Reference and Command Governors for Systems with Constraints: A Survey on Theory and Applications

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Reference and Command Governors for Systems with Constraints: A Survey on Theory and Applications

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Abstract

Reference and command governors are add-on control schemes which enforce state and control constraints on pre-stabilized systems by modifying, whenever necessary, the reference. This paper surveys the extensive literature concerning the development of such schemes for linear and nonlinear systems. The treatment of unmeasured disturbances and parametric uncertainties is also detailed. Generalizations, including extended command governors, feedforward reference governors, reduced order reference governors, parameter governors, networked reference governors, and decentralized/distributed reference governors, are discussed. Practical applications of these techniques are presented and surveyed as well. A comprehensive list of references is included. Connections with related approaches, including model predictive control and input shaping, are discussed. Opportunities and directions for future research are highlighted.

1 Introduction

With the advances in control theory, many effective techniques have become available for the design of feedback control laws with the desired stability, performance and disturbance rejection properties. The interest in treating requirements that have the form of *pointwise-in-time state and control* constraints has also been growing, given their importance for industrial applications. Examples of constraints in real-world applications include actuator magnitude and rate limits, bounds imposed on process variables to ensure safe and efficient system operation, and collision/obstacle avoidance requirements.

A control engineer faced with the task of satisfying constraints has several choices. One route is to re-design the controller within the Model Predictive Control (MPC) framework [35, 109, 160, 166, 176]. Another route is to augment a well-designed nominal controller, that already achieves high performance for small signals, with constraint handling capability for larger signals and transients that have the potential to induce constraint violation. This second route is attractive to practitioners who may be interested in preserving an existing/legacy controller or are concerned with the computational effort, tuning complexity, stability, robustness, certification issues, and in general other requirements satisfactorily addressed by the existing controller. Anti-windup compensation [8] and the augmentation of Lyapunov controllers with barrier functions [224] are examples of this second approach, and so are the reference governors (RGs) and command governors (CGs).

As its name suggests, the reference governor (see Figure 1) is an *add-on* scheme for enforcing pointwise-in-time state and control constraints by modifying the reference command to a well-designed (for small signals) closed-loop system. The reference governor plays the role of a pre-filter that, based on the current value of the *desired reference command* $r(t)$ and of the state (measurement or estimate) $x(t)$, generates a *modified reference command* $v(t)$ whenever propagating the reference command without modifications may lead to constraints violations.

The use of low pass pre-filters to enforce constraints is a classical control technique [228], which usually results in a modified reference which is always different from the actual reference (other than asymptotically). Instead, the reference governor

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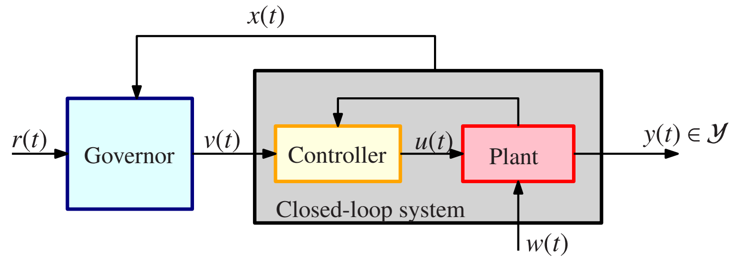


Fig. 1. Reference/command governor applied to closed-loop (Plant + Controller) system subject to constraints.

exploits state feedback, prediction, optimization, and set-invariance arguments [22, 26] to ensure that modifications of the command are performed only when necessary to avoid compromising system performance.

As an example, consider the application of a reference governor to the double integrator $x_1(t+1) = x_1(t) + 0.1x_2(t)$, $x_2(t+1) = x_2(t) + 0.1u(t)$, with state and control constraints, $|u(t)| \leq 0.1$, $|x_1(t)| \leq 1$, and $|x_2(t)| \leq 0.1$, and controlled through an LQ control law, $u(t) = 0.9170(v(t) - x_1(t)) - 1.6821x_2(t)$. The operation of this system is illustrated in Figure 2, for the two cases $v(t) = r(t)$ and $v(t)$ assigned by a reference governor. As explained, using a reference governor, the command and response are slowed down in order to keep constraints satisfied. However, the modification to the reference is much smaller than what would be done using a low pass filter. In fact, with the reference governor, the constrained variables ride the constraint boundary, which is usually impossible to achieve through a simple low pass filter.

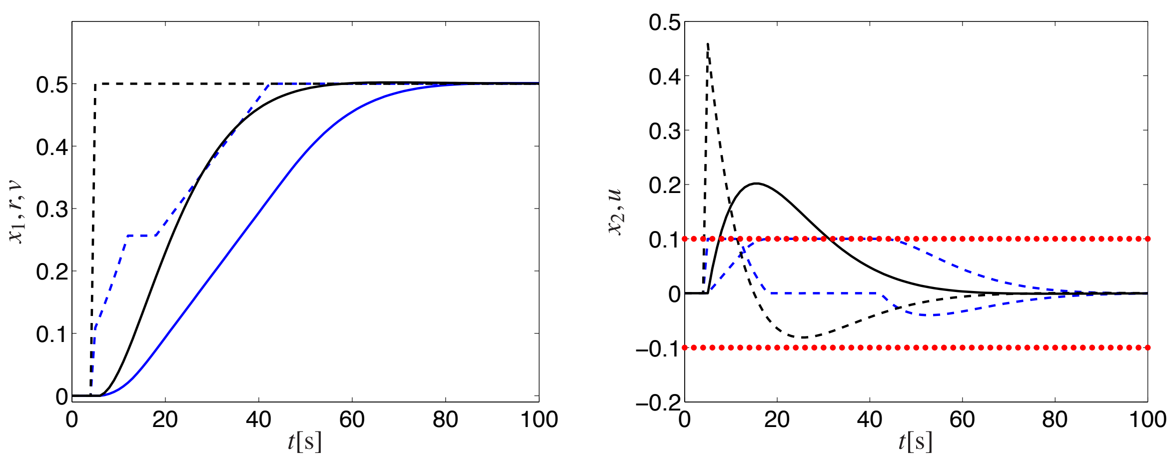


Fig. 2. Double integrator simulations. Left: Time histories of command (dash) and position responses (solid), without (black) and with (blue) reference governor. Right: Time histories of velocity (solid) and control input (dash) responses, without (black) and with (blue) reference governor; constraints in red (dot).

A number of governor schemes have been proposed in the literature. The range of potential options includes, among others, scalar and vector reference governors, command governors, extended command governors, incremental reference governors, feedforward reference governors, network reference governors, reduced order reference governors, distributed reference governors, parameter governors, and virtual state governors. While different in obtained properties and implementation aspects, the common intent of these governors is to preserve, whenever possible, the response of the closed loop system designed by conventional control techniques. Frequently (but not always), they achieve this by ensuring that the modified reference command is as close as possible to the original reference command subject to satisfying the constraints.

Reference governors were first proposed as continuous-time algorithms in [137]. Later, the discrete-time framework [103, 104] has emerged due to some advantages from an implementation standpoint. The static reference governor [104] used $v(t) = \kappa(t)r(t)$, where the parameter $\kappa(t)$, $0 \leq \kappa(t) \leq 1$, was maximized subject the condition $x(t+1) \in O_\infty$, where O_∞ is the maximal output admissible set [106] of all states that, with reference command equal to zero, do not lead to subsequent constraint violation. Because of the possibility of oscillations [104], the static reference governor was abandoned and replaced by a dynamic reference governor for which finite-time convergence for constant or nearly constant reference commands is ensured. Other formulations of reference and command governors have appeared in [14, 16–18, 54, 101], see also references therein. The developments included the treatment of linear systems with uncertainties and set-bounded disturbance inputs, the case of output feedback, and the implementation based on non-positively invariant sets. Extended command governors [105] provided a further generalization with the potential to achieve a larger constrained domain of attraction and faster response, at the price of increased computational complexity.

Several reference governors for nonlinear systems have also been developed, see e.g., [12, 20, 27, 100, 102, 173] and references therein. Some of these approaches exploit on-line prediction through simulations or level sets of Lyapunov functions to guard against constraint violation. The parameter governor has been proposed in [157] to adjust constant controller parameters or controller states based on prediction and optimization.

More recently, classical reference and command governor ideas have been extended in several directions related to the general area of cyber-physical systems (CPS), including the distributed control of large scale systems, modular control architectures, and network control systems.

The aim of this survey paper is to collect and systematize in a common framework the numerous contributions that at the current stage are dispersed in a number of different papers, to discuss the most recent results, to illustrate the impact on real world applications, and to provide a perspective on some open research directions. Accordingly, we will first survey the most important results concerning linear (Section 2) and nonlinear (Section 3) reference governor schemes. Then, we will describe the most recent research directions (Section 4) and we will provide an overview of the use of reference governors in different applications (Section 5). Discussions on connections with other control schemes (such as MPC) and on directions for future research are included at the end of the paper (Section 6).

Notation: $\mathbb{R}$ and $\mathbb{Z}_+$ denote the real and non-negative integer numbers, respectively. Relational operators $<, >, \leq, \geq$ are intended componentwise for vectors, while for matrices indicate (semi)definitenes. The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $\|x\| = \sqrt{x_1^2 + \ldots + x_n^2}$ whereas $\|x\|^2_\Psi$, with $\Psi = \Psi^T > 0$, denotes the quadratic form $x^T \Psi x$. For given sets $X, Y \subset \mathbb{R}^n$, $X \sim Y := \{x : x + y \in X, \forall y \in Y\}$ is the *Pontryagin (or Minkowski) set difference* and $X \oplus Y := \{x + y : x \in X, y \in Y\}$ the *Minkowski set sum*. (a, b) denotes the stacked vector $(a, b) = [a^T \ b^T]^T$.

2 Reference Governors for linear systems

In this section we will survey the theory of Reference and Command Governors for linear systems. In Subsection 2.1 we will show the basic theory for nominal linear systems assuming the state can be measured. Then we will show how this theory can be extended to the case of linear system subject to process disturbances and making use of noisy output measurements (Subsection 2.2). Subsections 2.2 and 2.4 present in detail the computational aspects related to the design and online implementation of the presented schemes.

2.1 Nominal Case - Reference and Command Governors

Reference governors for linear systems are designed based on their discrete-time models in the form,

$$\begin{aligned} x(t+1) &= Ax(t) + Bv(t), \\ y(t) &= Cx(t) + Dv(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $v(t) \in \mathbb{R}^m$ is the input vector, and $y(t) \in \mathbb{R}^p$ is the output vector. The model (1) represents a closed-loop system, thereby reflecting the combined *closed-loop* dynamics of the plant with a stabilizing controller. Consequently, the closed-loop system is assumed to be asymptotically stable, i.e., the matrix A is Schur (all eigenvalues are strictly in the interior of the unit disk). Constraints are imposed on the output variables $y(t)$,

$$y(t) \in Y \text{ for all } t \in \mathbb{Z}_+, \tag{2}$$

where $Y \subset \mathbb{R}^p$ is a specified set. Note that, since (1) is a model of the closed-loop system, (2) can represent constraints on either state or control variables. For instance, a control constraint $|u_1(t)| \leq 1$ where the control is generated by a state feedback law, $u = Kx$, can be restated as $|y_1(t)| \leq 1$ with $y_1 = \hat{e}_1 K x$, and $\hat{e}_1 = [1 \ 0 \ \cdots \ 0]$.

In this survey, an assumption on the convexity of Y is made throughout, because of theoretical and computational simplifications which occur when the set Y is convex.

