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Abstract

We study in this paper the problem of iterative feedback gains auto-tuning for a class of nonlinear systems. For the class of Input-Output linearizable nonlinear systems with bounded additive uncertainties, we first design a nominal Input-Output linearization-based robust controller that ensures global uniform boundedness of the output tracking error dynamics. Then, we complement the robust controller with a model-free *multi-parametric extremum seeking (MES)* control to iteratively auto-tune the feedback gains. We analyze the stability of the whole controller, i.e., robust nonlinear controller combined with the MES model-free learning algorithm. We use numerical tests to demonstrate the performance of this method on a mechatronics example.

I. Introduction

Currently, feedback controllers are used in a variety of systems. There are several types of feedback, e.g., state feedback vs. output feedback, linear vs. nonlinear etc. However, one common characteristic of all available feedback controllers is the fact that they all rely on some 'well chosen' feedback gains. The selection of these feedback gains is often done based on some desired performance. For instance, the gains can be chosen in such a way to minimize overshoot of a linear closed-loop system, settling-time can be another performance target, minimizing a given finite-time or asymptotic, state or output tracking error can be of interest in many applications, as well. Over the past years, there has been a myriad of results about feedback gains tuning. Maybe one of the most famous, and widely taught technique, would be the so-called Ziegler-Nichols rules for PID gains tuning for linear systems [1]. However, such rules apply for the particular class of linear systems, under linear PID feedback, and are considered to be heuristic in nature.

For more general cases of models and controllers, and a more systematic or autonomous way of tuning feedback gains, the control community started looking at an iterative procedure to auto-tune feedback gains for closed-loop systems. Indeed, in the seminal paper [2], the authors introduced the idea that the feedback controllers' parameters could be tuned iteratively to compensate for model uncertainties, and that the tuning could be based on measurements directly obtained from the system. This idea of iterative control tuning lead to the so-called 'iterative feedback tuning' (IFT) research field, where the goal is to iteratively auto-tune feedback gains of a closed-loop system based on the online optimization of a well defined performance cost function. There have been a lot of results about IFT in the past 20 years, and it is not the purpose of this paper to survey all the existing papers in the field. However, the existing results are mainly dedicated to linear systems controlled with linear feedbacks, e.g., [3], [4], [5], [6], [7], [8], [9], [10], [11], [12].

Based on these IFT algorithms for linear systems, some extensions to nonlinear systems have been proposed. For instance in [13], the author study the case of discrete nonlinear systems controlled with linear time-invariant output feedback. The effect of IFT algorithms developed originally for linear systems was studied on nonlinear systems by assuming local Taylor approximation of the nonlinear dynamics. However, the full analysis of the feedback loop, i.e., IFT of the linear controller applied to the nonlinear dynamics, was not reported in this paper. Other feedback gains iterative tuning algorithms were developed for nonlinear systems in [14], [15], [16]. The algorithms developed in these papers, first *assume that the closed-loop input and output signals remain bounded during the gains tuning*, and they rely on the numerical estimation of the gradient of a given cost function with respect to the controller gains, which necessitates to run the system $n+2$ *times for each learning iteration*, where n is the dimension of the tuned gain vector. This obviously can be a limiting factor if the number of tuned parameters is large. An alternative has been proposed in [17], where the proposed IFT algorithm requires only 2 experiments per learning iteration. However, this algorithm was proposed for SISO systems, where the nonlinear dynamics can be approximated with linear time-varying dynamics. The algorithms is of local nature designed around one test reference, and thus was not proven to be robust w.r.t. changes in the reference. Moreover, this IFT algorithm requires to solve a local identification problem (formulated as an optimal least-square problem) at each learning iteration. More recently, another approach has been proposed in [18] for SISO systems. This method, called virtual reference feedback tuning (VRFT), is based on the assumption of open-loop Lagrange stability of the system, since it requires an initial set of (noise-free) input-output data.

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In this work, we propose to study the problem of gains auto-tuning in the general setting of uncertain nonlinear systems (refer to [19], [20] for preliminary results), with a rigorous stability analysis of the full system, i.e., learning algorithm merged with the nonlinear controller and the nonlinear uncertain system. We consider here a particular class of nonlinear systems, namely, nonlinear models affine in the control input, which are linearizable via static state feedback. We consider bounded additive model uncertainties with known upper bound function. We propose a simple modular iterative gains tuning controller, in the sense that we first design a passive robust controller, based on the classical Input-Output linearization method complemented with a Lyapunov reconstruction-based control, e.g., [21], [22]. This passive robust controller ensures uniform boundedness of the tracking errors (Lagrange stability) and their convergence to a given invariant set, i.e., *we do not assume but rather guarantee Lagrange stability of the feedback system while tuning the gains*. Next, in a second phase, we add a multi-parametric extremum seeking algorithm to iteratively auto-tune the feedback gains of the passive robust controller, to optimize a desired system performance. The desired closed-loop performance is formulated in terms of a desired learning cost function minimization. One of the main advantages of using MES to minimize the learning cost function, is the fact that MES necessitates only *one evaluation of the cost function for each learning iteration*, i.e., one experiment per learning iteration.

This paper is organized as follows: First, some notations and definitions are recalled in Section II. Next, we present the class of systems studied here and formulate the control problem in Section III. The proposed control approach together with the closed-loop dynamic solutions' boundedness are presented in Section IV. Section V is dedicated to the application of the controller to a mechatronics example, namely, an electromagnetic actuator. Finally, the paper ends with a summarizing discussion in Section VI.

II. Notations and definitions

Throughout the paper we will use $|.|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $|x| = \sqrt{x^T x}$. We will use the notations $diag\{m_1, ..., m_n\}$ for $n \times n$ diagonal matrix, $z(i)$ denotes the ith element of the vector z. We use $\dot{(.)}$ for the short notation of time derivative and $f^{(r)}(t)$ for $\frac{d^r f(t)}{dt^r}$. $Max(V)$ denotes the maximum element of a vector V, and $sgn(.)$ denotes for the sign function. We denote by $\mathbb{C}^k$ functions that are k times differentiable, and by $\mathbb{C}^\infty$ a smooth function. A function is said analytic in a given set, if it admits a convergent Taylor series approximation in some neighborhood of every point of the set. An impulsive dynamical system is said to be well-posed, if it has well defined distinct resetting times, admits a unique solution over a finite forward time interval, and does not exhibits any Zeno solutions, i.e., an infinitely many resetting of the system in finite time interval [23]. Finally, in the sequel when we talk about error trajectories' boundedness, we mean uniform boundedness as defined in [21] (p.167, Definition 4.6) for nonlinear continuous systems, and in [23] (p. 67, Definition 2.12) for time-dependent impulsive dynamical systems.

III. Problem formulation

A. Class of systems

We consider here affine uncertain nonlinear systems of the form

$$\begin{array}{c} \dot{x} = f(x) + \Delta f(x) + g(x)u, \; x(0) = x_0, \\ y = h(x), \end{array} \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^{n_a}, y \in \mathbb{R}^m$ $(n_a \geq m)$, represent, the state, the input and the controlled output vectors, respectively. x_0 is a known initial condition, $\Delta f(x)$ is a vector field representing additive model uncertainties. The vector fields f, Δf, columns of g and function h satisfy the following assumptions.

Assumption 1: $f : \mathbb{R}^n \to \mathbb{R}^n$ and the columns of $g : \mathbb{R}^n \to \mathbb{R}^{n \times n_a}$ are $\mathbb{C}^\infty$ vector fields on a bounded set X of $\mathbb{R}^n$ and $h(x)$ is a $\mathbb{C}^\infty$ function on X. The vector field $\Delta f(x)$ is $\mathbb{C}^1$ on X.

Assumption 2: System (1) has a well-defined (vector) relative degree $\{r_1, \ldots, r_m\}$ at each point $x^0 \in X$, and the system is linearizable, i.e., $\sum_{i=1}^{i=m} r_i = n$ (see e.g., [24]).

Assumption 3: The uncertainty vector Δf is s.t. $|\Delta f(x)| \leq d(x) \; \forall x \in X$, where $d \; : \; X \to \mathbb{R}$ is a smooth nonnegative function.

Assumption 4: The desired output trajectories y_{id} are smooth functions of time, relating desired initial points y_{i0} at $t = 0$ to desired final points y_{if} at $t = t_f$, and s.t. $y_{id}(t) = y_{if}, \; \forall t \geq t_f, \; t_f > 0, \; i \in \{1, ..., m\}$.

B. Control objectives

Our objective is to design a feedback controller $u(x, K)$, which ensures for the uncertain model (1) uniform boundedness of a tracking error, and for which the stabilizing feedback gains vector K is iteratively auto-tuned, to optimize a desired performance cost function. We stress here that the goal of the gain auto-tuning is not stabilization but rather performance optimization. To achieve this control objective, we proceed as follows: We first design a 'passive' robust controller which ensures boundedness of the tracking error dynamics, and then we combine it with a model-free learning algorithm to iteratively auto-tune the feedback gains of the controller, and optimize online a desired performance cost function.

IV. Controller design

A. Step one: Passive robust control design

Under Assumption 2, and nominal conditions, i.e., $\Delta f = 0$, system (1) can be written as [24]

$$y^{(r)}(t) = b(\xi(t)) + A(\xi(t))u(t), \tag{2}$$

where

$$\begin{array}{l} y^{(r)}(t) \triangleq (y_1^{(r_1)}(t), \ldots, y_m^{(r_m)}(t))^T, \\ \xi(t) = (\xi^1(t), \ldots, \xi^m(t))^T, \\ \xi^i(t) = (y_i(t), \ldots, y_i^{(r_i-1)}(t)),\ 1 \le i \le m, \end{array} \tag{3}$$

and where b, A write as functions of f, g, h, and A is non-singular in X ([24], pp. 234-288).
At this point we introduce one more assumption on the system.

