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Design of Low Fuel Trajectory in Interior Realm as a Backup Trajectory for Lunar Exploration

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In case of a failure on a Hohmann-base translunar trajectory, a reconfiguration of the trajectory that utilizes the three body dynamics of the interior realm of Earth-moon system is proposed. The stable and unstable manifold of a periodic orbit around L1 point extended toward the Earth side have homoclinic intersections. In the proposed method, after detection of a failure on the nominal trajectory, the trajectory is modified by small maneuvers so that the spacecraft can be kicked back by the moon and transferred to the unstable manifold. Then the spacecraft is returned back to the moon side through the intersection with the corresponding stable manifold on the Earth side. The periodic orbit is again used as a parking orbit so that the amount of delta-v at the moon orbit insertion can be reduced. Since the required amounts of delta-v at each individual maneuver are small throughout the reconfigured trajectory, it can serve as a solution for a backup trajectory in case of amain engine failure. Also since the operability in interior realm is good and the trajectory takes time, it can give a great chance for diagnosis and repair of the failure before arriving at the moon.

Key Words: Lunar Exploration, Failure, Trajectory Design, Three Body Dynamics, Interior Realm

Nomenclature

v	:	velocity
Δv	:	velocity increment
x, y	:	position in two dimension
r_{12}	:	distance between Earth and Moon
r_1	:	distance to spacecraft from the Earth
r_2	:	distance to spacecraft from the Moon
μ	:	mass ratio of the Moon
E	:	Energy
CJ	:	Jacobi constant
a	:	Semimajor axis
ω	:	Angle of apoapse

1. Introduction

Lunar exploration is yet again attracting attention from all over the world. In the 21st century, it was kicked off by SMART-1 launched by ESA in 2003. Since then, Japan (SELENE in 2007), China (Chang'e in 2007 and 2010), India (Chandrayaan in 2008) and United States (ARTEMIS in 2008, LRO in 2009, GRAIL in 2011) have sent spacecrafts to the moon. Within ISECG, Japan (SELENE-2 and SELENE-3), United States (LADEE), ESA (Lunar Lander), India (Chandrayaan-2), and Russia (Luna-Resurs, Luna-Glob) are planning to launch spacecrafts to the moon for the next decade[1)].

In a practical mission design, it is desirable to consider a method of trajectory reconfiguration in case of failures like main engine anomaly, loss of fuel, missing of lunar orbit injection. In the development of SELENE, method of reducing the minimum required Δv to be captured by the moon in case of main engine anomaly was proposed[2)] where the nominal two-impulse Hohmann trajectory is reconfigured by adapting three-impulse Hohmann transfer. Also method of trajectory design which enables the spacecraft to re-encounters the moon even when the original orbit injection is not performed was proposed[2)].

On the other hand, spacecraft trajectory design based on the three body dynamics (primary body, secondary body, and spacecraft) is subject to considerable research. In the design, unique properties of the dynamics can be utilized, such as resonance of the spacecraft w.r.t. to motion of the secondary body or periodic orbit around the Lagrange points, where gravity forces from two bodies, centrifugal force and Coriolis force are balanced. The unique dynamics of three-body problem can be properly utilized to design transfer trajectory requiring lower Δv, compared to a Hohmann transfer two-body problem design, and then the mass budget is relaxed.

GENESIS launched by NASA demonstrated sampling of solar wind at L1 periodic orbit of Sun-Earth System in which the spacecraft reached the orbit along the stable manifold of the orbit, stabilized the orbit during the observation for two years, and returned back to the Earth through the unstable manifold of the orbit[4)]. ARTEMIS launched also by NASA demonstrated lunar transfer through L1 and L2 periodic orbits of Earth-moon system for the first time in the world[5)]. The trajectory design based on three body dynamics is already in a practical level.

While having a lower fuel nominal trajectory and equipping more scientific instruments on board is appealing, we explore the use of three body dynamics to develop a lower fuel "backup" trajectory in case of failures on a two body nominal trajectory. For example, if it turns out that sufficient amount of fuel to reach the lunar orbit by originally planned way is unavailable due to a failure, if another trajectory requiring lower amount of fuel is available, it will give a chance to achieve the goal. Since not only the sum but also the required amounts of Δv at each individual maneuver are small in these trajectories, it would help to achieve the goal when the attainable Δv by a single maneuver is significantly limited due to an anomaly of main engine.