The common feature of most reference governor schemes proposed in the literature is that they compute at each time instant a command $v(t)$ such that, if constantly kept from t onward, the ensuing output will always satisfy the constraints. More formally, define the *maximal output admissible set* O_∞ [106] as the set of all states x and inputs, v, such that the predicted response from the initial state x and if the input v is kept constant satisfies the constraints, i.e.,

$$O_\infty = \{(v,x) : \ \hat{y}(k|v,x) \in Y, \ \forall k \in \mathbb{Z}_+\}, \tag{3}$$

where for the system (1) the prediction $\hat{y}(k|x,v)$ is defined as

$$\begin{aligned} \hat{y}(k|v,x) &= CA^k x + C\sum_{j=1}^{k} A^{j-1}Bv + Dv \\ &= CA^k x + C(I-A)^{-1}(I-A^k)Bv + Dv. \end{aligned} \tag{4}$$

On the basis of the currently available state $x(t)$, reference governor schemes compute $v(t)$ so that

$$(v(t), x(t)) \in P, \tag{5}$$

where $P \subseteq O_\infty \subset \mathbb{R}^m \times \mathbb{R}^n$. Although in principle $P = O_\infty$ is an acceptable choice, this is usually avoided for computational reasons. The most usual choice for P is $P = \tilde{O}_\infty$, where $\tilde{O}_\infty$ is a slightly tightened version of O_∞ obtained by constraining the command v so that the associated steady state output $\bar{y}_v = (D + C(I-A)^{-1}B)v$ satisfies constraints with a nonzero (typically small) margin $\epsilon > 0$, i.e.,

$$\tilde{O}_\infty = O_\infty \cap O^\epsilon, \tag{6}$$

where

$$O^\epsilon = \{(v,x) : \ \bar{y}_v \in (1-\epsilon)Y\}. \tag{7}$$

Clearly, $\tilde{O}_\infty$ can be made arbitrary close to O_∞ by decreasing ϵ. It can be proven [106] that if A is Schur, (A,C) is observable, and Y is compact, then the set $\tilde{O}_\infty$ is *finitely determined*, i.e. there exists a finite index k^* such that

$$\tilde{O}_\infty = \{(v,x) \mid \hat{y}(k|v,x) \in Y, \ k = 0,\ldots,k^*\} \cap O^\epsilon. \tag{8}$$

Moreover, it is possible to prove [106] that $\tilde{O}_\infty$ is *positively invariant*, which means that if $(v(t),x(t)) \in \tilde{O}_\infty$ and $v(t)$ is applied to the system at time t, then $(v(t), x(t+1)) \in \tilde{O}_\infty$. Furthermore, if Y is convex, also $\tilde{O}_\infty$ is *convex*. For the double integrator example showed in Figure 2, the $\tilde{O}_\infty$ set is shown in Figure 3, together with some sections for different values of v that illustrate the allowed values of states for the corresponding values of the modified reference command.

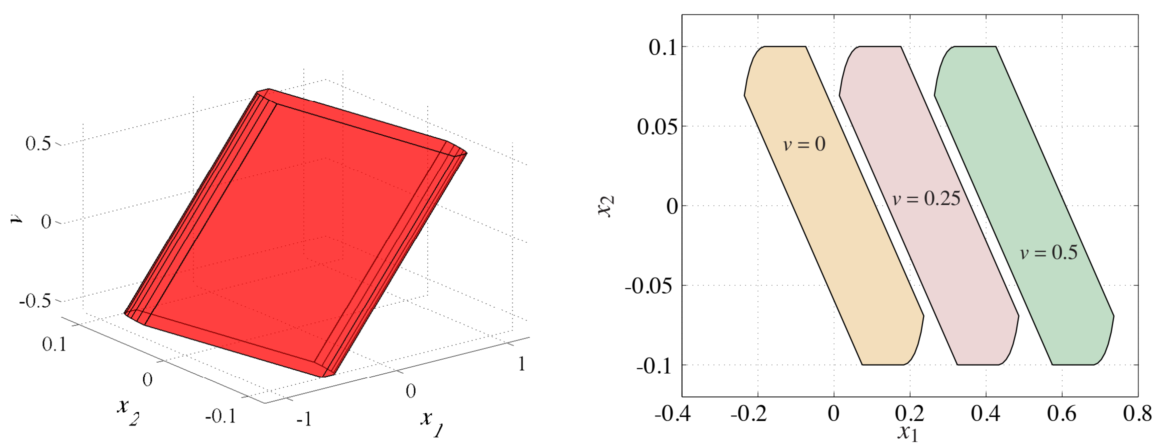


Fig. 3. $\tilde{O}_\infty$ for the double integrator example. Left: entire $\tilde{O}_\infty$. Right: sections of $\tilde{O}_\infty$ in the (x_1, x_2)- plane obtained for $v = 0, 0.25, 0.5$.

The scalar Reference Governor (RG), introduced in [17, 101, 103, 104], is based on the idea of computing at each time instant t and on the basis of the current state $x(t)$, a command $v(t)$ which is the best feasible approximation of the desired set-point $r(t)$

along the line segment connecting $v(t-1)$ and $r(t)$ that ensures $(v(t), x(t)) \in \tilde{O}_\infty$. More formally, the RG solves at each discrete time t, the following optimization problem,

$$\begin{aligned} \kappa(t) = \max_{\kappa \in [0,1]} \quad & \kappa \\ \text{s.t.} \quad & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & (v, x(t)) \in \tilde{O}_\infty, \end{aligned} \tag{9}$$

where $\kappa(t)$ is a scalar adjustable bandwidth parameter and $v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1))$ is the command to be applied to the system. If no danger of constraint violation exists, $\kappa(t) = 1$, and $v(t) = r(t)$ so that the reference governor does not interfere with the desired operation of the system. If $v(t) = r(t)$ would cause a constraint violation, the value of $\kappa(t)$ is decreased by the reference governor. In the extreme case, $\kappa(t) = 0$, $v(t) = v(t-1)$, which means that the reference governor momentarily isolates the system from further variations of the reference command to ensure safety, in terms of constraint enforcement.

Due to the positive invariance of $\tilde{O}_\infty$, $v(t) = v(t-1)$ always satisfies the constraints, which ensures *recursive feasibility* under the condition that at time $t = 0$ a command $v(0)$ such that $(v(0), x(0)) \in \tilde{O}_\infty$ is known. Response properties of the reference governor, including conditions for the *finite-time convergence* of $v(t)$ to $r(t)$, are detailed in [101, 103]. Essentially, if $r(t)$ remains constant for $t \geq t_0$ (i.e., $r(t) = \bar{r}, \forall t \geq t_0$) and $\bar{r}$ is strictly steady-state constraint admissible (i.e., if $\bar{r} \in O^\epsilon$), then $v(t)$ converges to $\bar{r}$ in *finite-time*. If $r(t)$ is not strictly steady-state constraint admissible, then $v(t)$ converges in a finite time to a constant command $\bar{v} \in O^\epsilon$ that is the closest feasible approximation of $\bar{r}$ along the line connecting $v(t_0)$ and $\bar{r}$. Under appropriate assumptions, similar finite-time convergence results can be proved for $r(t)$ varying in a vicinity of a constant value [101] and for $r(t)$ sufficiently slowly varying [123, 128]. Finite-time convergence is a desirable property indicating that after transients caused by large changes in $r(t)$, the reference governor becomes inactive and the nominal closed-loop system performance is recovered. For the above double integrator example, the state trajectories from Figure 2, and superimposed onto sections of $\tilde{O}_\infty$ for different values of v are shown in Figure 4.

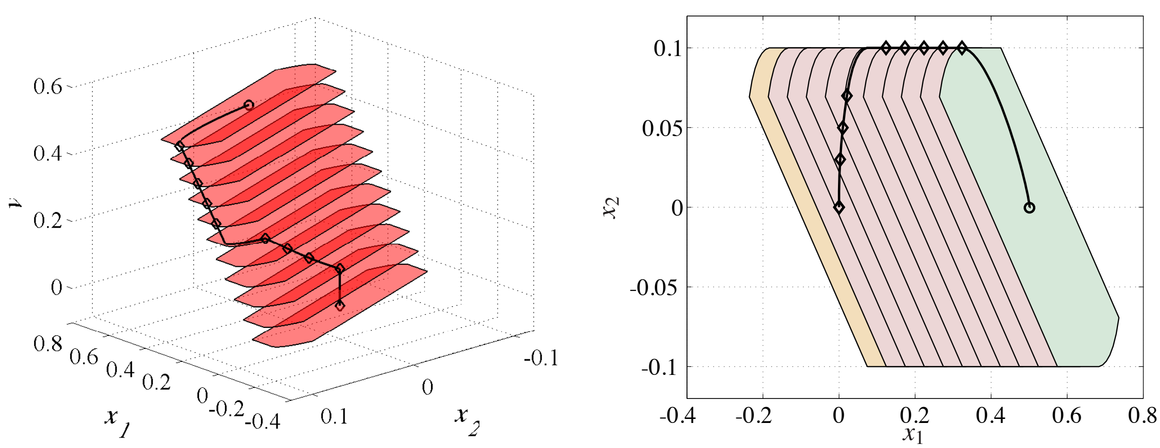


Fig. 4. Trajectory of the double integrator plotted on top of sections of $\tilde{O}_\infty$. The diamond markers indicate where the state trajectory first enters a shown section. Left: trajectories of states and command. Right: state trajectory projected onto the (x_1, x_2)-plane.

One of the main strengths of the scalar RG is that, since only the scalar parameter $\kappa(t)$ is optimized on-line, the computational effort is small. However, this is also its potential weakness. In fact, the command decision space is restricted from $\mathbb{R}^m$ to $\mathbb{R}$, potentially limiting the control performance in the case of systems with $m > 1$. A first scheme proposed to overcome this limitation is the *Vector Reference Governor*. The Vector Reference Governor uses a diagonal matrix $\mathbf{K}(t) \in \mathbb{R}^{m \times m}$ in place of a scalar $\kappa(t)$ to decouple the governing of different channels [103]. A further generalization is the so-called *Command Governor* (CG) proposed in [14, 54]. The CG directly uses $v(t) \in \mathbb{R}^m$ as an optimization variable, and solves at each time instant t the optimization problem

$$\begin{aligned} v(t) = \arg\min_{v} \quad & \|v - r(t)\|_Q^2 \\ \text{s.t.} \quad & (v, x(t)) \in \tilde{O}_\infty, \end{aligned} \tag{10}$$

with $Q > 0$. Compared with the scalar RG, CG can provide a faster response if $m \geq 2$. It also ensures recursive feasibility under the condition that an admissible command $v(0)$ such that $(x(0), v(0)) \in \tilde{O}_\infty$ *exists* at time $t = 0$. In contrast, in the scalar RG case a safe $v(0)$ needs *to be known*, and hence the constrained domain of attraction may be a function of the initialization procedure or requiring that the system is started in steady state. In addition, for a constant $r(t) = \bar{r}$, finite time convergence to $\bar{v} \in O^{\epsilon}$ which minimizes the norm $\|\bar{v} - \bar{r}\|_Q$, can be proved for CG [14]. The improved performance of CG as compared to the scalar RG comes at the price of increased computational effort due to solving the optimization problem (10). Both the scalar RG and the CG have been intensively studied in the last three decades and have been proposed for a significant number of real world applications (see Section 5).

A substantial amount of research on RG and CG for linear systems has been devoted to the computational aspects, some of which will be detailed in Section 2.4. Concerning computational aspects, it is worth mentioning that, if needed, a way to reduce the computational burden is through the use of simpler subsets $P \subset \tilde{O}_\infty$ in place of $\tilde{O}_\infty$. Choices for P not being positively invariant are possible, but in this case the optimization problem may not admit a solution at some step. In these situations, simply applying $v(t) = v(t-1)$ ensures that the constraints are still satisfied, since $(v(t-1), x(t-1)) \in P \subset O_\infty$ and hence $(v(t-1), x(t)) \subset O_\infty$, even if $(v(t-1), x(t)) \notin P$ due to the non-invariance of P. The subset P can be obtained from $\tilde{O}_\infty$ by the systematic elimination of *almost* redundant constraint and applying a pull-in procedure, see [101]. This strategy can lead to a ten-fold reduction in the on-line computing effort at the price of a loss in performance, see e.g., [228].