Assumption 5: We assume that the additive uncertainties Δf in (1) appear as additive uncertainties in the linearized model (2), (3), as follows

$$y^{(r)} = b(\xi) + \Delta b(\xi) + A(\xi)u, \tag{4}$$

where Δb is $\mathbb{C}^1$ on $\tilde{X}$, and s.t. $|\Delta b(\xi)| \le d_2(\xi)\ \forall \xi \in \tilde{X}$, where $d_2\ :\ \tilde{X} \to \mathbb{R}$ is a smooth nonnegative function, and $\tilde{X}$ is the image of the set X by the diffeomorphism $x \to \xi$ between the states of (1) and (2).
If we consider the nominal model (2), we can define a virtual input vector v as

$$b(\xi(t)) + A(\xi(t))u(t) = v(t). \tag{5}$$

Combining (2) and (5), we obtain the linear (virtual) Input-Output mapping

$$y^{(r)}(t) = v(t). \tag{6}$$

Based on the linear system (6), we write the stabilizing output feedback for the nominal system (4) with $\Delta b(\xi) = 0$, as

$$\begin{array}{c} u_{nom} = A^{-1}(\xi)(v_s(t,\xi) - b(\xi)),\ v_s = (v_{s1}, ..., v_{sm})^T, \\ v_{si}(t,\xi) = y_{i_d}^{(ri)} - K_{ri}^i(y_i^{(ri-1)} - y_{i_d}^{(ri-1)}) - ... - K_1^i(y_i - y_{i_d}), i \in \{1, ..., m\}. \end{array} \tag{7}$$

Denoting the tracking error vector as $e_i(t) = y_i(t) - y_{i_d}(t)$, we obtain the tracking error dynamics

$$e_i^{(r_i)}(t) + K_{r_i}^i e_i^{(r_i-1)}(t) + ... + K_1^i e_i(t) = 0,\ i = 1, ..., m, \tag{8}$$

and by tuning the gains $K_j^i,\ i = 1, ..., m,\ j = 1, ..., r_i$ such that all the polynomials in (8) are Hurwitz, we obtain global asymptotic convergence of the tracking errors $e_i(t),\ i = 1, ...m$, to zero. To formalize this condition let us state the following assumption.

Assumption 6: We assume that there exists a nonempty set $\mathcal{K}$ of again $K_j^i,\ i = 1, ..., m,\ j = 1, ..., r_i$, such that the polynomials (8) are Hurwitz.

Remark 1: Assumption 6 is well know in the Input-Output linearization control literature. It simply states that we can find gains that stabilize the polynomials (8), which can be done for example by pole placements (see Section V for an example).
Next, if we consider that $\Delta b(\xi) \neq 0$ in (4), the global asymptotic stability of the error dynamics will not be guarantied anymore due to the additive error vector $\Delta b(\xi)$, we then choose to use Lyapunov reconstruction technique (e.g., [22]) to obtain a controller ensuring practical stability of the tracking error. This controller is presented in the following Theorem.

Theorem 1: Consider the system (1) for any $x_0 \in \mathbb{R}^n$, under Assumptions 1, 2, 3, 4, 5 and 6, with the feedback controller

$$\begin{array}{c} u = A^{-1}(\xi)(v_s(t,\xi) - b(\xi)) - A^{-1}(\xi)\frac{\partial V}{\partial \tilde{z}}^T k\ d_2(e),\ k > 0,\ \ v_s = (v_{s1}, ..., v_{sm})^T, \\ v_{si}(t,\xi) = y_{i_d}^{(ri)} - K_{ri}^i(y_i^{(ri-1)} - y_{i_d}^{(ri-1)}) - ... - K_1^i(y_i - y_{i_d}). \end{array} \tag{9}$$

Where $K_j^i \in \mathcal{K},\ j = 1, ..., ri,\ i = 1, ..., m$, and $V = z^T P z,\ P > 0$ such that $P\tilde{A} + \tilde{A}^T P = -I$, with $\tilde{A}$ being an $n \times n$ matrix defined as

$$\tilde{A} = \begin{pmatrix} 0, 1, 0, ..., 0 \\ 0, 0, 1, 0, ..., 0 \\ \ddots \\ -K_1^1, ..., -K_{r1}^1, 0, ..., 0 \\ \ddots \\ 0, ..., 0, 1, 0, ..., 0 \\ 0, ..., 0, 0, 1, ..., 0 \\ \ddots \\ 0, ..., 0, -K_1^m, ..., -K_{rm}^m \end{pmatrix}, \tag{10}$$

and $z = (z^1, ..., z^m)^T,\ z^i = (e_i, ..., e_i^{r_i-1}),\ i = 1, ..., m,\ \tilde{z} = (z^1(r_1), ..., z^m(r_m))^T \in \mathbb{R}^m$. Then, the vector z is uniformly bounded and reached the positive invariant set $S = \{z \in \mathbb{R}^n |\ 1 - k\ |\frac{\partial V}{\partial \tilde{z}}| \geq 0\}$.

Proof: We know from the discussion above that for the system (1), under Assumptions 1, 2 and 6, in the nominal case, i.e., $\Delta f = 0$, the control (7) globally asymptotically (exponentially) stabilizes the linear error dynamic (8), which by the classical results in ([21], pp. 135-136), leads to the existence of a Lyapunov function $V = z^T P z$ s.t. the time derivative of V along the nominal system (1) (with $\Delta f = 0$), under the control law u_{nom} given by (7), satisfies

$$\dot{V}|_{((1),\Delta f=0)} \leq -|z|^2,$$

where $z = (z^1, ..., z^m)^T,\ z^i = (e_i, ..., e_i^{r_i-1}),\ i = 1, ..., m$, and $P > 0$ is the unique solution of the Lyapunov equation $P\tilde{A} + \tilde{A}^T P = -I$, wherein $\tilde{A}$ given by (10), has been obtained by rewriting the error dynamic in the control canonical form. Now, we will use the technique of Lyapunov reconstruction from nonlinear robust control, e.g., [22], to obtain the full controller (9). Indeed, if we compute the time derivative of V along the uncertain model (1), under Assumption 3, 5, and considering the augmented control law $u = u_{nom} + u_{robust}$, we obtain

$$\dot{V}|_{((1),\Delta f\neq 0)} \leq -|z|^2 + \frac{\partial V}{\partial \tilde{z}}.(A\ u_{robust} + \Delta b), \tag{11}$$

where $\tilde{z} = (z^1(r_1), ..., z^m(r_m))^T \in \mathbb{R}^m$.
Next, if we define u_{robust} as

$$u_{robust} = -A^{-1}(\xi)\frac{\partial V}{\partial \tilde{z}}^T\ k\ d_2(e),\ k > 0 \tag{12}$$

substituting (12) in (11), we obtain

$$\begin{aligned}\dot{V}|_{((1),\Delta f\neq 0)} &\leq -|z|^2 - |\tfrac{\partial V}{\partial \tilde{z}}|^2\ k\ d_2(e) + \tfrac{\partial V}{\partial \tilde{z}}\Delta b \\ &\leq -|z|^2 - |\tfrac{\partial V}{\partial \tilde{z}}|^2\ k\ d_2(e) + |\tfrac{\partial V}{\partial \tilde{z}}| d_2 \\ &\leq (1 - k\ |\tfrac{\partial V}{\partial \tilde{z}}|)\ |\tfrac{\partial V}{\partial \tilde{z}}|\ d_2\end{aligned} \tag{13}$$

which proves that V is decreasing as long as $1 - k\ |\frac{\partial V}{\partial \tilde{z}}| < 0$, until the error vector enters the positive invariant set $S = \{z \in \mathbb{R}^n |\ 1 - k\ |\frac{\partial V}{\partial \tilde{z}}| \geq 0\}$, which implies boundedness of V, and equivalently uniform boundedness of $|z|$ (which can be directly obtained via the inequality $\lambda_{min}(P)|z|^2 \leq V(z)$, e.g., [25]). ■

Remark 2: In the proof of Theorem 1, we use a smooth control term u_{robust} given by (12) to compensate for the effect of the uncertain term Δb. Indeed, the choice (12) leads to the right-hand side of inequality (13) where we can see that for a big enough values of k, the negative second term in the right-hand side of (13) (second line), can compensate for the third term (which is due to the uncertainty Δb). However, we could use a non-smooth control by choosing $u_{robust} = -A^{-1} sgn(\frac{\partial V}{\partial \tilde{z}})^T k\ d_2(e)$. In this case, the negative second term in the right-hand side of (13) will completely cancel the positive third term, and an asymptotic (Lyapunov) stability results will be achieved. Indeed, this type of discontinuous controller is well known to compensate for bounded uncertainties, e.g., refer to [22] and references therein, and would lead to an asymptotic stability result for the tracking error dynamics. However, it is a discontinuous control, and thus it is not advisable for real applications. Its regularization is often done by substituting the $sign$ function by a saturation function, e.g., [22], which leads to a practical stability result similar to the one obtained with the proposed u_{robust} term in Theorem 1.

Remark 3: Assumption 5 is needed in this work, because we are using an Input-Output linearization approach, complemented with a robust controller. This assumption is common in the literature of robust nonlinear control when Input-Output linearization by feedback is used, e.g., please refer to [26]. This assumption is satisfied under the so-called, matching conditions, which have been explicitly reported in ([26], p. 146).