The inspiration for this work is HITEN, the first Japanese lunar explorer, which was unable to transfer to the moon via originally planned orbit. A trajectory utilizing three body dynamics of both Sun-Earth system and Earth-moon system was proposed and successfully applied to achieve the mission goals. The spacecraft first went to the "exterior realm" of the Earth-moon system and was injected to the unstable manifold of L2 periodic orbit of Sun-Earth system. Then through the intersection point of the manifold with the stable manifold of L2 periodic orbit of Earth-moon system, it was injected to the stable manifold and then transferred back to the moon via channel around the L2 point[6]. The trajectory reconfiguration considered for SELENE[2] is similar to this in that it also utilizes the dynamics of the exterior realm of the Earth-moon system.

In this work, method of translunar trajectory reconfiguration which utilizes three body dynamics of the "interior realm" of Earth-moon system is proposed. As it in the reconfigured trajectory utilizing the dynamics of exterior realm, required amounts of Δv at each individual maneuver are significantly smaller than those of trajectory designed based on two body dynamics, although the total amount of necessary fuel does not change much. Another benefit of using interior realm is a good operability of the satellite where faster communication link and better observation are achieved. Since lower fuel trajectories have longer time of flight, it can enable opportunity for diagnosis and repair of the failure before arriving at moon.

2. Three Body Dynamics

Throughout this paper, we discuss the system within the planer circular restricted three-body problem (PCR3BP). The normalized equation of motion of spacecraft in the presence of the Earth and the moon revolving around each other in a plain can be described in the rotating frame as follows,

$$\ddot{x} - 2\dot{y} = -\overline{U}_x \tag{1}$$

$$\ddot{y} + 2\dot{x} = -\overline{U}_y \tag{2}$$

where

$$\overline{U}(x, y) = -\frac{1}{2}(x^2 + y^2) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} \tag{3}$$

The distance is normalized by the distance between Earth and the moon. The time is by the revolution period of the moon over 2π. The first term of (3) gives the potential energy due to the centrifugal force. The energy E is constant over time unless Δv is assigned. -2E is called Jacobi constant.

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \overline{U} \tag{4}$$

$$CJ = -2E \tag{5}$$

There exist five equilibrium points called Lagrange points, L1 to L5. We focus on the L1 point located between the Earth and the moon. The L1 point is of saddle x center type, and there exists a family of periodic orbits around the L1 point for certain energy values. There also exist a set of trajectories winding away from the periodic orbit and set of trajectories winding onto the periodic orbit, constructing the tube shaped unstable manifolds and stable manifolds toward both for the Earth and the moon side.

Fig. 1 shows the L1 point, L1 periodic orbit and the corresponding stable and unstable manifolds when CJ is 3.19. L1 periodic orbit provides a channel to connect the Earth side and the moon side. The size of L1 periodic orbit depends on CJ. Smaller the CJ is bigger is the size of the periodic orbit. The L1 periodic orbit exist when CJ is lower than 3.2 (CJ of L1 point). If CJ is bigger than the 3.2, the energy is so small that the channel is closed. If CJ is significantly smaller than 3.0 (CJ of L4 and L5 point), the energy is so big that the periodic orbit cannot exist and unique properties of the three body dynamics disappear.

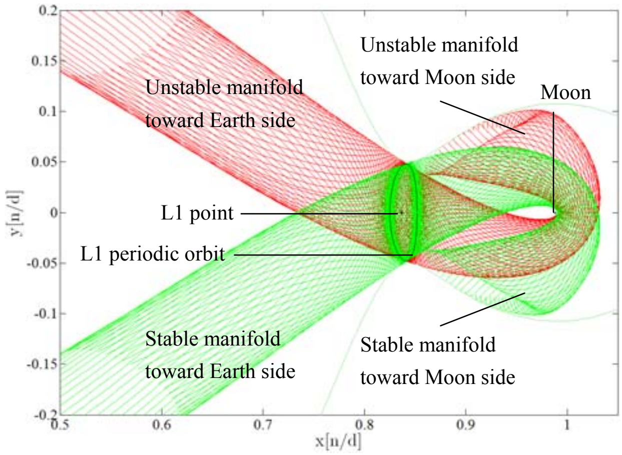

Fig. 1. L1 point, L1 periodic orbit and manifolds (CJ = 3.19)