Several variants of the RG and CG schemes have been proposed in the literature, such as the *Prioritized Reference Governor* [125]. The Prioritized Reference Governor enforces hard constraints and it satisfies soft constraints in the order of priority. The soft constraints are relaxed by slack variables and the penalty on the slack variables is added to the cost with lower weights corresponding to lower priority constraints.

A scheme that may further improve the performance over the CG by using an even larger decision space is the *Extended Command Governor* (ECG) [105]. The starting point of this approach is that the notion of prediction (4) can be seen as a special case of the more general prediction,

$$\hat{y}(k|\hat{v}(\cdot), x) \;=\; CA^k x + C\sum_{j=1}^{k} A^{j-1} B\hat{v}(j) + D\hat{v}(j), \tag{11}$$

where the command $\hat{v}(\cdot)$ is kept constant, i.e. $\hat{v}(\cdot) = v$. The idea of the ECG is to use, instead of a constant command, a command that is the sum of a constant part $\bar{\rho}$ and of a vanishing part $\hat{\mu}(\cdot)$,

$$\hat{v}(\cdot) = \bar{\rho} + \hat{\mu}(\cdot), \tag{12}$$

where the vanishing part $\hat{\mu}(\cdot)$ is the output of a fictitious autonomous system,

$$\chi(t+1) = A_\chi \chi(t), \tag{13}$$
$$\hat{\mu}(t) = C_\chi \chi(t), \tag{14}$$

where A_χ, C_χ are design parameters with A_χ being a Schur matrix. Since this system is fictitious, its initial state is an extra decision variable that can be used to improve the performance of the ECG.

The simplest way to use this idea consists in computing the maximal output admissible set $\tilde{O}_\infty$ (using the very same procedures developed for the classical RG/CG schemes) for the extended system

$$\begin{bmatrix} x(t+1) \\ \chi(t+1) \end{bmatrix} = \begin{bmatrix} A & BC_\chi \\ 0 & A_\chi \end{bmatrix} \begin{bmatrix} x(t) \\ \chi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \rho(t),$$

and then solve at each time step t the optimization problem

$$\begin{aligned} (\bar{\rho}(t), \chi(t)) = \; & \arg\min_{\bar{\rho}, \chi} \quad \tfrac{1}{2}\|\bar{\rho} - r(t)\|_Q^2 + \tfrac{1}{2}\|\chi\|_P^2 \\ & \text{s.t.} \quad \left(\bar{\rho}, \begin{bmatrix} x(t) \\ \chi \end{bmatrix}\right) \in \tilde{O}_\infty. \end{aligned} \tag{15}$$

where $P > 0$ satisfies the discrete-time Lyapunov inequality,

$$A_\chi^{\mathrm{T}} P A_\chi - P < 0.$$

The command that is applied based on the solution of the optimization problem (15) is

$$v(t) = \bar{\rho}(t) + C_\chi \chi(t).$$

The ECG enjoys the same theoretical properties as the CG while providing a faster response (especially in the case of systems with rate limited actuators) and enjoying a larger domain of attraction. Note that if $n_\chi = 0$ the ECG becomes a CG [105]. For what concerns A_χ and C_χ, in principle any arbitrary choice can be made as long as A_χ is a Schur. The first paper using this framework appears to be [14] where $A_\chi = \gamma I_m$ was chosen with $\gamma \in [0,\ 1)$. Another common choice for A_χ consists in the use of shift registers [105],

$$A_\chi = \begin{bmatrix} 0 & I_m & 0 & 0 & \cdots \\ 0 & 0 & I_m & 0 & \cdots \\ 0 & 0 & 0 & I_m & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \qquad (16)$$
$$C_\chi = \begin{bmatrix} I_m & 0 & 0 & 0 & \cdots \end{bmatrix},$$

where I_m is an $m \times m$ identity matrix. For the choice in (16), ECG can be re-formulated as a particular MPC scheme [105]. Another approach [130], motivated by [206], uses the Laguerre sequences,

$$A_\chi = \begin{bmatrix} \alpha I_m & \beta I_m & -\alpha\beta I_m & \alpha^2 \beta I_m & \cdots \\ 0 & \alpha I_m & \beta I_m & -\alpha\beta I_m & \cdots \\ 0 & 0 & \alpha I_m & \beta I_m & \cdots \\ 0 & 0 & 0 & \alpha I_m & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \qquad (17)$$
$$C_\chi = \sqrt{\beta} \begin{bmatrix} I_m & -\alpha I_m & \alpha^2 I_m & -\alpha^3 I_m & \cdots & (-\alpha)^{N-1} I_m \end{bmatrix},$$

where $\beta = 1 - \alpha^2$, and $0 \le \alpha \le 1$ is a design parameter. Note that for the choice $\alpha = 1$, (17) reduces to (16).

2.2 Disturbance, Noise and Output Feedback

An attractive feature of RG and CG schemes is that they can be easily modified to account for bounded unmeasured disturbances,

$$\begin{aligned} x(t+1) &= Ax(t) + Bv(t) + B_w w(t), \\ y(t) &= Cx(t) + Dv(t) + D_w w(t), \end{aligned} \qquad (18)$$

where $w(t)$ is a disturbance that is assumed to be unknown but bounded, i.e., $w(t) \in W$, $\forall t$ where W is a compact set containing the origin. The main difference with respect to the nominal case is that the *maximal output admissible set* (sometimes also called maximal constraint admissible set) is defined as follows [145]

$$O_\infty = \{(v, x) :\ \hat{y}_d(k|v, x) \subseteq Y,\ \forall k \in \mathbb{Z}_+\}, \qquad (19)$$

where $\hat{y}_d(k|x,v)$ is now a set-valued prediction of the output taking into account all possible realizations of the disturbance. Since (18) is linear, the set-valued ouput predictions (19) can be written as the Minkowski set-sum of a nominal prediction plus the set of all the possible effects of the disturbance on the output,

$$\hat{y}_d(k|v,x) = \hat{y}(k|v,x) \oplus \hat{\mathcal{Y}}_k, \tag{20}$$

where the nominal predictions $\hat{y}(k|v,x)$ are as in (4) and

$$\hat{\mathcal{Y}}_k = C\left(\bigoplus_{i=0}^{k-1} A^{k-i-1} B_w W\right) \oplus D_w W. \tag{21}$$

Using the sets $\hat{\mathcal{Y}}_k$ from (21) and the Pontryagin set difference, it is possible to rewrite the maximal output admissible set as

$$O_\infty = \{(v,x) : \ \hat{y}(k|v,x) \in Y \sim \hat{\mathcal{Y}}_k, \ \forall k \in \mathbb{Z}_+\}. \tag{22}$$

Since A is a Schur matrix and W is compact, with $0 \in W$, the monotonically increasing sequence $\hat{\mathcal{Y}}_{k+1} \supseteq \hat{\mathcal{Y}}_k$ converges to a finite set $\hat{\mathcal{Y}}_\infty = \lim_{k\to\infty} \hat{\mathcal{Y}}_k$. As a consequence, it is possible to prove [144] that if Y is convex and compact, (A,C) is observable, and W is compact, then the set $\tilde{O}_\infty = O_\infty \cap O^\epsilon$ is finitely determined, convex, compact, and positively invariant. This implies that all the strategies developed for disturbance-free linear systems can be extended to the case where bounded unmeasured disturbances are present, by simply computing $\tilde{O}_\infty$ while accounting for the disturbances as in (22) . Also, the same properties (recursive feasibility and finite time convergence) are retained.

Note that the idea of pre-stabilizing a system and then using the boundedness and convergence of the resulting sequence $\hat{\mathcal{Y}}_k$ to deal with bounded disturbances, first introduced for RG in [144], has subsequently become popular also in the so-called tube-MPC (see [138, 202] and references therein).

Using similar arguments, it is also possible to treat the case when the state of the system is not measured, but only a possibly noisy output is measured. More precisely, suppose that for system (18) only the output,

$$z(t) = C_z x(t) + \xi(t), \tag{23}$$

is measured, where $\xi(t)$ is an unknown bounded measurement error satisfying $\xi(t) \in \Xi, \forall t \in \mathbb{Z}_+$, and Ξ is a bounded convex set containing the origin. The approach proposed in [6] consists in using the set-valued recursions introduced in [108] to build a *set-membership state estimator*. To do so, it is assumed that a set $\hat{\mathcal{X}}(0|0)$ which contains the true initial state, $x(0)$, is known. At each time instant t, the set of plausible state estimates, $\hat{\mathcal{X}}(t|t)$, can be computed as the intersection of the set of plausible state predictions and of the set of states compatible with the output observations,

$$\hat{\mathcal{X}}(t|t) = \hat{\mathcal{X}}(t|t-1) \cap \hat{\mathcal{X}}_z(t), \tag{24}$$

where the predictions are

$$\hat{\mathcal{X}}(t|t-1) = A\hat{\mathcal{X}}(t-1|t-1) \oplus \{B_v v(t-1)\} \oplus B_w W, \tag{25}$$

and the states compatible with the output measurements are in the set

$$\hat{\mathcal{X}}_z(t) = \{C_z^T (C_z C_z^T)^{-1} z(t)\} \oplus \{x \in \mathbb{R}^n : -C_z x \in \Xi\}. \tag{26}$$

Once $\hat{\mathcal{X}}(t|t)$ is computed, it is possible to rewrite it as the Minkowskii set sum of a nominal state estimate, $\hat{x}(t) \in \hat{\mathcal{X}}(t|t)$, plus a set, $\hat{\mathcal{E}}(t)$, containing the origin, i.e., $\hat{\mathcal{X}}(t|t) = \{\hat{x}(t)\} \oplus \hat{\mathcal{E}}(t)$. A sequence of maximal output admissible sets can be defined as

$$O_\infty(t) = \{(v,\hat{x}) : \ \hat{y}(k|v,\hat{x}) \in Y \sim (\hat{\mathcal{Y}}_k \oplus CA^k \hat{\mathcal{E}}(t)), \ \forall k \in \mathbb{Z}_+\}. \tag{27}$$

It is possible to prove [6] that the sequence of sets, $\tilde{O}_\infty(t) = O_\infty(t) \cap O^\epsilon$ is positively invariant and each set in the sequence is finitely determined and convex. As a consequence, all schemes developed for nominal linear systems can be applied in the case of output feedback, by using the estimate, $\hat{x}$, in place of the (unknown) state, x. The feasibility and convergence properties are also preserved. However, as t increases, the complexity of the representation of the set $\hat{\mathcal{X}}(t|t)$ increases, complicating the computations of $\tilde{O}_\infty(t)$. Also, $O_\infty(t)$ depends on $\hat{\mathcal{E}}(t)$ which in turn depends on a priori unknown measurements, $z(t)$. Hence, in general, $O_\infty(t)$ must be computed online, which may cause significant computational challenges. For these reasons, it may be convenient to use simpler outer approximations of $\hat{\mathcal{X}}(t|t)$, at the price of an increased conservativeness.