B. Main result: Iterative auto-tuning of the feedback gains

In Theorem 1, we showed that the passive robust controller (9) leads to bounded tracking errors, which are attracted to the invariant set S for a given choice of the feedback gains $K_j^i,\ j = 1, ..., ri,\ i = 1, ..., m$. Next, to iteratively auto-tune the feedback gains of (9), we define a desired learning cost function, and use a multi-parametric extremum seeking to iteratively auto-tune the gains and minimize the learning cost function. We first denote the learning cost function to be minimized as $Q(\beta)$ where β represents the optimization variables vector, defined as

$$\beta = (\delta K_1^1, ..., \delta K_{r1}^1, ..., \delta K_1^m, ..., \delta K_{rm}^m, \delta k)^T, \tag{14}$$

Such that the updated feedback gains write as

$$\begin{aligned} K_j^i = K_{j-nominal}^i + \delta K_j^i,\ j = 1, ..., ri,\ i = 1, ..., m. \\ k = k_{nominal} + \delta k,\ \ k_{nominal} > 0,\end{aligned} \tag{15}$$

where $K_{j-nominal}^i,\ j = 1, ...ri,\ i = 1, ..., m$ are the nominal initial values of the feedback gains chosen such that Assumption 6 is satisfied.

Remark 4: The choice of the learning cost function Q is not unique. For instance, if the controller tracking performance at the specific time instants $It_f,\ I\in\{1,2,3,...\}$ is important for the targeted application (see the example presented in Section V), one can choose Q as

$$Q(\beta)=z^T(It_f)C_1z(It_f),\ \ C_1>0. \tag{16}$$

If another performance needs to be optimized over a finite time interval, for instance a combination of a tracking performance and a control power performance, then one can choose for example the cost function

$$Q(\beta)=\int_{(I-1)t_f}^{It_f}z^T(t)C_1z(t)dt+\int_{(I-1)t_f}^{It_f}u^T(t)C_2u(t)dt,\ I\in\{1,2,3,...\},\ C_1,\ C_2>0. \tag{17}$$

The gains' variation vector β is then used to minimize the cost function Q over the iterations $I\in\{1,2,3,\dots\}$. Other closed-loop performances, like over-shooting and settling time minimization, could be added in the learning cost function, as well. Following extremum seeking theory, e.g., [27], [28], the variations of the gains are defined as

$$\begin{array}{l}\dot{x}_{K_j^i}=a_{K_j^i}\delta_{K_j^i}\omega_{K_j^i}\sin(\omega_{K_j^i}t-\frac{\pi}{2})Q(\beta),\\ \delta\hat{K}_j^i(t)=x_{K_j^i}(t)+a_{K_j^i}sin(\omega_{K_j^i}t+\frac{\pi}{2}),\ j=1,...ri,\ i=1,...,m,\\ \dot{x}_k=a_k\delta_k\omega_ksin(\omega_kt-\frac{\pi}{2})Q(\beta),\\ \delta\hat{k}(t)=x_k(t)+a_ksin(\omega_kt+\frac{\pi}{2}),\end{array} \tag{18}$$

where $a_{K_j^i},\ \delta_{K_j^i},\ j=1,...ri,\ i=1,...,m,\ a_k,\ \delta_k$ are positive tuning parameters, and

$$\omega_1+\omega_2\neq\omega_3,\ \text{for}\ \omega_1\neq\omega_2\neq\omega_3,\ \forall\omega_1,\omega_2,\omega_3\in\{\omega_{K_j^i},\omega_k,\ j=1,...ri,\ i=1,...,m\}, \tag{19}$$

with $\omega_i>\omega^*,\ \forall\omega_i\in\{\omega_{K_j^i},\omega_k,\ j=1,...ri,\ i=1,...,m\}$, ω^* large enough.
To study the stability of the learning-based controller, i.e., controller (9), with the varying gains (15) and (18), we first need to introduce some additional assumptions.

Assumption 7: We assume that the learning cost function Q has a local minimum at β^*.

Assumption 8: We consider that the initial gain vector β is sufficiently close to the optimal gain vector β^*.

Assumption 9: The learning cost function is analytic and its variation with respect to the gains is bounded in the neighborhood of β^*, i.e., $|\frac{\partial Q}{\partial\beta}(\tilde{\beta})|\leq\Theta_2,\ \Theta_2>0,\ \tilde{\beta}\in\mathcal{V}(\beta^*)$, where $\mathcal{V}(\beta^*)$ denotes a compact neighborhood of β^*.
We can now state the following result.

Theorem 2: Consider the system (1) for any $x_0\in\mathbb{R}^n$, under Assumptions 1, 2, 3, 4, 5 and 6, with the feedback controller

$$\begin{array}{l}u=A^{-1}(\xi)(v_s(t,\xi)-b(\xi))-A^{-1}(\xi)\frac{\partial V}{\partial\tilde{z}}^Tk(t)\ d_2(e),\ k>0,\ \ v_s=(v_{s1},...,v_{sm})^T,\\ v_{si}(t,\xi)=\hat{y}_{id}^{(ri)}-K_{ri}^i(t)(y_i{}^{(ri-1)}-\hat{y}_{id}^{(ri-1)})-...-K_1^i(t)(y_i-\hat{y}_{id}),\ i=1,...,m.\end{array} \tag{20}$$

Where the state vector is reset following the resetting law $x(It_f)=x_0,\ I\in\{1,2,...\}$, the desired trajectory vector is reset following $\hat{y}_{id}(t)=y_{id}(t-(I-1)t_f),\ (I-1)t_f\leq t<It_f,\ I\in\{1,2,...\}$, and $K_j^i(t)\in\mathcal{K},\ j=1,...,ri,\ i=1,...,m$ are piecewise continuous gains switched at each iteration I, $I\in\{1,2,...\}$, following the update law

$$\begin{array}{l}K_j^i(t)=K_{j-nominal}^i+\delta K_j^i(t),\\ \delta K_j^i(t)=\delta\hat{K}_j^i((I-1)t_f),\ (I-1)t_f\leq t<It_f,\\ k(t)=k_{nominal}+\delta k(t),\ \ k_{nominal}>0\\ \delta k(t)=\delta\hat{k}((I-1)t_f),\ (I-1)t_f\leq t<It_f,\ I=1,2,3...\end{array} \tag{21}$$

where $\delta\hat{K}_j^i,\delta\hat{k}$ are given by (18), (19) and whereas the rest of the coefficients are defined similarly to Theorem 1. Then, the obtained closed-loop impulsive time-dependent dynamic system (1), (18), (19), (20) and (21), is well posed. The tracking error z is uniformly bounded, and is steered at each iteration I towards the positive invariant set $S_I=\{z\in\mathbb{R}^n|\ 1-k_I\ |\frac{\partial V}{\partial\tilde{z}}|\geq0\}$, $k_I=\beta_I(n+1)$, where β_I is the value of β at the Ith iteration. Furthermore, $|Q(\beta(It_f))-Q(\beta^*)|\leq\Theta_2(\frac{\Theta_1(\delta)}{\omega_0}+\sqrt{\sum_{i=1,...,m,\ j=1,...,ri}a_{K_j^i}{}^2+a_k{}^2}),\ \Theta_1,\ \Theta_2>0,$ for $I\to\infty$, where $\omega_0=Max(\omega_{K_1^1},...,\omega_{K_{rm}^m},\omega_k)$, $\delta=(\delta_{K_1^1},...,\delta_{K_{rm}^m},\delta_k)^T$, and Q satisfies Assumptions 7, 8 and 9. Wherein, the vector β remains bounded over the iterations s.t. $|\beta((I+1)t_f)-\beta(It_f)|\leq0.5t_fMax(\delta_{K_1^1}a_{K_1^1}{}^2,...,\delta_{K_{rm}^m}a_{K_{rm}^m}{}^2,\delta_ka_k^2)\Theta_2+t_f\omega_0\sqrt{\sum_{i=1,...,m\ j=1,...,ri}a_{K_j^i}{}^2+a_k{}^2},\ I\in\{1,2,...\}$, and satisfies asymptotically the bound $|\beta(It_f)-\beta^*|\leq\frac{\Theta_1(\delta)}{\omega_0}+\sqrt{\sum_{i=1,...,m\ j=1,...,ri}a_{K_j^i}{}^2+a_k{}^2},\ \Theta_1>0,$ for $I\to\infty$.