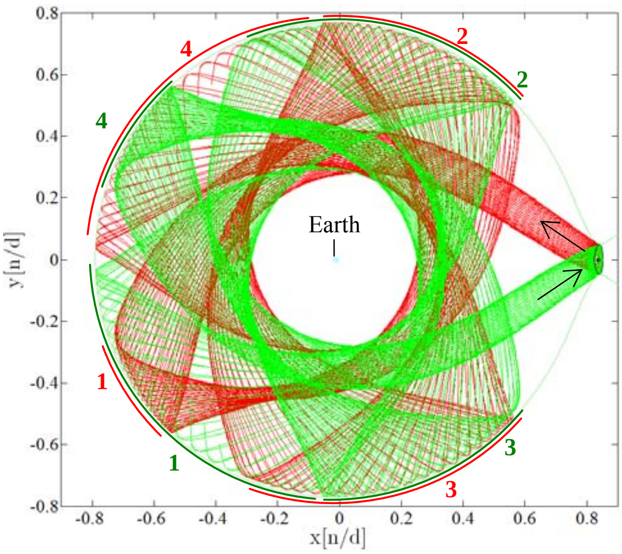

Fig. 2. Stable and unstable manifolds (CJ=3.19)

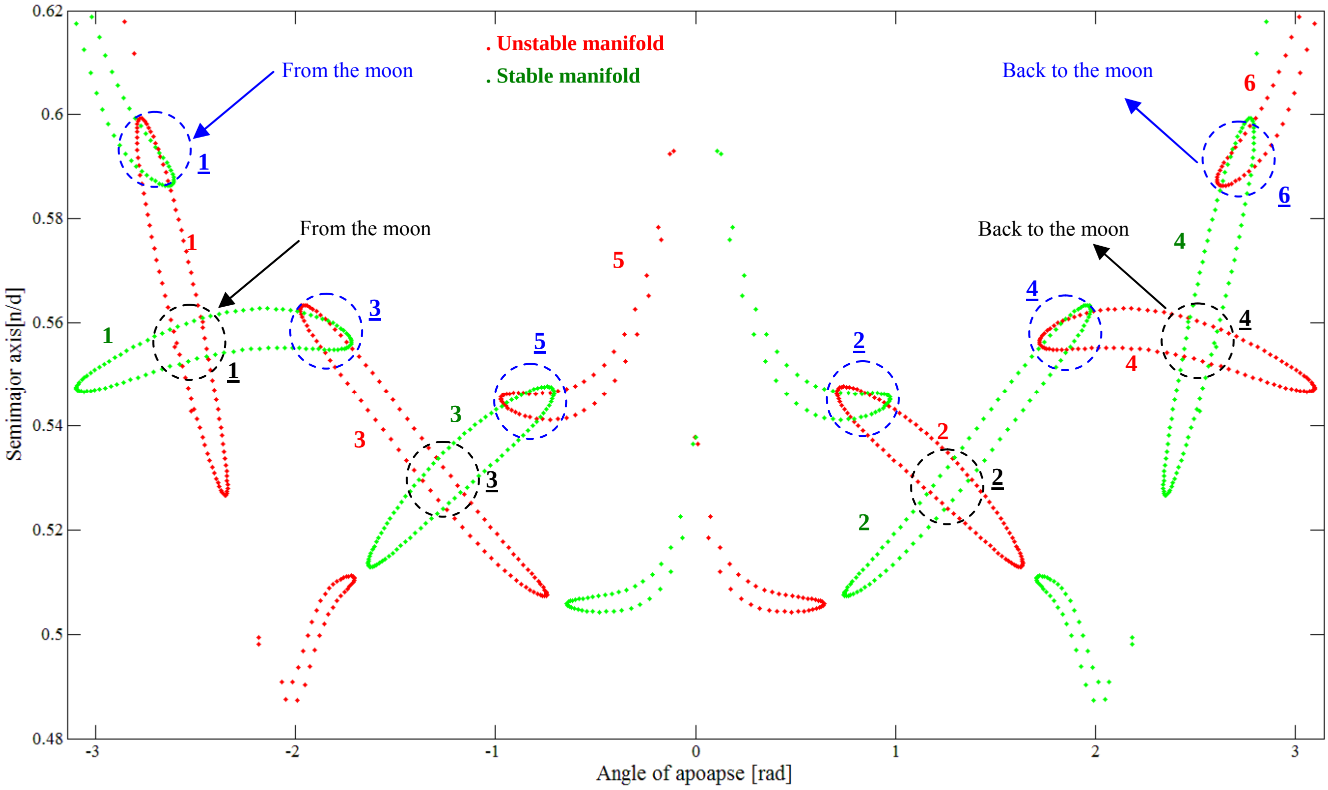

Fig. 3. Homoclinic intersection of the stable and unstable manifolds viewed in Poincare section (CJ = 3.19)

In Fig. 2 stable and unstable manifolds toward Earth side are numerically extended till they pass apoapses four times. The integers indicate the order of set of the apoapses which each manifold passes as time goes forward. Note that the fourth apoapse of stable manifold in Fig. 2 is the first apoapse in the backward integration since the stable manifold was drawn by giving each position of the periodic orbit a small perturbation with the direction of the eigenvector of the stable eigen value, then integrating backward time.