In [6], some sufficient conditions to ensure that these outer approximations still lead to a positively invariant, tightened version of $\tilde{O}_\infty$ have been highlighted. A very relevant case is when a Luenberger observer [37, 115] is used to estimate the state. In this case, starting from an initial estimate $\hat{x}(0)$ and given an initial uncertain set $\hat{\mathcal{E}}(0)$, the set-valued observer becomes

$$\begin{aligned} \hat{x}(t+1) &= A\hat{x}(t) + B_v v(t) + L(C_z\hat{x}(t) - z(t)), \\ \hat{\mathcal{E}}(t+1) &= A\hat{\mathcal{E}}(t) \oplus B_w W \oplus L\Xi, \end{aligned} \tag{28}$$

where L is the Luenberger observer gain that must be chosen so that $A + LC_z$ is a Schur matrix. Since $A + LC_z$ is Schur, the uncertainty set $\hat{\mathcal{E}}(t)$ remains bounded and will eventually converge,

$$\lim_{t\to\infty} \hat{\mathcal{E}}(t) = \lim_{t\to\infty} A^t\hat{\mathcal{E}}(0) \oplus \left(\bigoplus_{i=0}^{t-1} A^{t-i-1} (B_w W \oplus L\Xi) \right) = \bigoplus_{i=0}^{\infty} A^i (B_w W \oplus L\Xi) = \hat{\mathcal{E}}_\infty. \tag{29}$$

Note that, in the special case $\hat{\mathcal{E}}(0) = \hat{\mathcal{E}}_\infty$, the uncertainty set is stationary, i.e. $\hat{\mathcal{E}}(t) = \hat{\mathcal{E}}_\infty, \forall t \in \mathbb{Z}_+$, and O_∞ can be defined as

$$O_\infty = \{(v, \hat{x}) : \ \hat{y}(k|v, \hat{x}) \in Y \sim (\hat{\mathcal{Y}}_k \oplus C\hat{\mathcal{E}}_\infty), \ \forall k \in \mathbb{Z}_+\}. \tag{30}$$

In this case, the effect of the observation uncertainty can be taken into account with a simple Pontryagin set difference, and then $\tilde{O}_\infty = O_\infty \cap O^\epsilon$ can be computed off-line. Furthermore, with some further loss of performance, this approach can be used whenever $\hat{\mathcal{E}}(0) \subseteq \hat{\mathcal{E}}_\infty$.

Finally, a more extreme scenario that is worth mentioning is the case where no measurement is available [50]. This is equivalent to the case when observer (29) is used with $L = 0$. Reference/command governors not making use of any measurements are known in the literature as *feedforward* reference/command governors [95, 97]. The fact that reference governor schemes can be built even in the absence of measurements of the state (although at the cost of a conservatism that grows with the size of the disturbance set W) is due to the fact that since A is Schur, the nominal system is open loop detectable [50].

2.3 Design Aspects

For what concerns the design of reference governor schemes, the main aspect is the computation of $\tilde{O}_\infty$. In [106] results for the computation of the maximal output admissible set O_∞ were developed for autonomous asymptotically stable systems without reference. Such results do not apply to the case of governors, where the reference is modeled as a a constant control signal, and the system can be viewed as an augmented system with a marginally stable mode. However, in [145] it is shown that for every $\epsilon > 0$, $\tilde{O}_\infty$ is finitely determined.

To compute $\tilde{O}_\infty$ one must define

$$\tilde{O}_k := \left\{(v, x) : \hat{y}(i|v, x) \in Y \sim \hat{\mathcal{Y}}_k, i = 0, \ldots, k\right\} \cap O^\epsilon,$$

and increase k until k^*, which is the lowest k such that $\tilde{O}_k = \tilde{O}_{k+1}$, or in other words the lowest k such that the new constraint $\hat{y}(k+1|v, x) \in Y \sim \hat{\mathcal{Y}}_{k+1}$ is redundant with respect to the previous ones [52, 144]. From the computational viewpoint the two main difficulties to compute $\tilde{O}_\infty$ are: i) to verify that constraints are redundant, and ii) to perform the Pontryagin set differences.

To verify that a single constraint $h(y) \leq 0$ is redundant with respect to a set $\mathcal{H}$, one must verify that there exists no $y \in \mathcal{H}$, such that $h(y) > 0$. A possible way to do so is to solve the optimization problem,

$$\max_{y\in\mathcal{H}} h(y). \tag{31}$$

If the result of (31) is positive, the constraint is not redundant, otherwise it is redundant. Note that if $h(y)$ is convex, this maximization problem is, in general, non-convex. A special case where (31) results in a convex optimization problem is when $h(y)$ is affine.

Following from the definition, the Pontryagin set difference of a generic set $Y = \{y \,|\, h(y) \leq 0\}$ minus another set $\mathcal{Y}$ can be written as $Y \sim \mathcal{Y} = \{y \,|\, h_{\mathcal{Y}}(y) \leq 0\}$ where the function $h_{\mathcal{Y}}(y)$ is defined as

$$h_{\mathcal{Y}}(y) = \max_{\hat{y}\in\mathcal{Y}} h(y+\hat{y}). \tag{32}$$

If Y is convex, the Pontryagin set difference results in a convex set. While in the general case the computation of $h_{\mathcal{Y}}(y)$ may be involved, whenever $h(y)$ is an affine function, i.e., of the form, $h(y) = h^T y - \bar{h}$, the computations become much simpler as (32) translates into

$$\begin{aligned} h_{\mathcal{Y}}(y) &= \max_{\hat{y}\in\mathcal{Y}} h^T[y+\hat{y}] - \bar{h} \\ &= h^T y - \bar{h} + \max_{\hat{y}\in\mathcal{Y}} h^T \hat{y} \end{aligned} \tag{33}$$

with $\max_{\hat{y}\in\mathcal{Y}} h^T \hat{y}$ being an optimization problem that is convex when $\mathcal{Y}$ is convex, and is solved offline. Note that if $\mathcal{Y}$ is a polytope, then the maximum in (33) is achieved at one of the vertices.

It is clear from the above discussion that, whenever possible, it is convenient to define (or to approximate) the set Y in the constraints $y \in Y$ as a polytope described by a set of linear inequalities in the form

$$Y = \{y \in \mathbb{R}^p \mid Hy \leq \bar{h}\}, \tag{34}$$

for which it is relatively easy to determine offline a minimal representations of $\tilde{O}_\infty$ [106], [145], as a minimal set of linear inequalities of the form

$$\tilde{O}_\infty = \{(v,x): \; H_x x + H_v v \leq s\}, \tag{35}$$

since the above methods require only the solutions of linear programming problems. Similarly, if the disturbance set W is a polytope, the sets $\hat{\mathcal{Y}}_k,\ j \in \mathbb{Z}_+$ in (21) are also polytopes, and again the computations to construct O_∞ in (22) are usually manageable.

An alternative way to compute $\tilde{O}_\infty$ is through the method presented in [19] to compute polyhedral invariant sets under state constraints. Finally, it is worth to mention that beside computing $\tilde{O}_\infty$ explicitly offline, it is also possible to compute it (and k^*) implicitly online using explicit predictions and terminal invariant sets, as in MPC schemes.

2.4 Computational and Implementation Aspects

One of the main advantages of RG and CG with respect to other more complex constrained control strategies, such as MPC, is the limited computational footprint of the online implementation. Next, we describe some of the most common algorithms for evaluating the RG and CG laws for linear systems, with particular focus on the case of constraints defined by polyhedra.

While their formulations may be sightly different, the governors all seek the closest admissible command to the current reference, given the current state. This amounts to seeking $v(t)$ as the projection of the reference $r(t)$ onto the section of $\tilde{O}_\infty$ taken at the current state $x(t)$. In the case of Euclidean norm, the projection problem is

$$\begin{aligned} v(t) = \ \arg\min_{v} \quad & \tfrac{1}{2}\|r(t)-v\|_Q^2 \\ \text{s.t.} \quad & (v, x(t)) \in \tilde{O}_\infty, \end{aligned} \tag{36}$$

where $Q > 0$. If $\tilde{O}_\infty$ is convex, (36) is a convex optimization problem.

Different governors allow for different projections, by either limiting the directions of the projection, e.g., for the scalar RG (9) when $m > 1$, or by lifting the problem to higher dimensions by introducing auxiliary variables, e.g., the Extended Command Governor (15).

In the scalar RG (9) only one optimization variable is considered. If $\tilde{O}_\infty$ is convex, this results in a simple line-search problem. In fact, since $v = v(t-1)$ is admissible, the solution is in-between $v(t-1)$ and $r(t)$. In this case, a bisection search can be applied, which ensures that the error between the feasible solution v^ℓ at iterate ℓ and the optimal solution $\bar{v}$ satisfies $\|v^\ell - \bar{v}\| \leq 2^{-\ell}\|r - v(t-1)\|$. Thus, for a scalar RG a simple algorithm with linear convergence rate of $1/2$ is available. Each step of the search requires only computing the admissibility of a candidate solution v^ℓ, that is, checking whether the condition $(v, x(t)) \in \tilde{O}_\infty$ is satisfied. An even simpler algorithm can used in the case $\tilde{O}_\infty$ is polyhedral as in (35). In fact, in this case the problem reduces to finding the largest κ such that

$$H_x x(t) + H_v [v(t-1) + \kappa(r(t) - v(t-1))] \leq s,$$

which amounts to solving a series of linear inequalities in one variable that can be solved in closed form [101] as follows

$$\kappa(t) = \min\left\{ \min_{j\in J^+} \left\{ \frac{s_j - h_{x,j}^T x(t) - h_{v,j}^T v(t-1)}{h_{v,j}^T (r(t) - v(t-1))} \right\}, 1 \right\},$$

where $h_{x,j}^T$ and $h_{v,j}^T$ are the jth row of H_x and H_v, respectively, s_j is the jth element of s in (35), and $J^+ \subseteq \{1, 2, \ldots, n_c\}$ is the set of indices such that $h_{v,j}^T(r(t) - v(t-1)) > 0$. Note that in this case the computations to be performed online amount to a fixed number of simple operations that scales linearly with respect to the number of constraints in $\tilde{O}_\infty$ and with respect to the dimension of the system. Efficient computational strategies along these lines for a scalar RG and a mixture of affine, quadratic and convex constraints are defined and illustrated in [184].

More complex governors for linear systems , such as the CG, require the solution of Euclidean-norm projection problems with multiple optimization variables. When $\tilde{O}_\infty$ is polyhedral (35) such problem can be transformed into the parametric quadratic programming (pQP) problem,

$$\begin{aligned} v(t) = \ & \arg\min_v \quad \tfrac{1}{2} v^T Q v - r(t)^T Q v \\ & \text{s.t.} \quad H_v v \leq s - H_x x(t), \end{aligned} \tag{37}$$

where $Q > 0$, and the current state $x(t)$ and the current reference $r(t)$ are the parameters, that may change at every time step. Several classes of algorithms exist for solving (37). The interior point and active set methods [33, 234] typically exhibit faster convergence and reach the optimum in few iterations, however, each iteration may be fairly complicated, as it requires the solution of a linear system obtained from the KKT (Karush-Kuhn-Tucker) optimality conditions. Considering that in RG and CG (37) has only few variables, and that one of the attractive feature of governors is the reduced complexity of the computations, interior point and active set methods may be excessively complicated. Instead, first order methods [182] perform computationally simple iterations, although many of them may be required to reach the optimum. Thus, their code is much simpler than in the cases of interior point and active set methods, which allows for faster implementation and verification, and implementability even on platforms with limited computational capabilities such as in automotive systems or in factory automation. These benefits may offset the drawbacks associated with the slower convergence when compared to active set and interior point methods. In recent years, several low complexity first order methods tailored to real-time optimization-based control have been developed, mostly motivated by MPC. In [203] a first order method based on Nesterov's fast gradient algorithm [181] has been developed, which is very effective when only simple constraints, e.g., box constraints, are considered. However, when the constraints are non-simple, e.g., general polytopes like (35), the computational burden of the iteration becomes high. In [139], Lagrangian methods were developed, where at each iteration an unconstrained Lagrangian-based QP is solved and the Lagrange multipliers are adjusted. An alternative approach for solving the QP (37) is to solve its dual problem, which is the non-negative QP

$$\begin{aligned} \eta^* = \ & \arg\min_\eta \quad \tfrac{1}{2} \eta^T H_v Q^{-1} H_v^T \eta + (s - H_v r(t) - H_x x(t))^T \eta \\ & \text{s.t.} \quad \eta \geq 0, \end{aligned} \tag{38}$$

and then recover the solution of (37) as $v(t) = r(t) - Q H_v^T \eta^*$, where η^* is the optimizer of (38).