Proof: First, we discuss the well-posedness of the obtained closed-loop impulsive dynamical system. Indeed, the closed-loop system (1), (18), (19), (20) and (21), can be viewed as an impulsive time-dependent dynamical system ([23], pp. 18-19), with the trivial resetting law $\Delta x(t)=x_0,$ for $t=It_f,\ I\in\{1,2,...\}$. In this case the resetting times given by $It_f,\ t_f>0\ I\in\{1,2,...\}$, are well defined and distinct. Furthermore, due to Assumption 1 and the smoothness of (20) (within each iteration), this impulsive dynamic system admits a unique solution in forward time, for any initial condition $x_0\in\mathbb{R}^n$

([23], p. 12). Finally, the fact that $t_f \neq 0$ excludes a Zeno behavior over a finite time interval (only a finite number of resets are possible over a finite time interval). Next, let us consider the system (1) with the initial condition x_0 (or equivalently the initial tracking error $z_0 = h(x_0) - y_d(0)$), under Assumptions 1, 2, 3, 4, 5 and 6, with the feedback controller (20), (21), for a given time-interval $(I^{'} - 1)t_f \leq t < I^{'} t_f$, for any given $I^{'} \in \{1, 2, ...\}$. Based on Theorem 1, there exists a Lyapunov function $V_{I'} = z^T P_{I'} z$, such that $\dot{V}_{I'} \leq (1 - k_{I'} \, |\frac{\partial V_{I'}}{\partial \tilde{z}}|) \, |\frac{\partial V_{I'}}{\partial \tilde{z}}| \, d_2$, where $P_{I'}$ is solution of the Lyapunov equation $P_{I'} \tilde{A}_{I'} + \tilde{A}^T_{I'} P_{I'} = -I$, wherein $\tilde{A}$ given by (10), with the gains for the iteration $I^{'}$

$$\begin{array}{l} K^i_{I'\,j}(t) = K^i_{j-nominal} + \delta K^i_j(t), \\ \delta K^i_j(t) = \delta \hat{K}^i_j((I^{'} - 1)t_f), \; (I^{'} - 1)t_f \leq t < I^{'} t_f, \\ k_{I'}(t) = k_{nominal} + \delta k(t), \;\; k_{nominal} > 0, \\ \delta k(t) = \delta \hat{k}((I^{'} - 1)t_f), \; (I^{'} - 1)t_f \leq t < I^{'} t_f, \; I^{'} = 1, 2, 3... \end{array} \tag{22}$$

which shows that z, starting from z_0 (for all the iterations $I^{'} \in \{1, 2, ...\}$) is steered $\forall t \in [(I^{'} - 1)t_f, \; I^{'} t_f[$, towards the invariant set $S_{I'} = \{z \in \mathbb{R}^n | \; 1 - k_{I'} \; |\frac{\partial V_{I'}}{\partial \tilde{z}}| \geq 0\}$, and this is valid for all $I^{'} \in \{1, 2, 3...\}$. Furthermore, since at each switching point, i.e., each new iteration $I^{'}$, we reset the system from the same bounded initial condition z_0, we can conclude about the uniform boundedness of the tracking error z. Next, we use to the results presented in [29], that characterize the learning cost function Q behavior along the iterations. First, based on Assumptions 7, 8 and 9, the extremum seeking (ES) nonlinear dynamics (18), (19), can be approximated by a linear averaged dynamic (using averaging approximation over time ([29], p 435, Definition 1)). Furthermore, $\exists \Theta_1, \; \omega^*$, such that for all $\omega_0 = Max(\omega_{K^1_1}, ..., \omega_{K^m_{rm}}, \omega_k) > \omega^*$, the solution of the averaged model $\beta_{aver}(t)$ is locally close to the solution of the original ES dynamics, and satisfies ([29], p. 436)

$$|\beta(t) - d(t) - \beta_{aver}(t)| \leq \tfrac{\Theta_1(\delta)}{\omega_0}, \; \Theta_1 > 0, \; \forall t \geq 0,$$

with $d_{vec}(t) = (a_{K^1_1} sin(\omega_{K^1_1} t - \frac{\pi}{2}), ..., a_{K^m_{rm}} sin(\omega_{K^m_{rm}} t - \frac{\pi}{2}), a_k sin(\omega_k t - \frac{\pi}{2}))^T$, and $\delta = (\delta_{K^1_1}, ..., \delta_{K^m_{rm}}, \delta_k)^T$. Moreover, since Q is analytic it can be approximated locally in $\mathcal{V}(\beta^*)$ with a quadratic function, e.g., Taylor series up to second order. This together with the proper choice of the dither signals as in (18), and the dither frequencies satisfying (19), allows to prove that β_{aver} satisfies ([29], p. 437)

$$lim_{t \to \infty} \beta_{aver}(t) = \beta^*,$$

which together with the previous inequality leads to

$$\begin{array}{c} |\beta(t) - \beta^*| - |d(t)| \leq |\beta(t) - \beta^* - d(t)| \leq \frac{\Theta_1(\delta)}{\omega_0}, \; \Theta_1 > 0, \; t \to \infty, \\ \Rightarrow |\beta(t) - \beta^*| \leq \frac{\Theta_1(\delta)}{\omega_0} + |d(t)|, \; t \to \infty. \end{array}$$

This finally implies that

$$\begin{array}{c} |\beta(t) - \beta^*| \leq \frac{\Theta_1(\delta)}{\omega_0} + \sqrt{\sum\limits_{i=1,...,m, \; j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}, \; \Theta_1 > 0, \; t \to \infty, \\ \Rightarrow |\beta(It_f) - \beta^*| \leq \frac{\Theta_1(\delta)}{\omega_0} + \sqrt{\sum\limits_{i=1,...,m, \; j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}, \; \Theta_1 > 0, \; I \to \infty. \end{array}$$

Next, based on Assumption 9, the cost function is locally Lipschitz, with the Lipschitz constant $max_{\beta \in \mathcal{V}(\beta^*)} |\frac{\partial Q}{\partial \beta}| = \Theta_2$, i.e., $|Q(\beta_1) - Q(\beta_2)| \leq \Theta_2 |\beta_1 - \beta_2|, \; \forall \beta_1, \beta_2 \in \mathcal{V}(\beta^*)$, which together with the previous inequality leads to

$$|Q(\beta(It_f)) - Q(\beta^*)| \leq \Theta_2 (\frac{\Theta_1(\delta)}{\omega_0} + \sqrt{\sum_{i=1,...,m, \; j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}), \; \Theta_1, \; \Theta_2 > 0, \; I \to \infty.$$

Finally, we show that the ES algorithm (18), (19) is a gradient-based algorithm, as follows: from (18), and if we denote $X = (x_{K^1_1}, ..., x_{K^m_{rm}}, x_k)^T$, we can write

$$\dot{X} = (a_{K^1_1} \delta_{K^1_1} \omega_{K^1_1} sin(\omega_{K^1_1} t - \frac{\pi}{2}), ..., a_{K^m_{rm}} \delta_{K^m_{rm}} \omega_{K^m_{rm}} sin(\omega_{K^m_{rm}} t - \frac{\pi}{2}), a_k \delta_k \omega_k sin(\omega_k t - \frac{\pi}{2}))^T Q(\beta). \tag{23}$$

Based on Assumption 9, the cost function can be locally approximated with its first order Taylor development in $\mathcal{V}(\beta^*)$, which leads to

$$\dot{X} \simeq \tilde{d}_{vec} \big(Q(\tilde{\beta}) + \overline{d}^T_{vec} \frac{\partial Q}{\partial \beta}(\tilde{\beta}) \big), \; \tilde{\beta} \in \mathcal{V}(\beta^*), \tag{24}$$

where $\tilde{d}_{vec} = (a_{K^1_1} \delta_{K^1_1} \omega_{K^1_1} sin(\omega_{K^1_1} t - \frac{\pi}{2}), ..., a_{K^m_{rm}} \delta_{K^m_{rm}} \omega_{K^m_{rm}} sin(\omega_{K^m_{rm}} t - \frac{\pi}{2}), a_k \delta_k \omega_k sin(\omega_k t - \frac{\pi}{2}))^T$, and $\overline{d}_{vec} = (a_{K^1_1} sin(\omega_{K^1_1} t + \frac{\pi}{2}), ..., a_{K^m_{rm}} sin(\omega_{K^m_{rm}} t + \frac{\pi}{2}), a_k sin(\omega_k t + \frac{\pi}{2}))^T$.

Next, by integrating (24), over $[t, \; t + t_f]$ and neglecting the terms inversely proportional to the high frequencies, i.e., terms on $\frac{1}{\omega_i}$'s (high frequencies filtered by the integral operator), we obtain

$$X(t + t_f) - X(t) \simeq -t_f R \frac{\partial Q}{\partial \beta}(\tilde{\beta}), \tag{25}$$

with $R = 0.5\ diag\{\delta_{K_1^1}\omega_{K_1^1}{a_{K_1^1}}^2, ..., \delta_{K_{rm}^m}\omega_{K_{rm}^m}{a_{K_{rm}^m}}^2, \delta_k\omega_k a_k^2\}$.
Next, from (14) and (18), we can write $|\beta(t+t_f) - \beta(t)| \leq |X(t+t_f) - X(t)| + |\overline{d}_{vec}(t+t_f) - \overline{d}_{vec}(t)|$, which together with (25), with the bound $|\frac{\overline{d}_{vec}(t+t_f)-\overline{d}_{vec}(t)}{t_f}| \leq |\dot{\overline{d}}_{vec}| \leq \omega_0\sqrt{\sum_{i=1,...,m\ j=1,...,ri} {a_{K_j^i}}^2 + {a_k}^2}$, and Assumption 9, leads to the inequality

$$|\beta((I+1)t_f) - \beta(It_f)| \leq 0.5 t_f Max(\delta_{K_1^1}\omega_{K_1^1}{a_{K_1^1}}^2, ..., \delta_{K_{rm}^m}\omega_{K_{rm}^m}{a_{K_{rm}^m}}^2, \delta_k\omega_k a_k^2)\Theta_2$$
$$+ t_f\omega_0\sqrt{\sum_{i=1,...,m\ j=1,...,ri} {a_{K_j^i}}^2 + {a_k}^2},\ I \in \{1, 2, ...\}.$$

■

Remark 5: - The gains' asymptotic convergence-bounds presented in Theorem 2, are correlated to the choice of the first order multi-parametric extremum seeking (18), however, these bounds can be easily changed by using other MES algorithms, e.g., [30], [31]. This is due to the modular design of the controller (20), (21), which uses the passive robust part to ensure boundedness of the tracking error dynamics, and the learning part to optimize the learning cost function Q.
- In Theorem 2, we show that in each iteration I, the tracking error vector z is directed toward the invariant set S_I. However, due to the finite time-interval length t_f of each iteration, we cannot guarantee that the vector z enters S_I in each iteration (unless we are in the trivial case where $z_0 \in S_I$). All what we guarantee is that the vector norm $|z|$ starts from a bounded value $|z_0|$ and remains bounded during the iterations with an upper-bound which can be estimated as function of $|z_0|$ by using the bounds of the quadratic Lyapunov functions $V_I,\ I = 1, 2, ...$, i.e., a uniform boundedness result ([23], p. 6, Definition 2.12).