The intersection of the unstable manifold with a section at first apoapse overlaps that of the first apoapse of stable manifold. Also the third apoapse of unstable manifold seems to be partly overlapped by it. Fig. 2 shows intersection of these two manifolds projected on configuration space. Poincare sections in Fig. 3 provide a better view of the homoclinic intersection or Homoclinic connection[7].

Fig. 3 shows a Poincare section of the stable and unstable manifolds projected at the apoapse surface of sections. The integers colored by red and green indicate the order of the apoapses of unstable and stable manifold, corresponding to the integers in Fig. 2. Each state of the manifold at apoapse is converted to two dimensional variables, angle of apoapse ω and semimajor axis a[8] by following way.

$$\omega = \arctan(y/x) \tag{6}$$

$$a = \frac{1}{-r^2 + 2(\dot{x}y - \dot{y}x) - (\dot{x}^2 + \dot{y}^2) + \dfrac{2}{r}} \tag{7}$$

where $r = \sqrt{x^2 + y^2}$.

In PCR3BP, the motion of spacecraft is not exactly an ellipse. The semimajor axis in this context is the semimajor axis if the spacecraft trajectory is approximated by ellipse around the apoapse, in other words, semimajor axis of the osculating elliptical orbit. The equation (7) is derived from the definition of normalized energy in two body dynamics,

$$2E = -\frac{1}{a} = \dot{r}^2 - \frac{2}{r} + \frac{h^2}{r^2}, \tag{8}$$

where $h = r^2 + r^2\left(\dfrac{-\dot{x}y}{r^2} + \dfrac{x\dot{y}}{r^2}\right)$

is the sum of angular momentum due to the rotation of the frame and that of the spacecraft in the rotating frame.

In figure, the overlaps of projections on first apoapse of the stable manifold with the first and third apoapse of the unstable manifold are the intersections encircled by black dash line with 1 and blue dash line with 3 each other in Fig. 3. If the spacecraft is located on the unstable manifold which is going away from the moon side, if the state of the spacecraft on the unstable manifold is within the intersection with the stable manifold indicated by black 1, it can again return back to the moon side through the stable manifold by revolving around the Earth for four times. The transfer is indicated by the transition of the intersections in the Poincare section encircled by black dash line with 1 to 4. Similarly, if the state of the spacecraft is within the intersection indicated by blue 1, it can again return back to the moon by revolving around the Earth for six times. The trajectory is indicated by the transition of the intersections encircled by blue dash line with 1 to 6.

3. Trajectory Design Concept

The base idea of our trajectory reconfiguration is to utilize the homoclinic intersection of the unstable and stable manifold of the Earth side to re-target the moon. We assume the original mission plan is to transfer the spacecraft to the moon by Hohmann-base trajectory. Once a failure is detected on the nominal trajectory and it turns out that originally planned insertion to the moon is impossible, our idea is to give small maneuvers so that the spacecraft is away from the trajectory and transferred it to the unstable manifold of the L1 periodic orbit. Through the homoclinic intersection and the corresponding stable manifold, the spacecraft is again able to go back to the moon side.

At least two maneuvers are necessary to transfer the spacecraft from the nominal trajectory to the unstable manifold. The first maneuver is to deflect the spacecraft away from the nominal trajectory and the second maneuver is to connect to the trajectory on the unstable manifold. Then spacecraft ballistically revolves around the Earth several times (four or six times according to the way of intersection explained in the previous section) till it reaches the last apoapse of the stable manifold prior to the L1 periodic orbit shown in Fig. 2 and Fig. 3 indicated by integer “4” colored by green. Very small maneuvers can be applied at apses to adjust the state at the last apoapse before arriving. From the last apoapse, we retarget the moon again. The schematic of the trajectory reconfiguration is shown in Fig. 4.