The advantage of solving (38) in place of (37) is that the constraints in the former problem are simple and easily handled non-negativity constraints, while the disadvantages are that the problem is not strongly convex but only convex, i.e., $H_v Q^{-1} H_v^T \geq 0$, and that the number of variables in (38) is equal to the number of constraints in (37), which, for governors, tends to be significantly larger than the number of variables in (37). However, since, in general, the QPs for governor applications tend to be of small dimensions, even in terms of the absolute number of constraints, and only few constraints are active at any time due to the few variables, dual first order methods may be quite appealing for governor implementation. Among these, in [196] a dual fast gradient method has been proposed. In [197] a dual projected gradient method which allows for fixed point implementation was introduced, and in [65] a dual multiplicative update method was developed. Recently, a number of algorithms based on the alternating direction method of multipliers (ADMM) [32] for solving QPs in the context of MPC have been developed [99, 107, 200, 201]. The methods in [99, 107] require the QP to be strongly convex. In such a case, the governor problem has non-simple constraints, thus the iterations are computationally demanding. Instead, the methods in [200, 201] do not require strong convexity, so that the reference governor problem can be re-formulated by introducing a vector of non-negative slack variables, one per constraint, to convert inequality into equality constraints. With such a transformation, the problem is no-longer strongly convex but has only equality and simple (i.e., non-negativity) constraints, which results in computationally inexpensive ADMM iterations. Thus, the methods in [200, 201] provide algorithms with linear convergence for advanced governors, such as CG, with simple code and iterations.

Yet another alternative is to compute the solution of (37) explicitly offline as a function of the parameters r, x by using multiparametric programming [13, 15]. It was shown in [15, 140, 142] that for QPs, the parametric solution amounts to a piecewise affine function of the parameters. For the governor, the parametric solution results in

$$v = \begin{cases} F_1^{PWA}x + E_1^{PWA}r + G_1^{PWA} & \text{if} \quad H_1^{PWA}x + M_1^{PWA}r \le K_1^{PWA}x \\ & \vdots \\ F_\varsigma^{PWA}x + E_\varsigma^{PWA}r + G_\varsigma^{PWA} & \text{if} \quad H_\varsigma^{PWA}x + M_\varsigma^{PWA}r \le K_\varsigma^{PWA}x \end{cases} \tag{39}$$

being a piecewise affine function defined over ς polyhedral regions that partition the state-reference space into $\varsigma \in \mathbb{Z}_+$ regions. For a governor, there is always a region $\bar{\imath} \in \mathbb{Z}_+$ $\bar{\imath} \le \varsigma$ that corresponds to $\tilde{O}_\infty$, and for such region, $H_{\bar{\imath}}^{PWA} = H_x$, $M_{\bar{\imath}}^{PWA} = H_v$, $K_{\bar{\imath}}^{PWA} = s$, and $F_{\bar{\imath}}^{PWA} = 0$, $E_{\bar{\imath}}^{PWA} = I$, $G_{\bar{\imath}}^{PWA} = 0$, so that there, $v = r$. Once the parametric solution is computed offline, the online computations reduce to first identifying the current active region, i.e., the (unique) region to which x, r belong, through the evaluation of the inequalities in (39), and then to evaluating corresponding affine function. Given the simplicity of operations and the sequential nature of the algorithm, it is relatively simple to bound the worst case number of operations at each control cycle. The disadvantages of the explicit implementation are similar to the ones in MPC: The reconfigurability to model and constraints changes is lost and the computational footprint for large problems can exceed that of the online optimizer-based solution. We note that the offline elimination of almost redundant constraints [101] can facilitate this explicit implementation.

3 Reference governors for nonlinear systems

In the case of nonlinear systems, the design of the reference governor can be based on model linearizations or can be performed based on the nonlinear system model. Several promising approaches along these lines are surveyed below.

3.1 Implementing linear model-based reference/command governor on a nonlinear system

As demonstrated in Section 2, effective reference/command governor design techniques are available when the system model is linear, as in (1) or (18). However, many systems in real world applications are nonlinear. A "practical" way to deal with nonlinear systems consists in designing a governor based on an approximated linear model of the plant. In this case, however, the difference between the responses of the nonlinear system to which the reference governor is applied and of the linearized model on which the design is based, may need to be compensated for.

One approach to achieve such compensation is by treating the difference between the linearized and the nonlinear models as a set-bounded additive disturbance. However, this approach can be conservative. Heuristic techniques proposed in [228] treat such a disturbance as a constant over the prediction horizon. They proceed by extending the linear system model (1) as follows

$$\begin{aligned} x(t+1) &= Ax(t) + Bv(t), \\ w(t+1) &= w(t), \\ y(t) &= Cx(t) + Dv(t) + w(t), \end{aligned} \tag{40}$$

where the extra state $w(t)$ is a fictitious state added to approximately compensate the difference between the linear and the nonlinear system. At this point, the maximal output admissible set $\tilde{O}_\infty$ is computed for (40) and, at time instant t, in the reference/command governor algorithm computes v so that

$$\left(v, \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}\right) \in \tilde{O}_\infty,$$

where $w(t)$ can be computed in different ways, depending on the application. For instance, $w(t)$ may be set equal to the difference between the deviation of the current output of the nonlinear system from the nominal value (corresponding to the linearization point) and the output predicted according to the linear system model based on the state at the previous step, i.e., $\hat{y}(t|t-1) = C(Ax(t-1) + Bv(t-1)) + Dv(t-1)$. While this approach has been very useful in some practical applications (see e.g., [228], [130]), it does not guarantee strong properties such as recursive feasibility.

Another approach to use linear theory in a nonlinear context is by exploiting feedback linearization [113]. Feedback linearization renders the state dynamics linear and hence the prediction of the state evolution becomes computationally straightforward. While this general approach is attractive, note that, after a feedback linearizing transformation, constraints, even if originally convex, can become non-convex. However, in some special cases, the resulting set $\tilde{O}_\infty$ can be a convex set so that the linear reference governor theory can be directly applied. In the case $\tilde{O}_\infty$ is non-convex, a possible sub-optimal approach is to approximate it through convex regions, mixed-logical-dynamic (MLD) constraints of *if-then-else* type and concave constraints, see [123, 132] for details.

3.2 Embedding a nonlinear system into a family of linear systems

A different approach to the reference management of nonlinear systems consists in embedding a nonlinear system into a family of linear systems.

Reference [5] proposes a command governor design based on embedding the nonlinear system model into a family of Linear Time Varying polytopic uncertain models. A similar idea is used in [58], where the nonlinear system is embedded into a Linear Parameter Varying (LPV) system that is controlled through a reference governor and a gain-scheduled tracking algorithm. Similar methods to construct the maximal output admissible set and, if needed, a stabilizing controller, are presented in [61].

Nonlinear systems can also be treated by embedding a nonlinear model into a family of switched linear models dependent on the applied reference v. A variety of schemes have been developed that exploit such an embedding in the reference governor design, see [83, 84, 90] for specific examples. Strategies for the design of reference governors for piecewise affine models have been proposed in [27].

3.3 Reference governor design based on nonlinear models

Several techniques are available for designing reference governors based directly on nonlinear models. The most common approaches and their extensions are discussed next.

In a typical setup, nonlinear reference governor schemes deal with a pre-compensated system described by a nonlinear discrete-time system model in the form,

$$x(t+1) = f(x(t), v(t), w(t)), \tag{41}$$

and subject to constraints

$$y(t) = h(x(t), v(t)) \leq 0. \tag{42}$$

In (41), $w(t)$ is a disturbance that is assumed to be unknown but bounded, i.e., $w(t) \in W$, for all t where W is a compact set containing the origin. The equilibrium associated with a constant reference $v(t) = v$, if no disturbance is present, is denoted by $\bar{x}_v$, i.e., $\bar{x}_v = f(\bar{x}_v, v, 0)$. It is typically assumed that for any v, the equilibrium $\bar{x}_v$ is asymptotically stable if $w = 0$ or Input-to-State (ISS) stable [119] with respect to w.

It appears that the first reference governor specifically designed for (disturbance-free) nonlinear systems is [12]. This scheme is similar to the linear reference governor and finds at each time instant "the best" admissible reference along the line segment connecting the previously applied reference $v(t-1)$ and the desired $r(t)$ such that the predicted state response does not violate constraints:

$$\begin{aligned} \kappa(t) = \max_{\kappa\in[0,1]} \quad & \kappa \\ \text{s.t.} \quad & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & \hat{x}(t+k+1|t) = f(\hat{x}(t+k|t), v), \\ & h(\hat{x}(t+k|t), v) \leq 0, \quad \forall k \geq 0, \end{aligned}$$

where $\hat{x}(t+k|t)$ is the predicted trajectory propagating from $\hat{x}(t|t) = x(t)$, which is computed by simulating the nonlinear model. Once again, the reference to be applied is $v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1))$.

Similarly to the linear case, restricting the set of equilibria to the strictly steady-state admissible ones, and assuming that for any strictly steady-state admissible v the condition $h(x,v) \leq 0$ defines a compact set in x, the above infinite horizon problem can be rewritten as a finite horizon problem

$$
\begin{aligned}
\kappa(t) = \max_{\kappa \in [0,1]} \quad & \kappa \\
\text{s.t.} \quad & v = v(t-1) + \kappa(r(t) - v(t-1)), \\
& \hat{x}(t+k+1|t) = f(\hat{x}(t+k|t), v), \\
& h(x(t+k|t), v) \leq 0, k = 0, \ldots, k^*, \\
& h(\bar{x}_v, v) \leq -\varepsilon,
\end{aligned}
\tag{43}
$$

where $\varepsilon > 0$ is sufficiently small and $k^* \in \mathbb{Z}_+$ is sufficiently large to satisfy the finite determination-like property in [12], i.e., if the constraints are satisfied up to k^*, they will be satisfied after k^*. Note that the online optimization problem in (43) is a *scalar* optimization problem in the parameter $\kappa \in [0,1]$. While, in general, it is non-convex, it is still relatively simple and can be solved by bisections or grid search, while checking the feasibility of $\kappa = 1$ first.

Based on arguments in [12], it is possible to prove that the strategy (43) ensures *recursive feasibility* if a feasible $v(0)$ is known at time $t = 0$. For a constant reference $r(t) = \bar{r}, t \geq 0$, asymptotic convergence of $v(t)$ to $\bar{r}$ can be proved if $\bar{x}_v$ is a continuous function of v and if any reference contained in the segment between $v(0)$ and $\bar{r}$ is strictly steady-state admissible, i.e., $h(\bar{x}_{v(0)+\kappa(\bar{r}-v(0))}, v(0) + \kappa(\bar{r} - v(0))) \leq -\varepsilon, \forall \kappa \in [0,1]$. These results can be strengthened to finite-time convergence under the assumptions and small modifications presented in [100]. Furthermore, as shown in [94, 185], the approach can be extended to the case of non-scalar v in which a continuous curve of strictly steady-state admissible references connecting $v(0)$ and $\bar{r}$ is known. In this case, the reference governor can operate by optimizing v along this curve rather than along the line segment between $v(0)$ and $\bar{r}$, while guaranteeing the usual recursive feasibility and convergence properties. Such a strategy is useful, for instance, in obstacle avoidance scenarios.