Remark 6: In Assumption 6, we assume the existence of a set $\mathcal{K}$ of gains, such that the tracking errors (8) are stable. However, nothing is said in algorithm (18), (19), and (21), about the search excursion boundaries. Indeed, in algorithm (18), (19), if we keep the search amplitudes $a_{K_j^i},\ a_k$ small, we can expect the gains search to be around the initial point. However, to force the gains to remain within a desired set, we need to modify the extremum seeking to include input saturations. Few options have been recently reported in the ES community literature. For instance, we can bound the gains search within some desired min-max bounds by using the modified ES algorithm presented in [32], where the time derivations of the internal ES algorithm variables $x_{K_j^i},\ x_k$, i.e., the first and third equations in (18), are modified with some properly defined saturated functions of the learning cost (for the detailed equations, please refer to ([32], p. 1709)). Another option would be to use the modified dither-based ES algorithms presented in [33], where two approaches have been proposed to constraint the optimal variables search within desired upper-lower limits. The first approach is based on a penalty formulation, where the learning cost function in (18) is augmented with penalty terms to take into account the variables constraints. The second approach is based on adding an anti-windup term to the original dither-based ES algorithm.

Remark 7: Dither-based ES algorithms of the form (18), have been well studied and their main characteristics in terms of domain of attraction, speed of convergence, and accuracy, have been correlated to the choice of their search amplitudes a_is, their search frequency ω_is, and their integrators amplitudes δ_is. For example in [28], [34], it is shown that the convergence speed of a sinusoidal dither-based ES algorithm of the form (18) is proportional to the constant $0.5\delta\omega a^2$. This gives a clear indication of the effect of δ, ω, and a on the ES algorithm convergence speed. The precision of the convergence to the optimum is function of δ and a, and the convergence error tends to zero if $a,\ \delta$ tend to zero. Moreover, the domain of attraction of the ES algorithm can be enlarged arbitrarily (semi-global convergence), by reducing a, δ, and ω [34]. However, that might lead to a slowdown in convergence to the optimum, which is a well know characteristic of learning algorithms, where there is often a tradeoff between exploitation and exploration.

Remark 8: We want to compare here our approach with the available IFT algorithms for nonlinear systems. First of all, the available IFT algorithms for nonlinear systems, assume the existence of a controller that makes the closed-loop (Lagrange) stable. Whereas, we force the closed-loop to be always Lagrange stable, with a proper design of the feedback control, and its interaction with the gains tuning algorithm. Besides this important stability argument, the main difference between our algorithm and the available IFT methods, is in the number of experiments needed in each learning iteration. Indeed, the available nonlinear IFT algorithms in [13], [14], [15], [16] require $n+2$ experiments for each learning iteration, to estimate the gradient of the learning cost function with respect to an n-dimensional vector of feedback gains to be tuned. An alternative is proposed in [17], where only 2 experiments are needed per learning iteration, to approximate the gradient of the cost function. However, in this case the algorithm requires further to solve a local identification by least square optimization at each learning iteration. Moreover, the algorithm is based on a local approximation of the nonlinear dynamics with time-varying linear dynamics, and thus has no proven robustness to changes in the reference, e.g., when starting from a non-zero initial condition far from the reference. This available IFT algorithms are different from our algorithm in two ways. First, our algorithm only requires one experiment for each learning iteration. Second, in these available IFT algorithms, the system needs to be run several times per each learning iteration, with several different reference trajectories for the closed-loop system to track. This means that the system cannot be operational during the tuning, since it has to be probed with different reference signals (aside from its useful reference trajectory). On the contrary, with our algorithm, the system can be operational all the time, since we do not require to probe the system with different references. Instead, we just let the system run with its (useful) reference, to realize its task, and we use its output to auto-tune the feedback gains in realtime. This characteristic is important since it allows our

IFT algorithm to continue fine-tuning the gains while the system is operational. This can be useful in case of system aging, where some part of the system's dynamics are slowly changing, and the gains are tracking that change, or in the case of a more abrupt fault on the system, where the gains have to be re-tuned in realtime to preserve some performance for the faulty system.
In the next section, we propose to illustrate this approach on a mechatronics system.

V. The case of electromagnetic actuators

We apply here the method presented above to the case of an electromagnetic actuator. This system requires accurate control of the moving armature between two desired positions. The main objective, known as 'soft landing' of the moving armature, is to ensure small contact velocity between the moving armature and the fixed parts of the actuator. This motion is usually of iterative nature, since the actuator has to iteratively open and close to achieve a desired cyclic motion of a mechanical part, attached to the actuator, e.g., an engine-valve system in automotive applications.

A. System modelling

Following [35], [36], we consider the following nonlinear model for electromagnetic actuators

$$\begin{array}{l} m\frac{d^2x_a}{dt^2} = k_s(x_0 - x_a) - \eta\frac{dx_a}{dt} - \frac{ai^2}{2(b+x_a)^2}, \\ u = Ri + \frac{a}{b+x_a}\frac{di}{dt} - \frac{ai}{(b+x_a)^2}\frac{dx_a}{dt},\ 0 \leq x_a \leq x_f, \end{array} \tag{26}$$

where x_a represents the armature position, physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx_a}{dt}$ represents the armature velocity, m is the armature mass, k_s the spring constant, x_0 the initial spring length, η the damping coefficient (assumed to be constant), $\frac{ai^2}{2(b+x_a)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b are two constant parameters of the coil, R the resistance of the coil, $L = \frac{a}{b+x_a}$ the coil inductance, $\frac{ai}{(b+x_a)^2}\frac{dx_a}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. In this model we do not consider the saturation region of the flux linkage in the magnetic field generated by the coil, since we assume a current and armature motion ranges within the linear region of the flux.

B. Passive robust controller

In this section, we first design a nonlinear passive robust control based on Theorem 1. Following Assumption 4, we define a desired armature position trajectory x_{ref}, s.t. x_{ref} is smooth (at least C^2) function satisfying the initial/final constraints: $x_{ref}(0) = 0,\ x_{ref}(t_f) = x_f,\ \dot{x}_{ref}(0) = 0,\ \dot{x}_{ref}(t_f) = 0$, where t_f is a desired finite motion time, and x_f is a desired final position.
We consider the dynamical system (26), with bounded parametric uncertainties on the spring coefficient δk_s, with $|\delta k_s| \leq \delta k_{s_{max}}$, and the damping coefficient $\delta\eta$, with $|\delta\eta| \leq \delta\eta_{max}$, such that $k_s = k_{s_{nominal}} + \delta k_s$, $\eta = \eta_{nominal} + \delta\eta$, where $k_{s_{nominal}}$, $\eta_{nominal}$ are the nominal values of the spring stiffness and the damping coefficient, respectively. If we consider the state vector $x = (x_a,\ \dot{x}_a,\ i)^T$, and the controlled output x_a, the uncertain model of the electromagnetic actuator can be written in the form of (1), as

$$\begin{array}{l} \dot{x} = \begin{pmatrix} \dot{x}_a \\ \ddot{x}_a \\ \dot{i} \end{pmatrix} = \begin{pmatrix} x_2 \\ \frac{k_{s_{nominal}}}{m}(x_0 - x_1) - \frac{\eta_{nominal}}{m}x_2 - \frac{ax_3^2}{2(b+x_1)^2} \\ -\frac{R(b+x_1)}{a}x_3 + \frac{x_3x_2}{b+x_1} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\delta k_s}{m}(x_0 - x_1) + \frac{\delta\eta}{m}x_2 \\ 0 \end{pmatrix} + \\ \qquad \begin{pmatrix} 0 \\ 0 \\ \frac{b+x_1}{a} \end{pmatrix} u, \\ y = x_1. \end{array} \tag{27}$$