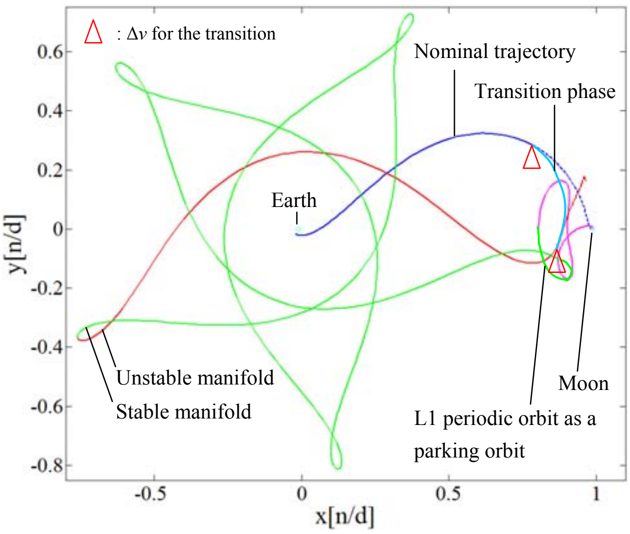

Fig. 4. The schematic of the trajectory reconfiguration

In the transfer from the last apoapse to the moon, L1 periodic orbit can be utilized as a parking orbit, yielding a significant reduction of the required amount of Δv at the moon orbit insertion. This is one of the benefits of using stable manifold as a transfer trajectory. Fig. 5 shows a two impulse Hohmann-base trajectory to the moon in three body dynamics in rotating frame. The initial orbit is Earth parking orbit of perigee height 1000 km and apogee height 36000 km. The targeted orbit is of periliune height 100 km and apolune height 11741 km in similar to those of SELENE. As the flight result shows[9] the Δv at the moon orbit insertion in this case takes about 290 m/s. On the other hand, Fig. 6 shows a transfer trajectory to the moon from the L1 periodic orbit as a parking orbit. Very small maneuver (0.3 m/s) is applied at the periodic orbit to deflect the spacecraft away from it and get the spacecraft on the unstable manifold tube extended toward the moon side. Then the spacecraft is ballistically transferred to the region near to the moon, and the second maneuver is applied to insert it to the target orbit. The CJ of the periodic orbit is 3.19. The trajectory takes about 12 days. The Δv at the insertion to the same target orbit of perilune and apolune height takes only 88.3 m/s. This would offer hope when the attainable Δv by single maneuver is significantly limited due to an anomaly of main engine.

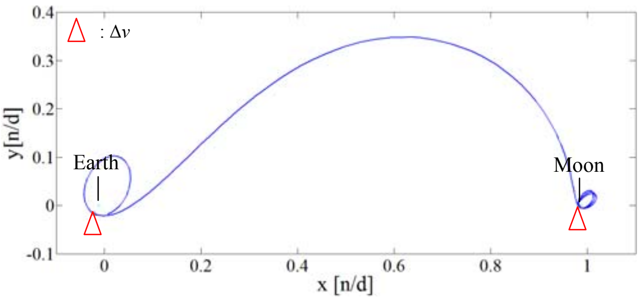

Fig. 5. Hohmann transfer in rotating frame (Δv for insertion is 290 m/s)

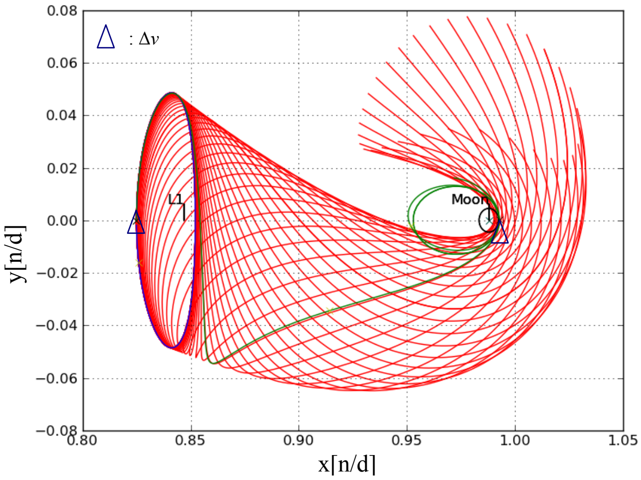

Fig. 6. Transfer trajectory to the moon from the L1 periodic orbit (CJ = 3.19, Δv for insertion is 88.3 m/s)

In the design of backup trajectory described above, we decompose the whole trajectory into four phases and optimize them respectively by a given order. The phases are transition phase from the nominal trajectory to a trajectory on the unstable manifold, the ballistic revolution phase around the Earth till the spacecraft reaches at the last apoapse of the stable manifold, insertion phase from the last apoapse to the L1 periodic orbit, and the insertion phase from the L1 periodic orbit to the moon.