The main difficulty in the implementation of this method concerns the determination of k^*. If a Lyapunov function $V(x,v)$ is known, the invariance properties of the level sets of the Lyapunov function can be used for the implicit computation of k^* online. The idea is to stop further predictions at the first time instant k such that the condition $V(\hat{x}(t+k|t), v) \leq V_{min}$ is satisfied, where V_{min} satisfies

$$
\begin{cases} V(x,v) \leq V_{min}, \\ h(\bar{x}_v, v) \leq -\varepsilon \end{cases} \Rightarrow h(x,v) \leq 0.
$$

Such a value of V_{min} can be pre-computed off-line by solving the following optimization problem,

$$
\begin{aligned}
V_{min} = \min_{v,x} \quad & V(x,v) \\
\text{s.t.} \quad & h(\bar{x}_v, v) \leq -\varepsilon, \\
& h(x,v) \geq 0.
\end{aligned}
\tag{44}
$$

Note that V is not required to be a strict Lyapunov function (i.e., only invariance of the level sets is required). Methods to estimate k^* offline can be devised using similar invariance ideas.

A different approach to nonlinear reference governor design consists in explicitly making use of the invariance of the level sets of Lyapunov functions [102, 173] to build a reference governor scheme. Specifically, given a Lyapunov function $V(x,v)$, suppose it is possible to find a bound $\Gamma(v)$ such that

$$
V(x,v) \leq \Gamma(v) \;\Rightarrow h(x,v) \leq 0. \tag{45}
$$

Then a Lyapunov function-based reference governor is based on the solution of the following optimization problem,

$$\begin{aligned} \kappa(t) = \max_{\kappa\in[0,1],v} \quad & \kappa \\ \text{s.t.} \quad & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & V(x(t), v) \le \Gamma(v), \\ & h(\bar{x}_v, v) \le -\varepsilon. \end{aligned} \tag{46}$$

This scheme shares the very same properties of recursive feasibility and convergence with (43). In terms of online implementation, (46) is less computationally demanding than (43) as it does not involve any explicit prediction. Moreover, it has inherent robustness properties in that it is able to deal with any system such that the set $\{x|V(x,v) - \Gamma(v) \le 0\}$ is invariant and constraint admissible. On the other hand, (46) is typically more conservative and the response of $v(t)$ may be slow. Note that, changing the constraints from (42) into $V(x(t), v(t)) \le \Gamma(v)$ through (45) may be viewed as a form of constraint re-modeling where the output constraints are replaced by a constraint on the value of the Lyapunov function. Note also that (43) and (46) can be combined using $\Gamma(v)$ in place of V_{min} to have a more efficient implicit determination of k^*.

From the design viewpoint, assuming a Lyapunov function V is available, one of the main issues in implementing (46) concerns the off-line determination of $\Gamma(v)$. Note first that $\Gamma(v) = V_{min}$ with V_{min} as in (44) is always a feasible (although potentially a conservative) value for $\Gamma(v)$. A notable case where the computation of $\Gamma(v)$ is easy is presented in [94] where it is shown that, in the case of a linear constraint in the form $h_x^T x + h_v^T v \le h$, and for a Lyapunov function that is lower-boundable by a quadratic form, i.e., $V(x,v) \ge (x-v)^T P(x-v)$, for $P = P^{\mathrm{T}} > 0$, a feasible $\Gamma(v)$ can be computed in closed form as

$$\Gamma(v) = \frac{(h_x^T \bar{x}_v + h_v^T v + h)^2}{h_x^T P^{-1} h_x}. \tag{47}$$

Note that $\Gamma(v)$ as in (47) is optimal in the case $V(x,v) = (x-v)^T P(x-v)$. In the case of multiple constraints, $\Gamma(v)$ can be computed as the minimum of the bound computed for each constraint. Other special cases for which the computation of $\Gamma(v)$ is easy are reported in [187].

Finally, note that the approach based on (46) can be used even in the cases where disturbances are present, provided $V(x,v)$ is an ISS-Lyapunov function. In this case, there exists a value V_{ISS} such that $V(x,v) > V_{ISS}$ implies $V(f(x,v,w),v) - V(x,v) < 0, \forall w \in W$. The method (46) can be used for the disturbed system under the only conditions that ε is sufficiently large to ensure that $V_{min} \ge V_{ISS}$, with V_{min} defined as in (44).

In [100], a generalization of both prediction-based and Lyapunov function-based nonlinear reference governors applicable to constrained systems with disturbances (41)-(42) is presented. The basis of this method is to define a continuous function $S(x,v)$ so that, for any $(x(t), v)$ such that $S(x(t), v) \le 0$, if v is kept constant from t onward, the trajectory $\hat{x}(t+k|t), k \ge 0$ is

- *safe:* constraints are never violated, i.e. $h(\hat{x}(t+k|t), v) \le 0, \forall k \ge 0$;
- *strongly returnable:* there exists a finite integer k^*, which may depend on $x(t)$, such that $S(\hat{x}(t+k^*|t), v) < 0$, which means that after a finite time k^* the trajectory returns to the interior of the set $\{(x,v) : S(x,v) \le 0\}$.

With S defined, $\kappa(t)$ can be chosen at each time instant t by solving the following scalar optimization problem,

$$\begin{aligned} \kappa(t) = \max_{\kappa\in[0,1]} \quad & \kappa \\ \text{s.t.} \quad & S(x(t), v(t-1) + \kappa(t)(r(t) - v(t-1)) \le 0, \\ & h(\bar{x}_v, v) \le -\varepsilon. \end{aligned} \tag{48}$$

If no feasible solution to (48) exists, $\kappa(t) = 0$ is used. Note that the prediction-based reference governor (43) may be viewed as a particular case of (48) with

$$\begin{aligned} S(x,v) = \max_{\substack{0\le k\le k^* \\ i=1,\dots,n_y}} \quad & \{h_i(\hat{x}(t+k|t))\} \\ \text{s.t.} \quad & x(t|t) = x, \end{aligned}$$

where n_y is the dimension of the output vector. The Lyapunov function-based reference governor (46) may also be viewed as a particular case of (48) with $S(x,v) = V(x,v) - \Gamma(v)$. For such generalized nonlinear reference governor, recursive feasibility and finite-time convergence properties hold under suitable assumptions, see [100].

Again, as in the previous scheme, the main difficulty of the method is to construct the function $S(x,v)$. However it should be remarked that the fact that sublevel sets of $S(x,v)$ must be only strongly returnable rather than positive invariant (as in the case of a Lyapunov function) may simplify the computation of S. In fact for any S such that $\bar{x}_v \in \{x : S(x,v) < 0\}, \forall v$, strong returnability is ensured due to the fact that $\bar{x}_v$ is asymptotically stable (and thus attractive).

To give an example of how S can be computed, consider the case of a system for which it is possible to find a function $\delta(\epsilon)$ such that for any v, $\|x - \bar{x}_v\| \leq \epsilon$, implies $\|h(\bar{x}_v, v) - h(\hat{x}(t+k|t), v)\| \leq \delta(\epsilon), \forall k \geq 0$, then S can be built as $S(x,v) = h(\bar{x}_v, v) - \delta(\|x - \bar{x}_v\|)$. Note that although possibly quite conservative, this type of a set does not need to be based explicitly on the detailed system dynamic model and, in principle, could be constructed from intrinsic properties of the system (e.g., energy conservation principles, physics laws, etc.), or experimentally, using estimation/learning procedures.

Another approach to construct S in the disturbance-free case uses off-line simulations of the closed-loop system and generates S as a classifier so that $S(x(t), \bar{v}) \leq 0$ distinguishes safe pairs of states and constant reference commands from the unsafe ones. Machine learning techniques [175, 193] can be used to compute such a classifier from simulation data. Note that combining/aggregating several of such classifiers, $S_j(x,v)$, $j = 1, \cdots, J$, i.e., using $S(x,v) = \max_{j=1,\dots,J} S_j(x,v)$, is possible. This "union of classifiers" approach has the potential to significantly simplify the machine learning task by aggregating several locally valid classifiers [100]. In the case with disturbances, the offline computations are considerably more involved as the classification of pairs of states and constant reference commands into safe and unsafe pairs involves evaluating multiple scenarios or solving optimal control problems with respect to $w(\cdot)$.

3.4 Explicit Reference Governor

A different approach to the reference management for constrained nonlinear systems is the *Explicit Reference Governor* recently proposed in [94, 188], which represents an alternative to optimization-based solutions. The Explicit Reference Governor manages the reference of pre-compensated continuous-time systems in the form

$$\dot{x} = f(x,v), \tag{49}$$

subject to constraint (42). The main idea is to ensure constraints enforcement by continuously manipulating the reference v so that the current state $x(t)$ always belongs to a safe invariant set centered at the steady-state $\bar{x}_{v(t)}$. To build the scheme, first a Lyapunov function $V(x,v)$ and $\Gamma(v)$ as in (46) are computed. Then, the derivative of $v(t)$ is manipulated accordingly to the following static state-feedback control law

$$\dot{v} = k_\Gamma \left[\Gamma(v) - V(x,v)\right] \frac{r - v}{\max\{\|r - v\|, \varepsilon\}} \tag{50}$$

where $k_\Gamma > 0$ and $\varepsilon > 0$ are arbitrary scalars. It is possible to prove [94] that, if at time $t = 0$ an admissible $v(0)$ is known and if the reference is steady-state admissible, feasibility and convergence are ensured under the same conditions as for the other nonlinear schemes presented in Section 3.3. Variants of (50) ensuring convergence for any r are presented in [188]. Further extensions of the method can be found in [94, 186, 187]. In terms of performance, the Explicit Reference Governor is usually more conservative than the optimization-based schemes. However, it is, in general, simpler to implement and less computationally demanding.

3.5 Parameter governor

Parameter governors [154, 155, 157] are schemes inspired by the the reference governor philosophy that adjust parameters, $\theta(t) \in \Theta$, of a nominal control law so as to optimize over a finite horizon the predicted system response subject to constraints. Parameters are assumed to remain constant over the prediction horizon. The cost minimized at each time instant is of the form

$$J(t) = \|\theta\|^2_{\Psi_\theta} + \sum_{k=0}^{T} \Omega(x(t+k|t), \theta(t), r(t)), \tag{51}$$

and penalizes the system response as well as the parameter deviations. The assumption of constant parameters over the prediction horizon reduces computational and implementation effort, and simplifies the analysis. In fact, Θ may consist of a finite number of elements and so the evaluation of (51) and of the constraints reduces to a finite set of simulations.

Specific parameter governor schemes considered in [157] include the feedforward [1] governor and the gain governor. For these schemes terminal set conditions need not to be imposed to ensure stability, provided the horizon is chosen sufficiently long and in agreement with the appropriate assumptions. In the feedforward governor approach of [157], a disturbance-free system is considered with an integrator included as a part of the overall system,

$$\begin{aligned} x(t+1) &= f(x(t), u(t)), \\ x_i(t+1) &= x_i(t) + z(t) - r, \\ z(t) &= h_z(x(t)), \end{aligned} \tag{52}$$

where z is an output which is supposed to track the reference, r. The control law includes an integral action and an adjustable feedforward offset $\theta(t)$,

$$u(t) = u_e(r) - \epsilon x_i(t) + \bar{u}_{fb}(x(t), r) + \theta(t), \tag{53}$$

where $\epsilon > 0$ is a small gain, and $x_e(r)$, $u_e(r)$ denote, respectively, the equilibrium values of state and control variables corresponding to the given r. Due to the use of integral action, if θ is constant, then as $t \to \infty$ it follows, under suitable assumptions, that $z(t) \to r$, $x_p(t) \to x_e(r)$, $u(t) \to u_e(r)$. The small gain integral control leads to dynamics decomposition into slow and fast modes [60]. The fast dynamics can be made to avoid constraint violations by changing $\theta(t)$; consequently, the constraints will be satisfied if the slow manifold is well within the constraint admissible region. The feedforward governor of [157] can thus handle large reference changes and recover a large set of initial states. Note that the cost (51) is modified to include the penalty on the integral states.