First, Assumption 1 is clearly satisfied over a nonempty bounded states set X. As for Assumption 2, it is straightforward to check that if we compute the third time-derivative of the output x_a, the control variable u appears in a nonsingular expression, which implies that $r = n = 3$. Assumption 3 is also satisfied since $|\Delta f(x)| \leq \frac{\delta k_{s_{max}}}{m}|x_0 - x_1| + \frac{\delta\eta_{max}}{m}|x_2|$.
Next, following the Input-Output linearization method, we can write

$$y^{(3)} = x_a^{(3)} = -\frac{k_{s_{nominal}}}{m}\dot{x}_a - \frac{\eta_{nominal}}{m}\ddot{x}_a + \frac{Ri^2}{(b+x_a)m} - \frac{\delta k_s}{m}\dot{x}_a - \frac{\delta\eta}{m}\ddot{x}_a - \frac{i}{m(b+x_a)}u, \tag{28}$$

which is of the form of equation (4), with $A = -\frac{i}{m(b+x_a)}$, $b = -\frac{k_{s_{nominal}}}{m}\dot{x}_a - \frac{\eta_{nominal}}{m}\ddot{x}_a + \frac{Ri^2}{(b+x_a)m}$, and the additive uncertainty term $\Delta b = -\frac{\delta k_s}{m}\dot{x}_a - \frac{\delta\eta}{m}\ddot{x}_a$, such that $|\Delta b| \leq \frac{\delta k_{s_{max}}}{m}|\dot{x}_a| + \frac{\delta\eta_{max}}{m}|\ddot{x}_a| = d_2(x_a, \dot{x}_a)$. Let us define the tracking error vector $\mathbf{z} := (z_1,\ z_2,\ z_3)^T = (x_a - x_{ref},\ \dot{x}_a - \dot{x}_{ref},\ \ddot{x}_a - \ddot{x}_{ref})^T$, where $\dot{x}_{ref} = \frac{dx_{ref}(t)}{dt}$, and $\ddot{x}_{ref} = \frac{d^2x_{ref}(t)}{dt^2}$. Next, using Theorem 1, we can write the following robust passive controller

$$\begin{array}{c} u = -\frac{m(b+x_a)}{i}(v_s + \frac{k_{s_{nominal}}}{m}\dot{x}_a + \frac{\eta_{nominal}}{m}\ddot{x}_a - \frac{Ri^2}{(b+x_a)m}) + \frac{m(b+x_a)}{i}\frac{\partial V}{\partial z_3}k_s(\frac{\delta k_{s_{max}}}{m}|\dot{x}_a| + \frac{\delta\eta_{max}}{m}|\ddot{x}_a|),\\ v_s = x^{(3)}_{ref}(t) + K_3(x_a^{(2)} - x^{(2)}_{ref}(t)) + K_2(x_a^{(1)} - x^{(1)}_{ref}(t)) + K_1(x_a - x_{ref}(t)),\\ k > 0,\ K_i < 0, i = 1, 2, 3. \end{array} \tag{29}$$

Where $V = z^T P z,\ P > 0$ solution of the equation $P\tilde{\tilde{A}} + \tilde{\tilde{A}}^T P = -I$, with

$$\tilde{\tilde{A}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ K_1 & K_2 & K_3 \end{pmatrix}, \tag{30}$$

where $K_1,\ K_2,\ K_3$ are chosen such that $\tilde{\tilde{A}}$ is Hurwitz.

Remark 9: With regard to Assumption 6, about the existence of a non-empty set of gains $\mathcal{K}$, such that $\tilde{\tilde{A}}$ is Hurwitz, we can easily characterize $\mathcal{K}$ is this case. Indeed, if we want to place the eigenvalues of $\tilde{\tilde{A}}$ at the values $s_1,\ s_2,\ s_3$, such that $s_{1_{min}} \leq s_1 \leq s_{1_{max}},\ s_{1_{min}} < s_{1_{max}} < 0,\ s_{2_{min}} \leq s_2 \leq s_{2_{max}},\ s_{2_{min}} < s_{2_{max}} < 0$, and $s_{3_{min}} \leq s_3 \leq s_{3_{max}},\ s_{3_{min}} < s_{3_{max}} < 0$, by direct match of the coefficients of the characteristic polynomial: $s^3 - K_3 s^2 - K_2 s - K_1 = 0$, with the desired characteristic polynomial: $\prod_{i=1}^{i=3}(s - s_i) = 0$, we can write

$$\begin{array}{l} K_1 = \prod_{i=1}^{i=3} s_i,\\ K_2 = -\sum_{i,j\in\{1,2,3\}, i\neq j} s_i s_j,\\ K_3 = \sum_{i=1}^{i=3} s_i, \end{array} \tag{31}$$

which allows as to write the set $\mathcal{K}$ as

$$\begin{array}{c} \mathcal{K} = \{(K_1, K_2, K_3) | s_{i_{min}} s_{j_{min}} s_{r_{max}} \leq K_1 \leq s_{i_{max}} s_{j_{max}} s_{r_{min}},\ i \neq j \neq k, i, j, r \in \{1, 2, 3\}\\ -\sum_{i,j\in\{1,2,3\}, i\neq j} s_{i_{min}} s_{j_{min}} \leq K_2 \leq -\sum_{i,j\in\{1,2,3\}, i\neq j} s_{i_{max}} s_{j_{max}},\\ \sum_{i=1}^{i=3} s_{i_{min}} \leq K_3 \leq \sum_{i=1}^{i=3} s_{i_{max}}\}. \end{array}$$

C. Learning-based auto-tuning of the controller gains

We use now the results of Theorem 2, to iteratively auto-tune the feedback gains of the controller (29). Considering a cyclic behavior of the actuator, with each iteration happening over a time interval of length t_f, we define the following learning cost function

$$Q(\beta) = C_1 z_1(It_f)^2 + C_2 z_2(It_f)^2 + C_3 \int_{(I-1)t_f}^{It_f} u^T(t)u(t)dt, \tag{32}$$

where $I = 1, 2, 3...$ is the number of iterations, $C_1,\ C_2,\ C_3 > 0$, and $\beta = (\delta K_1,\ \delta K_2,\ \delta K_3,\ \delta k)^T$, such as the feedback gains write as

$$\begin{array}{l} K_1 = K_{1_{nominal}} + \delta K_1,\\ K_2 = K_{2_{nominal}} + \delta K_2,\\ K_3 = K_{3_{nominal}} + \delta K_3,\\ k = k_{nominal} + \delta k, \end{array} \tag{33}$$

where $K_{1_{nominal}},\ K_{2_{nominal}},\ K_{3_{nominal}},\ k_{nominal}$, are the nominal initial values of the feedback gains in (29).

The learning cost function (32), has been chosen based on the desired overall performance of the controlled system. Indeed, in this case the main control objective is 'soft-landing', which explains the two first terms in (32). As for the third term in the learning cost function, it has been added to limit the required closed-loop control voltage.

Following (18), (19), and (21) the variations of the estimated gains are given by

$$\begin{array}{l} \dot{x}_{K_1} = a_{K_1}\delta_1\omega_1 sin(\omega_1 t - \frac{\pi}{2})Q(\beta),\\ \delta\hat{K}_1(t) = x_{K_1}(t) + a_{K_1} sin(\omega_1 t + \frac{\pi}{2}),\\ \dot{x}_{K_2} = a_{K_2}\delta_2\omega_2 sin(\omega_2 t - \frac{\pi}{2})Q(\beta),\\ \delta\hat{K}_2(t) = x_{K_2}(t) + a_{K_2} sin(\omega_2 t + \frac{\pi}{2}),\\ \dot{x}_{K_3} = a_{K_3}\delta_3\omega_3 sin(\omega_3 t - \frac{\pi}{2})Q(\beta),\\ \delta\hat{K}_3(t) = x_{K_3}(t) + a_{K_3} sin(\omega_3 t + \frac{\pi}{2}),\\ \dot{x}_k = a_k\delta_4\omega_4 sin(\omega_4 t - \frac{\pi}{2})Q(\beta),\\ \delta\hat{k}(t) = x_k(t) + a_k sin(\omega_4 t + \frac{\pi}{2}),\\ \delta K_j(t) = \delta\hat{K}_j((I-1)t_f),\ (I-1)t_f \leq t < It_f,\ j \in \{1, 2, 3\},\ I = 1, 2, 3...\\ \delta k(t) = \delta\hat{k}((I-1)t_f),\ (I-1)t_f \leq t < It_f,\ I = 1, 2, 3... \end{array} \tag{34}$$

where $a_{K_1},\ a_{K_2},\ a_{K_3},\ a_k,\ \delta_i\ i \in \{1, 2, 3, 4\}$ are positive, and $\omega_p + \omega_q \neq \omega_r,\ p, q, r \in \{1, 2, 3, 4\}$, for $p \neq q \neq r$.

Parameter	Value
m	0.27 $[kg]$
R	6 $[\Omega]$
η	7.53 $[kg/sec]$
x_0	8 $[mm]$
k	158 $[N/mm]$
a	14.96×10^{-6} $[Nm^2/A^2]$
b	4×10^{-5} $[m]$

TABLE I: Numerical values of the mechanical parameters

D. Simulation results

We show here the behavior of the proposed approach on the electromagnetic actuator example presented in [37], where the model (26) is used with the numerical values of Table I. The desired trajectory has been selected as the $5th$ order polynomial $x_{ref}(t) = \sum_{i=0}^{5} a_i (t/t_f)^i$, where the a_i's have been computed to satisfy the boundary constraints $x_{ref}(0) = 0, x_{ref}(t_f) = x_f, \dot{x}_{ref}(0) = \dot{x}_{ref}(t_f) = 0, \ddot{x}_{ref}(0) = \ddot{x}_{ref}(t_f) = 0$, with $t_f = 1\ sec$, $x_f = 0.5\ mm$.
Furthermore, to make the simulation case more challenging we assume an initial error both on the position and the velocity $z_1(0) = 0.01\ mm$, $z_2(0) = 0.01\ mm/sec$. Note that these values may seem small, but for this type of actuators it is usually the case that the armature starts form a predefined static position constrained mechanically, so we know that the initial velocity is zero and we know in advance very precisely the initial position of the armature. However, we want to show the performances of the controller on some challenging cases. We also added a saturation block on the closed-loop control voltage, with an upper limit of 40 volts. We performed several tests, with different initial conditions and initial values for the nominal gains. They all showed a performance improvement after few iterations, i.e., learning cost function decrease after few iterations. We cannot report here all the cases, however, we chose to report two main cases. One in nominal noise-free conditions, where we assume perfect measurements of the armature position and velocity. The second test considers noisy measurements conditions. We report below the numerical results obtained in both cases.