4. Optimization

Since we assume that the backup trajectory is used when a failure like main engine anomaly or loss of fuel occurs, the amount of Δv at each maneuver and also their sum should be smaller than that of the moon insertion maneuver of the nominal trajectory, 290 m/s.

The last three phases of the backup trajectory are well researched. In the previous work, we showed that targeting the L1 periodic orbit from inside the tube of stable manifold (third phase) needs only around 2 m/s[8]. Typical example of targeting the moon from the L1 periodic orbit (forth phase) is shown in Fig. 6. Although the required Δv depends on CJ of the L1 periodic orbit, it is significantly smaller than 290 m/s.

In this work, we first show the existence of the first phase of the proposed backup trajectory. Based on an iterative search among different orbit topologies, we found that a three-impulse maneuver can provide a transfer from Hohmann-base nominal trajectory to the unstable manifold with acceptable level of Δv. In particular, the first maneuver is assigned on the nominal trajectory to increase the energy and have the spacecraft go to the far side of the moon. Then it is kicked back by the moon. The second maneuver is assigned near the moon to adjust the direction of the trajectory. The third maneuver is assigned at the first perigee after the kick to reduce the energy and connect the trajectory to the unstable manifold.

We optimize this trajectory by multiple shooting method given a fixed initial point on the nominal trajectory. The starting point is 3.5 days after leaving the Earth parking orbit, 1.7 days before arrival at the moon in nominal plan..

The trajectory to be optimized starts from the fixed initial point, and ends at the first apogee after the kick back. The parameters are time of the three maneuvers, amount of the three maneuvers for each direction, states of the points where the maneuvers are assigned, and the state of the end point as shown in the equation (9). The objective function is square sum of the Δv at the three maneuvers as shown in the equation (10). In addition to equality constraints for continuity and apse condition shown in the equation (11) and (12), inequality constraints are assigned to the range of time, Δv, and CJ.

$$z = [t_1, X_1, \Delta v_1, t_2, X_2, \Delta v_2, t_3, X_3, \Delta v_3, t_4, X_4] \quad (9)$$

where $X_i = [x_i, y_i, v_{xi}, v_{yi}], \Delta v_i = [\Delta v_{xi}, \Delta v_{yi}]$

$$J = \sum_{i=1}^{3} \left(\Delta v_{xi}^2 + \Delta v_{yi}^2\right) \quad (10)$$

$$f(X_{i-1} + [0,0,\Delta v_{i-1}], t_i) - X_i = 0 \quad (i = 1 \ldots 4) \quad (11)$$

where X_0 is state of the initial point, and $\Delta v_0 = [0,0]$

$$x_i \cdot v_{xi} + y_i \cdot v_{yi} = 0 \quad (i = 2 \ldots 4) \quad (12)$$

The most important inequality constraint is CJ at the end point. It dominates required amount of Δv throughout the trajectory. Since the CJ of Hohmann-base nominal trajectory is very low (energy is very high comparing with energy of typical low fuel trajectory), higher the CJ at the end point is (lower the energy) larger the require amount of Δv is.

Fig. 7 and fig. 8 show the optimized trajectory when the minimum of CJ at the end point is constrained to be 2.9. Fig. 9 is a zoom up of the trajectory in inertial frame.

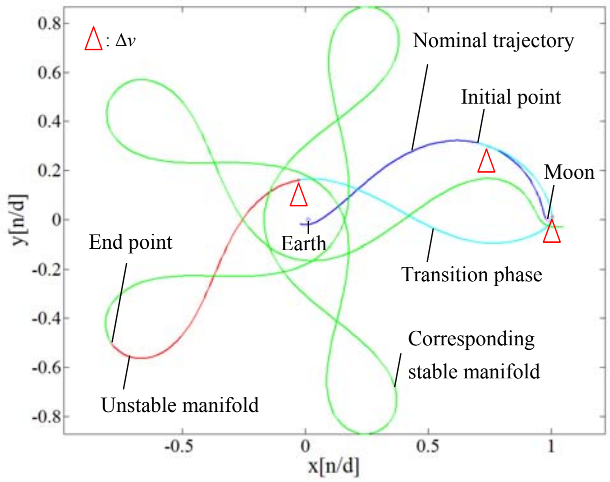

Fig. 7. Optimized first phase of backup trajectory in rotating frame (The minimum CJ at the end point is constrained to be 2.9)