3.6 Other developments

Other approaches to the design of reference governors and related schemes for nonlinear systems have been presented in the literature. Developments are reported in [79] (and earlier in [78], for linear systems) concerning nonlinear reference governors for systems subject to input constraints only. In [114] an output feedback reference governor for nonlinear systems is presented. Reference [7] presents the conditions for a nonlinear command governor where $\|r - v\|^2$ is minimized by solving at each time instant a (possibly non-convex) optimization problem. In [5] a similar nonlinear command governor is considered and compared with the reference governor based on embedding the nonlinear model into a family of Linear Time Varying polytopic uncertain systems. In the incremental reference governor approach of [227] in order to reduce the computational effort, the solution of (43) is distributed over time by checking the feasibility of a single value of $v(t)$ that differs from $v(t-1)$ by a fixed and "small enough" step size. In [151] a predictor-corrector form of Newton's method, with one iteration/update per time step, is proposed to be applied to the parameteric root-finding problem in the command governor case. Landing reference governors have been proposed for systems with terminal constraints, e.g., for reaching a desired position with a small velocity in mechanical systems [122, 146, 149]. In [215], the robust reference governor approach is introduced in the context of a fuel cell constrained control application. There, the case when disturbances $w(t)$ are constant parameters is treated, and Taylor series approximation (first order with a quadratic bound on omitted terms) is employed in the implementation to predict constraint violation.

4 Recently proposed reference governor schemes

Several special reference governor schemes and design procedures have recently been proposed. They include reduced order reference governors, virtual state governors, reference governors for fault handling, reference governors for decentralized systems, and reference governors for systems controlled through communication networks.

4.1 Reduced order reference governors

Reduced order governors [123, 131, 134] enforce the constraints on systems with a slow and a fast dynamics based on a reduced order model including only the "slow" dynamics, while accounting for the contributions of fast states and observer errors in

[1] Although it has a very similar name, the (parameter) feedforward governor presented in this section should not be confused with the (sensorless) feedforward command governor presented in Section 2.2

an implicit way. The design of these governors exploits a transformation of the system into a "slow" subsystem and a "fast" subsystem, the outputs of which are additively combined. The governor design is based on the reduced order model reflecting the dynamics of "slow" states only. The constraints are tightened and an ancillary constraint on the variation of the reference $\Delta v(t) = v(t) - v(t-1)$ is imposed to ensure that the contributions of the fast dynamics are appropriately bounded. Specifically, when the system is linear, the reduced order model output is controlled to satisfy the constraint,

$$y_r(k|t) \in Y \sim E_y, \quad \text{for all } k \geq 0,$$

while $\Delta v(t)$ is constrained to guarantee that the deviations of the fast states satisfy $x_f(t) \in E_x$, where the sets E_y and E_x are design parameters. By appropriately constraining $\Delta v(t)$ not only the contributions of fast states but also of the observer errors can be made to satisfy a given bound over the prediction horizon. Under suitable assumptions, constraint enforcement, recursive feasibility and desirable response properties, such as finite time convergence to steady-state admissible constant commands, can be guaranteed. In [162], it is shown that if the model can be represented as a second order system plus time delay, an efficient and fast implementation of the reference governor computations is possible.

4.2 Network reference governor handling variable time-delays

Reference governor-based approaches for networked control systems, where the governor and the plant are connected via a non-ideal communication network, have been initially proposed in [11,56,57,141], and more recently in [68–70]. References [11,56, 57] focus on cases when the communication between the plant and the CG is synchronized, which results in the communication network introducing delays that are multiple of the sampling period. Such delays, which can represent also packet drops when they become infinitely long, are dealt with by different types of redundancy strategies, based either on the best case assumptions, when resynchronization strategies are applied to recover in case of command delay/loss, or on the worst case assumptions, when commands that are are robust with respect to any possible sequence of data losses are selected.

References [68–70] focus on the case when the plant and the RG/CG are not synchronized by a common clock, thus resulting in asynchronous communication and the delay being real valued. In such a case, modifications to the O_∞ set construction are exploited for robustly dealing with the network induced delay. In [68–70], the command $v(t)$ is transmitted through a communication channel that has variable real-valued time delay, $\delta(t) \in [0, \bar{\delta}]$. When the delay $\delta(t)$ is smaller than the reference governor update period, T_s, the effect of the delay on the state is shown in [69] to satisfy the relation,

$$x(t+1) = Ax(t) + Bv(t) + R(\delta(t))\Delta v(t), \tag{54}$$

where $\Delta v(t) = v(t+1) - v(t)$, $R(\delta) = -\int_0^\delta e^{A_c(T_s-\tau)} B_c d\tau$, and (A_c, B_c) are the matrices of the continuous-time system model. Consequently, the effective disturbance introduced by the delay in (54) is proportional to the change in the command, $\Delta v(t)$. Thus, the network reference governor manipulates the reference change to ensure that the delay-induced disturbance does not cause a constraint violation. With some slightly conservative approximation, the network reference governor reduces to solving a quadratic program. The case of longer-than-sampling period time delay is dealt with by augmenting the design with a simple acceptance/rejection logic located at the plant site, which provides guarantees in terms of constraint satisfaction and convergence in probability of the command to the reference.

While originally developed for network control systems, the network reference governor can prove useful in any application when the time delay is time varying. Several extensions of these results are developed in [69, 70], including delays in both command and measurement channels, the output feedback case, and longer (random, possibly unbounded) delay for which a simple command acceptance/rejection logic is implemented at the plant side. A further extension developed in [70] considers slowly-varying delays with known bound on the rate of change.

4.3 Virtual state governor for integrating existing controllers

The *virtual state governor* [34] is a strategy for modular control system design that aims at integrating multiple actuators, each equipped with an assigned non-modifiable feedback control law, while enforcing constraints and minimizing the use of those actuators that are "expensive" to operate. This problem is of interest in automotive applications, for instance in cornering control [73], engine control [66, 74], energy management in hybrid powertrains [72], and in aerospace applications such as spacecraft attitude control [34].

Given a constrained plant with two actuator groups, each with a pre-assigned controller $u_1(t) = K_1(t)x(t)$, $u_2(t) = K_2x(t)$, the virtual state governor produces the "virtual states" $x_1, x_2 \in \mathbf{R}^n$, $(x_1(t)\ x_2(t)) = \kappa(x)$ that are provided to the pre-assigned controllers in place of the system state x. Thus, the virtual state governor modulates the effect of the controllers by modifying the state from which the feedback is computed. The virtual states are obtained by decomposing the actual state in a way that minimizes the usage of the expensive actuator while ensuring constraint satisfaction. Such a decomposition is computed by solving a quadratic program based on the maximal output admissible sets for the plant in closed loop with each single controller and a Lyapunov function of the loop involving the expensive actuator.

4.4 Governors and fault tolerant control

A further recent use of reference and command governor schemes is in the development of fault-tolerant systems. The main idea behind these schemes is that in the presence of constraints, after the occurrence of a fault (e.g. the loss of an actuator), a system is not able to achieve the same nominal performance that it had before the fault occurrence. Consequently, it may not be enough to reconfigure the control feedback, but also the control objectives (i.e., the reference) should be modified. Reference and command governor schemes represent a quite natural solution to this problem.

The first contributions on the subject appear to be [42, 43] where the reference-offset governor was first introduced. The reference-offset governor consists in the joint use of a command governor and a parameter governor, where the latter is the feedforward governor [157]. In this approach, the system (1) is extended with a fictitious input $\theta(t)$,

$$x(t+1) = Ax(t) + Bv(t) + B_\theta\theta(t),$$

the corresponding $\tilde{O}_\infty$, is computed and the problem

$$\begin{aligned}(v(t), \theta(t)) = \ & \arg\min_{v,\theta} \quad \|v - r(t)\|_Q^2 + \|\theta\|_{Q_\theta}^2, \\ & \text{s.t.} \quad (v, \theta, x(t)) \in \tilde{O}_\infty,\end{aligned}$$

with $Q > 0$ and $Q_\theta > 0$ is solved. The idea behind this scheme is that, after a fault event, the parameter $\theta(t)$ can be used to add an offset to the nominal system that automatically counteracts some of the effects of the fault. A similar approach has been proposed in [59]. A number of papers exploiting this idea have appeared. In [39] and [92] the reference-offset governor was used within a distributed one-master-to-many-slaves framework and was augmented with the capability to manage communication latencies between the master and the slaves. In [30] the reference-offset governor was applied in a scheme where a fault detection and identification module informs the governor of the changes of the system model due to the fault. This idea has been extended in [28, 29, 31, 230] where both the reference governor and the stabilizing feedback control are reconfigured on the basis of the identified fault. The idea of changing the model and the constraints after a fault has been formalized in [88, 89] within the framework of the hybrid command governor introduced in [83, 84].

4.5 Decentralized command governors for large scale and multi-agent systems

Several recent research efforts have been devoted to the development of *decentralized command governors* applied to systems consisting of N dynamically coupled subsystems, which are subject to local and global constraints. The initial solutions made use of the feedforward command governor approach developed in [95] that allows to reformulate the decentralized reference management problem as a static problem of determining at each decision step local references $v_i(t)$, $i = 1, \ldots, N$, such that the aggregated vector $v(t) = (v_1(t), \ldots, v_N(t))$ belongs to the static set O^ϵ and such that the variation of $v(t)$ within two update times is constrained in the set of admissible variations ΔV, i.e., $v(t+1) - v(t) \in \Delta V$. Following this philosophy, both sequential and parallel approaches have been proposed. Sequential approaches [45] are schemes where only one agent at a time is allowed to modify its command. Parallel approaches [46] are schemes where all the agents are allowed to move the command at the same time, while accounting for the worst case choices of others. This second approach has proven to work well when the aggregated command v is far from the boundary of O^ϵ, but unsatisfactorily when v is close to its boundary. For this reason, hybrid approaches switching between parallel and sequential modes have been proposed in [96] and [222]. An interesting aspect of these schemes is that, although ensuring constraints satisfaction, they may experience problems with convergence to "good approximations" of the desired reference signals $r_i(t), i = 1, \ldots, N$. In fact, as shown in [45], these schemes may experience convergence of the commands $v_i(t), i = 1, \ldots, N$ to Nash equilibria that are not Pareto-optimal. In [47], [220] and [49] this phenomenon has been carefully investigated and some algorithms to check the existence of these anomalies and to eliminate them are provided. Following the same idea of feedforward distributed command governor strategies, decentralized schemes making use of the state have been recently presented in [221], [48] and [223]. Finally, it is worth mentioning special decentralized command governor schemes [51], [213] that have been developed for the case of independent systems subject to coupling constraints. In this case, using colorability theory, it is possible to partition the agents into sets of agents that are not directly coupled by a constraint. Following this idea, a sequential scheme has been proposed where all agents belonging to the same set can move at the same time independently one from each other, while the remaning agents keep the previous command.

4.6 Other developments

Further interesting results have been presented in recent years. Among them, we mention the approaches in [216] that combine reference governors and controller switching to improve performance. A related strategy is also used in [156]. In [171] a reference governor with the added capability of resetting internal closed-loop system states to avoid constraint violation is developed. With this approach the constrained domain of attraction, i.e., the set of states recoverable without constraint violation, is enlarged. This scheme is referred to as the Controller State and Reference Governor. References [123,126,128,129] exploit contractive, rather than just invariant, sets and passivity to handle systems with time-varying reference commands and time-varying constraints and to treat the reference governor placement inside rather than outside of the control loop. The papers [190, 191] propose the so called adaptive reference governor, in which $v(t+k|t)$ is parameterized in terms of future values of the disturbance and a semi-definite programming problem is solved to achieve a less conservative design. The papers [207, 236] propose another command modification scheme to improve transient performance of an adaptive controller and to enforce constraints. Recently, in [126], a reference governor for constrained control on manifolds, and in particular on the special orthogonal group of dimension 3, $SO(3)$, has been proposed.