- *Test one (noise-free)*: In this test, we select the nominal feedback gains as: $K_1 = -500,\ K_2 = -125,\ K_3 = -26,\ k = 1$, which satisfy Assumption 5.

We compare the performance of the passive robust controller (29) with the fixed nominal gains, to the performance of the learning controller (29),(33), (34), which is implemented with the learning cost function (32), where $C_1 = 500,\ C_2 = 500,\ C_3 = 10^{-4}$. As with any optimal control method, there is no ready-to-use recipe to choose the cost function weights C_i, but the system and the targeted performance can direct the weights' choice. For instance, in our case we give equal weights to the first and second terms, which are the main 'soft-landing' performance terms. The last control-effort minimization term, has smaller weight for two main reasons; the first one is that, it is not the main goal of the feedback control, and the second is that the third term is an integral of the control power over the motion time interval $[t_0,\ t_f]$, which is intrinsically higher than the two first terms, i.e., the position and velocity errors at a single impact time. For these reasons, we choose smaller weight for the third term to balance the learning cost function. The learning frequencies for each feedback gain are chosen as $\omega_1 = 7.5\ rad/sec$, $\omega_2 = 5.3\ rad/sec$, $\omega_3 = 5.1\ rad/sec$, $\omega_4 = 6.1\ rad/sec$. This choice is motivated by two things: First the frequencies have to satisfy condition (19). Second, all the frequencies need to be high enough to ensure convergence (based on averaging arguments), e.g., [27], and low enough comparatively to closed-loop system dynamics, to ensure a time-scale septation, i.e., [28], [34]. We point out here that to accelerate the learning convergence rate, which is related to the choice of the coefficients δ_is , e.g., [28], we have chosen the following values $\delta_1 = 2$, $\delta_2 = 2.8$, $\delta_3 = 2.9$, and $\delta_4 = 2.4$. Finally, the dither signals' amplitudes were selected as $a_{K_1} = 80$, $a_{K_2} = 160$, $a_{K_3} = 6$, and $a_k = 0.1$. The choice of the amplitudes a_is changes the speed and precision of convergence (see Remark 7). Indeed, small values for the amplitudes lead to a more precise estimation of the optimum (which is also clear from the estimation bounds proposed in Theorem 2), but slow down the convergence, as discussed in Remark 7. We choose the amplitude a_k to be smaller than the other amplitudes because its corresponds to the robustness gain k, which we want to keep small, since it is multiplied by the uncertainty upper-bound term $d_2(x_a, \dot{x}_a)$ which can have large values (see equation (28)). Comparatively, the remaining gains are only multiplied by the tracking error (see equation (29)), which are smaller (providing that the initial tracking error is small).

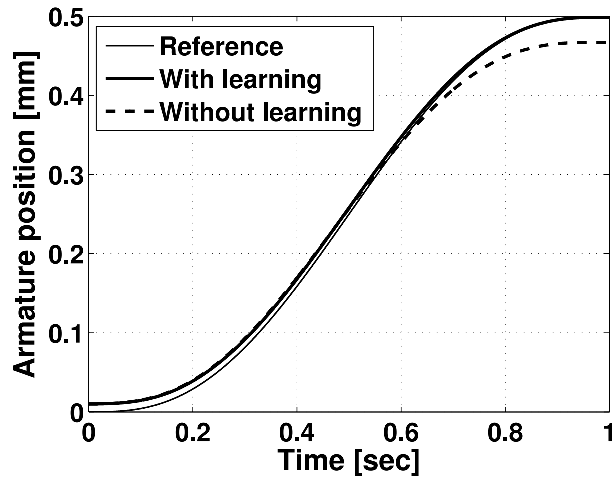
We show on figures 1(a), 1(b) the performance of the position and the velocity tracking, with and without the learning algorithm. We see clearly the effect of the learning algorithm, which makes the tracking performance better by properly tuning the feedback gains. The associated coil current and voltage signals are also reposted on figures 2(a) and 2(b), respectively. We notice a slight voltage saturation towards the end of the motion, where it reaches the voltage limit set by the saturation function value that we fixed at $40\ V$. We also report on figure 3, the cost function value along the learning iterations. We see a clear decrease of the cost function which reaches a local optimum after about 21 iterations. We underline here that we decided to stop the learning iterations, when the value of the cost function is lower than 10th of its initial value. Indeed, in practical applications we do not need to continue changing the gains values. After the cost function has dropped below a desired threshold, we can just select the associated gains as the optimal gains for the feedback system, and apply them to the system. One can then monitor the value of the learning cost function, if it rises suddenly for any reason, e.g., a fault in the system, the learning is then resumed to re-tune the gains to adapt to the new closed-loop dynamics. We report the learned feedback gains on figures 4(a), 4(b), 4(c), and 4(d), respectively. We can notice that the best value of the gains is not necessarily the largest value. Indeed, the learning cost function (32), is not a trivial function of the gains, i.e., one cannot have a simple intuition about how to adjust the gains, for the desired soft impact performance with control voltage saturations. This shows the usefulness of these type of IFT algorithms, which can help in tuning the feedback gains, even when the relationship between the desired performance and the feedback gains is not trivial.

In a second test presented below, we wanted to evaluate the algorithm when the measurements are corrupted with measurement noise, which is the case in most practical applications.

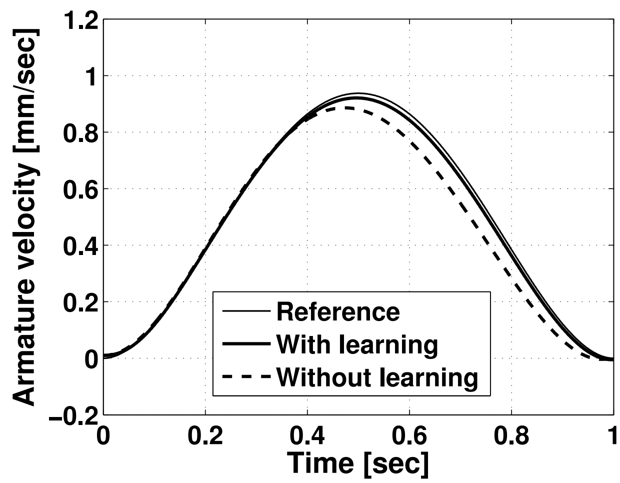
- *Test two (noisy measurements)*: We want to test here the robustness of the MES-based IFT algorithm w.r.t. measurement noise. We consider additive measurement noise on both the armature position and velocity. We introduced uniform additive position measurement noise of excursion $\pm 10^{-2}$, and additive velocity measurement noise of excursion $\pm 10^{-3}$. We report the obtained convergence results on figures 5, 6(a), 6(b), 6(c) and 6(d). These results show the robustness of the dither-based MES algorithm to additive measurement noise, as documented in the ES literature, e.g., [38]. Indeed, we see on figure 5 that the learning cost function decreases, at almost the same convergence rate. We see on figures 6(a), 6(b), 6(c) and 6(d) that the gains converge to slightly different values, which was expected because the noise appears in the measurements and in the feedback control, since the feedback control is based on the measurements. Thus the noise affects all the terms of the learning cost function. However, the main goal of improving the closed-loop performance, is still achieved as shown by the minimization of the learning cost function on figure 5.

VI. Conclusion

In this work, we have studied the problem of iterative feedback gains tuning for nonlinear systems with bounded additive uncertainties. First, we have used Input-Output linearization with static state feedback method and 'robustified' it with respect to the additive model uncertainties, using Lyapunov reconstruction techniques, to ensure uniform boundedness of a tracking error vector. Second, we have complemented the Input-Output linearization controller with a model-free learning algorithm to iteratively auto-tune the control feedback gains, and optimize a desired performance of the system. The learning algorithm used here is based on multi-parametric extremum seeking theory. The full controller, i.e., the learning algorithm together with the passive robust controller, forms an iterative feedback gains auto-tuning (IFT) controller. We have reported some numerical results obtained on an electromagnetic actuator example. Future investigations will focus on extending this method to the more general case of additive as well as multiplicative uncertainties, i.e. with uncertainties on $f(x)$, and $g(x)$. For example, this could be done following the same approach but with a different robust nonlinear controller, obtained from the multiplicative fault tolerant controller proposed in [22]. Another improvement direction could target the convergence rate and domain of attraction of the auto-tuning algorithm, for example, by using different ES algorithms with semi-global convergence properties, e.g., [39], [30], [40]. Furthermore, we saw that the ES-based IFT proposed here requires less experiments per learning iteration compared to the available IFT algorithms, which require a number of experiments that is directly proportional to the number of tuned gains. However, the number of parameters necessary to tune the algorithm with extremum seeking also becomes very large as the number of gains increase significantly. To remedy this, one research direction would be to try other types of model-free learning algorithms, e.g., machine learning and reinforcement learning algorithms.