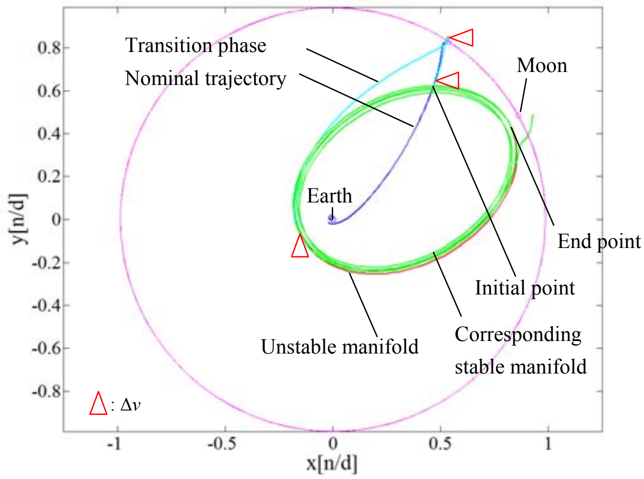

Fig. 8. Optimized first phase of backup trajectory in inertial frame (The minimum CJ at the end point is constrained to be 2.9)

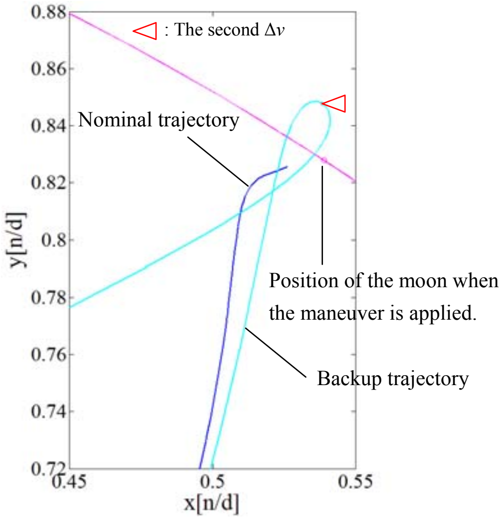

Fig. 9. Zoom-up of the trajectory in inertial frame

The corresponding stable manifold is also described. The first Δv is 89.7 m/s, the second Δv is 19.3 m/s, and the third Δv is 87.9 m/s. These are significantly smaller than 290 m/s and might be attainable even when the output of main engine is limited due to a failure.

On the other hand, expected amount of total required fuel for whole the backup trajectory is almost same with that of nominal trajectory. This is because sum of the three maneuvers (196.9 m/s) and the Δv in the forth phase with L1 periodic orbit of CJ 3.19 (88.3 m/s) is 285.2 m/s, and another small maneuvers are necessary for increasing CJ from that of the end point to 3.19 in the second phase and for targeting L1 periodic orbit in the third phase. Global optimization to reduce the total amount of Δv throughout the backup trajectory is currently underway and will be pursued in a future publication.

We optimized the trajectories with different minimum CJ values at the end point. Fig. 10 shows the value CJ at initial point (step 0), the three point where maneuvers are assigned (step 1, 2, 3), and the end point (step 4) for the different inequality constraints. The CJ of step 1 to 3 are calculated for state immediately before the maneuvers are assigned. For all cases, the CJ is reduced after the first maneuver so that the spacecraft can go to far side of the moon. At the third maneuver, CJ is increased to their minimum bound at the end point so that the required amount of Δv at the maneuver can be minimized.

The amount of Δv at the three maneuvers for different inequality constraints are shown in fig. 11. As mentioned above, the smaller the CJ at the end point is, smaller the required amount of Δv. However, the amount of Δv for increasing CJ to 3.19 in the second phase might be larger. Sum of Δv at the three maneuvers is 168.8 m/s (CJ min = 2.8), 196.9 m/s (CJ min = 2.9), 233.9 m/s (CJ min = 3.0), 281.0 m/s (CJ min = 3.1).

