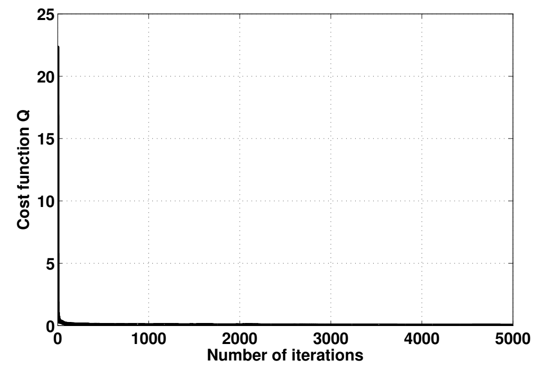


(b) Obtained Armature Velocity vs. Reference Trajectory

Fig. 1. Obtained trajectories vs. Reference Trajectory- Case with uncertain $k,\ \eta,\ f_d$



(a) Cost function- zoom



(b) Cost function

Fig. 2. Cost function- Case with uncertain $k,\ \eta,\ f_d$

scheme requires a lot of iterations to improve performance. Furthermore, we purposely tested the challenging case of three simultaneous uncertainties, which makes the space search for the learning algorithm large (note that this case of multiple uncertainties could not be solved with other classical model-based adaptive controller [27], due to some intrinsic limitations of model-based adaptive controller). However, In real-life applications uncertainties accumulate gradually over a long period of time, while the learning algorithm keeps tracking these changes continuously. Thus, the extremum seeking algorithm will be able to improve the controller performance in much fewer iterations. Finally, the control voltage is depicted on Figure 4, which shows an initial high value due to the relatively large simulated initial condition error on the armature position.

V. Conclusion

In this paper we have studied the problem of learning-based adaptive control for nonlinear systems with parametric uncertainties. We argued that for the class of nonlinear systems which can be rendered iISS w.r.t. the parameter estimation errors, by a robust feedback controller, it is possible to combine the iISS feedback controller with a model-free ES algorithm to obtain a learning-based adaptive controller. We showed a detailed application of this approach on a mechatronic example and reported encouraging numerical results. In this preliminary paper, we introduced the idea in a general setting, however, further investigation are needed to analyze specific nonlinear systems classes which can be stabilized (in the iISS sense) w.r.t. to the estimation error of the uncertain parameters, and show for these specific classes a constructive control design approach, in the context of the learning-based adaptive control presented here. Further work will also deal with using different ES algorithms with less restrictive conditions on the dither signals amplitude and frequencies, e.g.[8], [4], together with investigating other model-free learning algorithms such us reinforcement learning algorithms, and comparing the obtained controllers to the available classical adaptive controllers.

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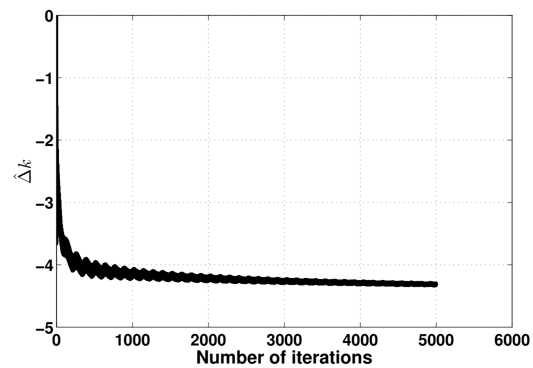
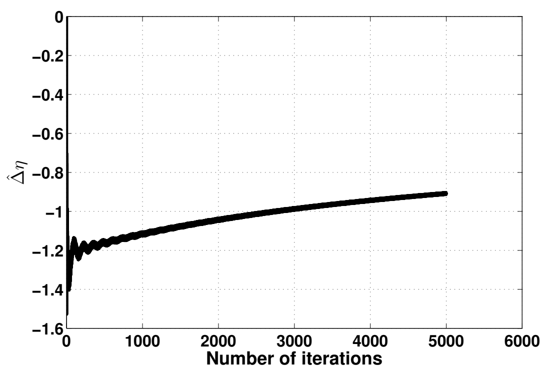
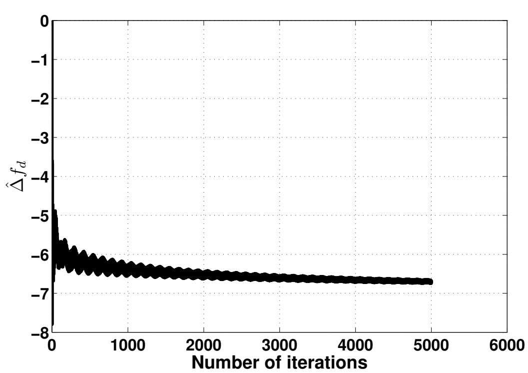


(a) Parameter k estimate



(b) Parameter η estimate



(c) Parameter f_d estimate

Fig. 3. Parameters estimates- Case with uncertain k, η, f_d

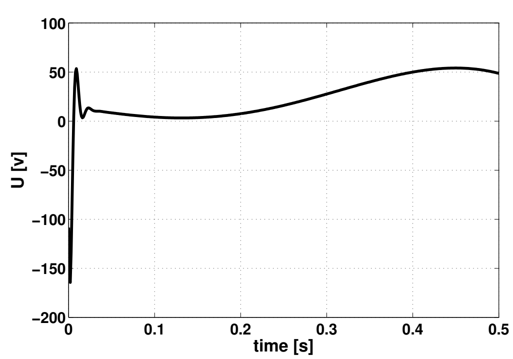


Fig. 4. Control voltage- Case with uncertain k, η, f_d

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