(a) Obtained armature position vs. reference trajectory - Controller (29)



(b) Obtained armature velocity vs. reference trajectory - Controller (29)

Fig. 1: Obtained outputs vs. reference trajectory - Controller (29) without learning (dashed line), with learning (bold line)

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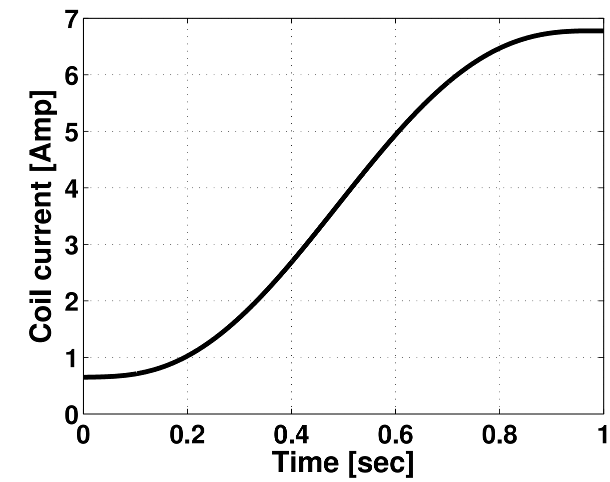
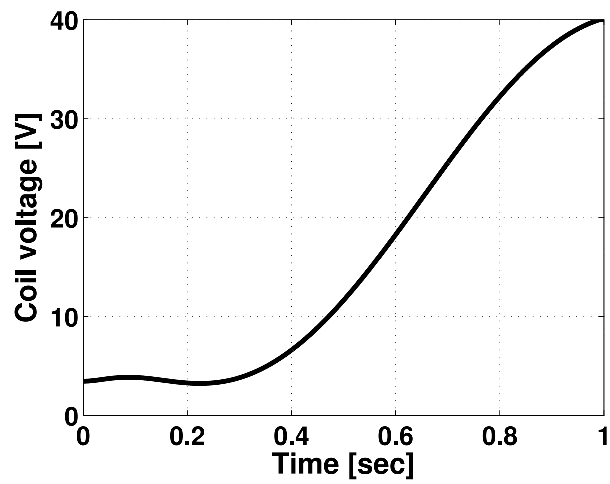


(a) Obtained coil current



(b) Control voltage

Fig. 2: Coil voltage and current - Controller (29)

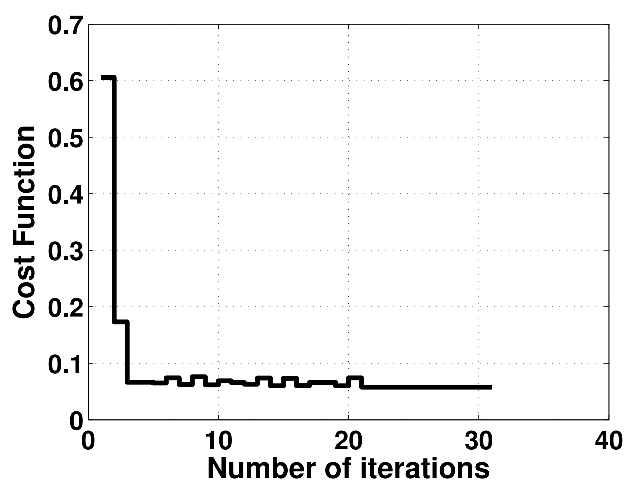
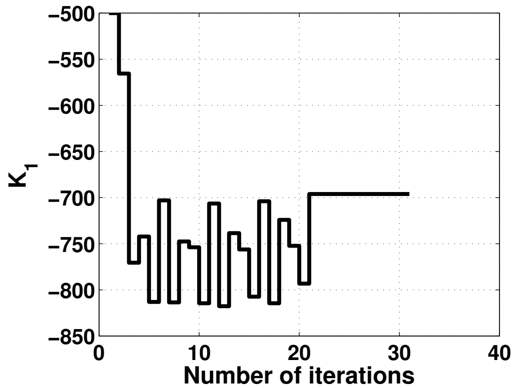
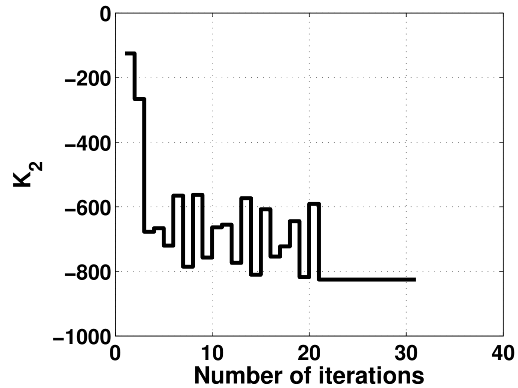


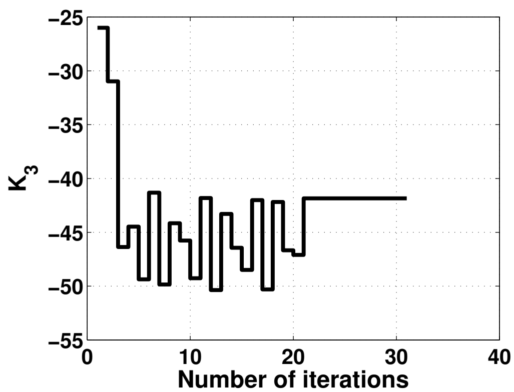
Fig. 3: Learning cost function vs. learning iterations- Controller (29)



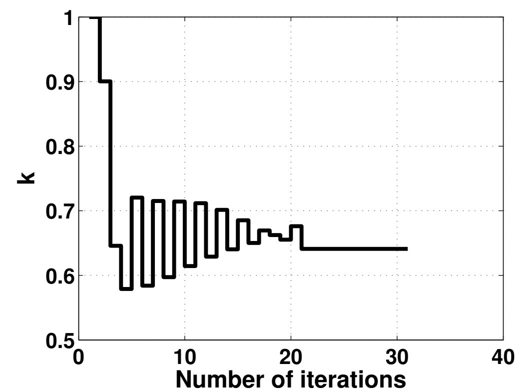
(a) K_1 vs. learning iterations



(b) K_2 vs. learning iterations



(c) K_3 vs. learning iterations



(d) k vs. learning iterations

Fig. 4: Gains learning- Controller (29)-Test 1

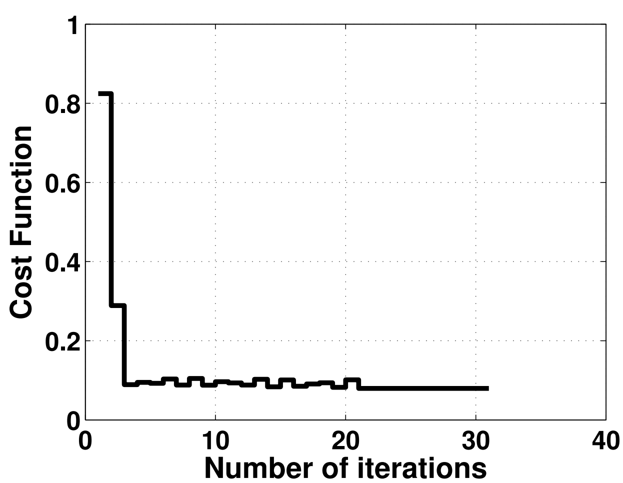
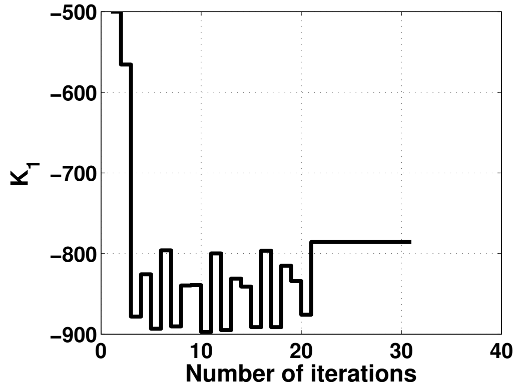
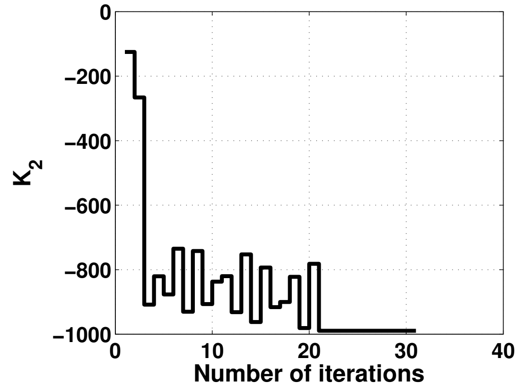


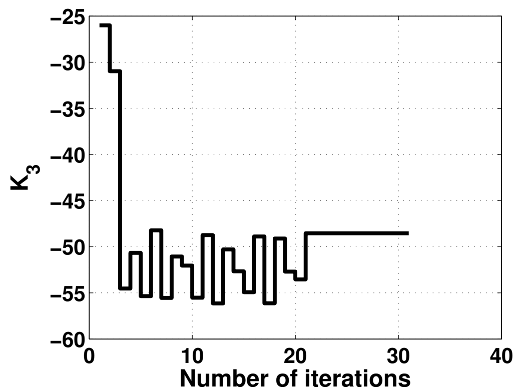
Fig. 5: Cost function vs. learning iterations for two different sets of initial gains



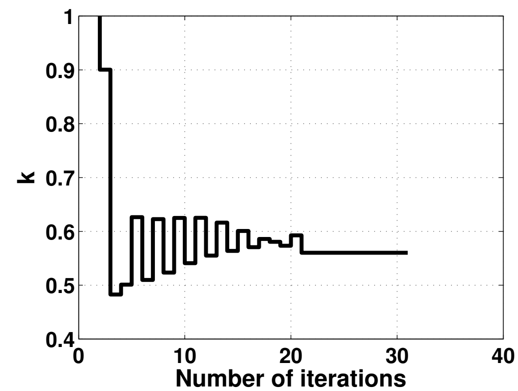
(a) K_1 vs. learning iterations



(b) K_2 vs. learning iterations



(c) K_3 vs. learning iterations



(d) k vs. learning iterations

Fig. 6: Gains learning- Controller (29)- Test 2

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