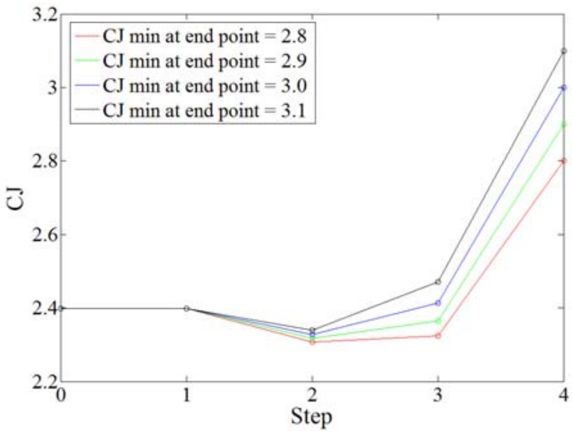

Fig. 10. History of CJ different minimum of CJ at the end point

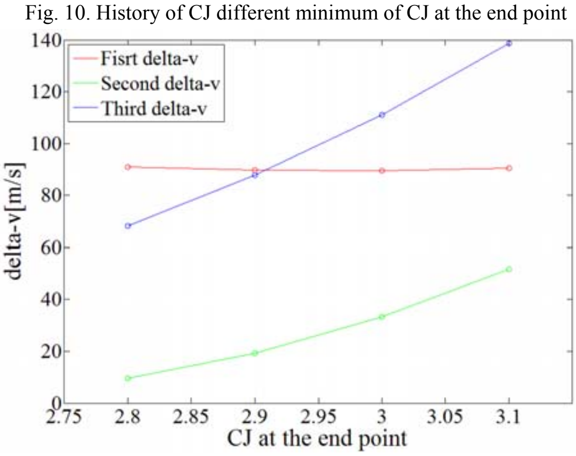

Fig. 11. Amount of Δv for different minimum of CJ at the end point

5. Conclusions

In case of a failure on a Hohmann-base nominal translunar trajectory, a trajectory reconfiguration that utilizes three body dynamics of the interior realm of the Earth-moon system is proposed. The reconfigured trajectory, 'back up trajectory' in other word, utilizes the unique dynamics of the stable and unstable manifold of L1 periodic orbit. When a failure is detected, the spacecraft once returns to the Earth side, and then through the homoclinic interaction of the manifolds it again targets the moon.

In this work, we showed the existence of such a back up trajectory with acceptable level of amount of Δv. Global optimization of the whole trajectory is currently under investigation.

Although the total amount of necessary fuel does not change much, the low fuel trajectory in interior realm as a back up trajectory provides the following two benefits. The first benefit is the reduction of the required amount of Δv at each individual maneuvers. This would offer hope if the attainable Δv by single maneuver is significantly limited due to an anomaly of main engine. In particular, the proposed three-impulse maneuver to target the spacecraft to unstable manifold from the nominal trajectory requires 89.7 m/s, 19.3 m/s, 87.9 m/s respectively. These are significantly smaller than that of the moon insertion maneuver of the nominal trajectory, 290 m/s, and might be attainable even when the output of main engine is limited due to a failure.

The second benefit, which is not available in a trajectory reconfiguration utilizing the exterior realms, is good operability of the satellite where faster communication link and better observation are achieved. Since lower fuel trajectory takes time in general, it can give a great chance for diagnosis and repair of the failure before arriving at the moon.

References

1) ISECG (International Space Exploration Coordination Group): Global Exploration Roadmap, 2011.
2) Kawakatsu, Y.: SELENE Translunar Trajectory Reconfiguration Plan Provided for the case of Main Engine Anomaly, *Proceedings of The 20th International Symposium on Space Flight Dynamics*, 2007.
3) Kawakatsu, Y.: Study on Lunar Approach Strategy Tolerant of a Lunar Orbit Injection Failure, *Trans. JSASS Space Tech. Japan*, **5,** pp.1-7, 2007.
4) Burnett, D.S. et al,: The GENESIS Discovery Mission: Return of Solar Matter to Earth, *Space Science Reviews*, **105**, pp.509-534, 2003.
5) Folta, D.C. et al: Application of multi-body dynamical environments: The ARTEMIS transfer trajectory design, *Acta Astronautica*, **73**, pp. 237-249, 2012.
6) Belbruno, E.A. and Mikker, J.K.: Sun-perturbed Earth-to-Moon transfers with ballistic capture, *Journal of Guidance, Control and Dynamics*, Vo. 16, No.5, pp.770-775, 1993.
7) Koon, W. et al,: Dynamical Systems, the Three-Body Problem and Space Mission Design, *Marsden Books*, pp. 60-72, 2008.
8) Grover, P. and Anderson, C: Optimal three body assist and manifold transfers in end-to-end lunar mission design, *Proceedings of the 22nd AAS/AIAA Space Flight Mechanics*, 2012.
9) Matsumoto, S. et al: Flight Result of Selenological and Engineering Explorer "KAGUYA" on Lunar Orbit, *The 22nd International Symposium on Space Flight Dynamics*, 2009.