5 Applications

The key advantages of the governors are the capabilities of guaranteeing the enforcement of constraints with limited computational effort. Thus, the areas where these have been found most success are those involving applications with (*i*) relatively fast dynamics, (*ii*) computational platforms with limited capabilities, and (*iii*) the need to operate the system in the entire operating range, i.e., including close to the constraints. These areas include especially automotive, aerospace, precision mechatronics and factory automation, and power grids.

5.1 Automotive Applications

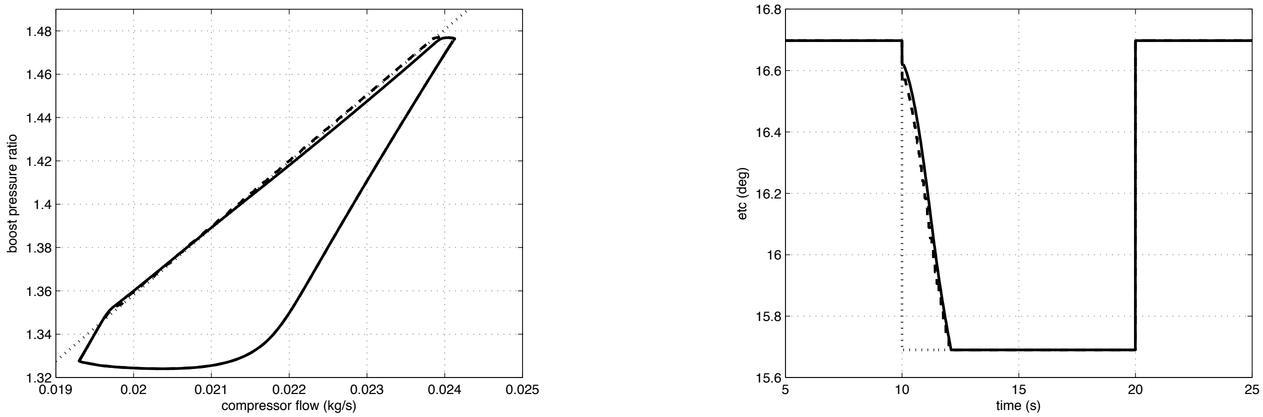
Due to constant pressure to reduce costs and satisfy increasingly more stringent regulations, (*i*)-(*iii*) above are of significant relevance in automotive applications. As a consequence, several reference governor applications have been reported in the automotive domain.

References [130], [123], [124] address compressor surge constraint handling in turbocharged gasoline engines. These engines are downsized and provide improved fuel efficiency, yet have tighter operating constraints. Reference governor techniques are applied to modify the electronic throttle (ETC) command and wastegate commands in [130] and air charge command in [123, 124]. Experimental results of reference governor implementation in a vehicle are reported in [123, 124]. To further reduce the computational footprint of the algorithm, in [131] the reduced order reference governor is applied, taking advantage of the different time scales in the engine dynamics. The validation results based on the nonlinear model and the observer for unmeasured states, see Figure 5, are presented in [131].

Other effective applications or governors for powertrain control include constraint handling in diesel engines [178–180], HCCI engines [118], free piston engines [238, 239], and the application to engine speed control [3, 147] of both reference and command governors.

For non-conventional powertrains, significant developments were pursued for fuel cells [98, 161, 215, 228], where the main constraints are on keeping the oxygen over hydrogen ratio sufficiently high to prevent oxygen starvation, on avoiding compressor surge and choke regions, and on avoiding compressor voltage saturation. The model can be up to 10^{th} order, and in [228] the procedure in Section 3.1 is exploited for compensating the mismatch between the linearized model and the actual nonlinear system. Parameter uncertainties in temperature and humidity are handled using the robust reference governor in [215]. The reference governor in [228] was implemented in the production microcontroller obtaining computation time of 1.3ms for an update rate of 10ms and requiring 4kB of ROM occupancy. For hybrid electric and full electric vehicles, reference governors were applied to handling constraints in electric batteries in [177, 212].

For chassis and vehicle dynamics control, [148] applies the reference governor and the extended command governor to vehicle roll control, by modify steering angle and operating the brakes so that constraints on the load transfer ratio, which represents a measure of roll instability, are enforced. Both scalar reference governor and extended command governor prevent rollover, the latter having higher computational burden, but much larger domain of recoverable states. As illustrated by Figures 6(a)- 6(b)

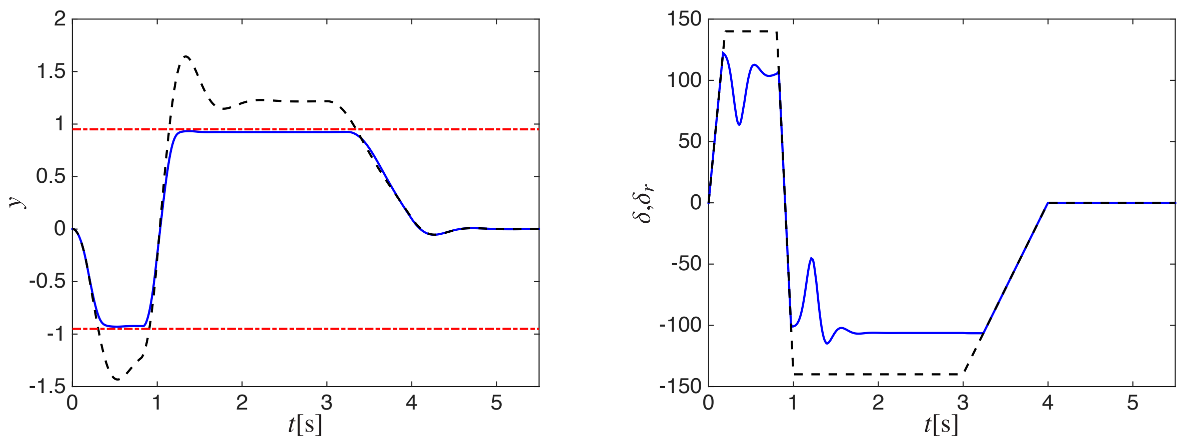


(a) Response of boost pressure ratio versus compressor flow from the nonlinear model plotted on a compressor map for the reduced order (solid) and the full order (dashed) reference governors.

(b) Throttle response from the nonlinear model for the reduced order (solid) and the full order (dashed) reference governors.

Fig. 5. Reference governor application to turbocharged gasoline engine.

ECG is robust to a mismatch between the model and actual system dynamics. Further validation results of these approaches based on nonlinear vehicle models are presented in [21].



(a) Load transfer ratio $y(t)$ with ECG designed based on $v = 22.5$ m/s model (solid) and without ECG (dashed) for a vehicle maneuver at 30 m/s. Constraints shown by dash-dotted lines.

(b) Reference steering angle $\delta_r(t)$ (dashed) and steering angle $\delta(t)$ (solid) commanded by ECG designed for $v = 22.5$ m/s, for a vehicle maneuver at 30 m/s .

Fig. 6. Extended command governor for vehicle rollover prevention

Other applications in active and passive vehicle safety include vehicle cornering and yaw stability control in [27], where reference governor is developed based on a piecewise affine system model, control of steer-by wire systems [75, 237] where a command governor with a particular cost function is used, and belt restraint systems [229].

5.2 *Precision mechatronics and manufacturing*

Precision mechatronics deals with challenges that are similar to the ones in the automotive domain, i.e., fast dynamics, platforms with low computational power, and tight operating constraints. In precision mechatronics, reference governors have been exploited for various applications of electromagnetic actuators [122, 132, 146, 149, 173]. In particular, the limited coil current leads to a force constraint expressed by a concave nonlinear function, while the soft landing constraint is of MLD type [62], and can be handled effectively using the techniques in [132]. Additional applications of reference governors to mechatronic systems include cable robots [192], disk drives [111], rotary cranes [117], and electrostatically actuated membrane mirrors [153].

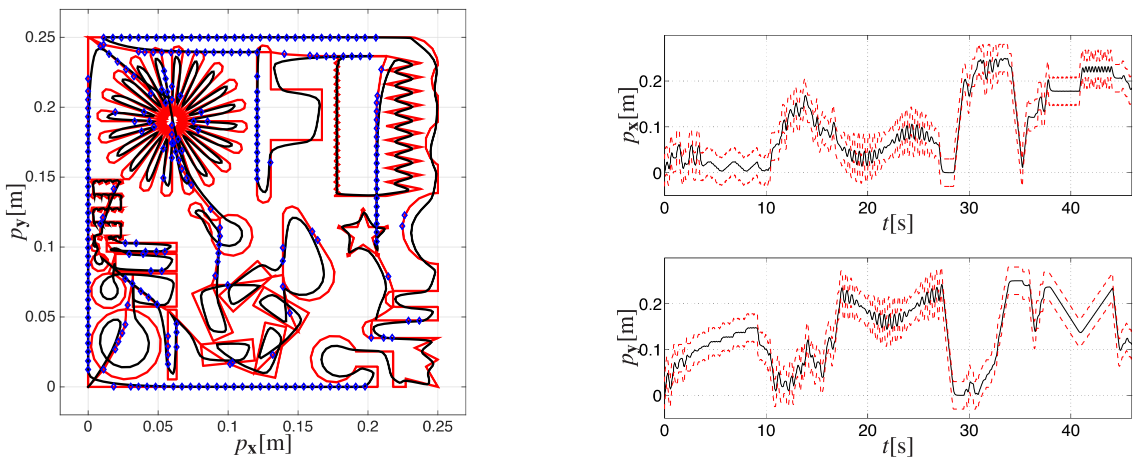


Fig. 7. Dual-stage machine motion control by reference governor and MPC. Left: spatial pattern (red), trajectory (black), points where reference governor reduces processing speed to ensure feasibility (blue diamonds). Right: trajectory (black solid) and constraints due processing point and fast stage range (red dash), i.e., $v(t) \pm s_f$.

Recently, reference governors have been applied to motion control of dual-stage processing machines for precision manufacturing [67, 112]. Dual-stage machines feature for each processing axis a fast stage, with large bandwidth and high accelerations but limited processing range, and a slow stage, with small bandwidth and limited accelerations, but large range. The two stages need to be coordinated to follow as fast as possible a given pattern, which results in a workpiece being properly processed with the worktool, e.g., a drill, a cutter, or a mill, mounted on the machine. Due to the stage dynamics time-scale separation, the problem reduces to controlling the slow stage such that the distance between its position (p_s) and the point to be currently processed (q) is smaller than the fast stage range (s_f), i.e., the reference dependent constraints $\|p_s(t) - r(t)\|_\infty \leq s_f$ must be enforced. Since the pattern cannot be changed, the reference governor selects how many points are to be processed in the next sampling period, thus choosing $v = r_i$, where $\{r_i\}_i$ are the processing points. This effectively operates a time-scaling of an ideal reference trajectory to ensure constraint satisfaction. While it can be used alone, in [67] the reference governor is used in combination with a Model Predictive Controller (MPC), which also minimizes secondary objectives such as acceleration-induced vibrations and/or energy consumption. From the previously predicted terminal state $x_{N|t-1}$, the reference governor extends the previous reference trajectory by a new point $v_{N|t}$, ensuring that for $v_{k|t}$, $i = 0, \ldots, N$ the MPC, where the terminal set constraint $(v_{N|t}, x_{N|t}) \in O_\infty$, is enforced, processes correctly the considered portion of the pattern, and is recursively feasible. Due to the reference dependent constraints, the reference governor is necessary to ensure recursive feasibility, and it also provides a feasible solution that can be used to guarantee real-time computation for processors with limited computational capabilities. The trajectory obtained from the combination of reference governor and MPC with only one step prediction horizon for a pattern designed by a real CAD-CAM system is shown in Figure 7. Thanks to the reference governor, despite the very short MPC horizon, all the constraints are enforced.

5.3 Aerospace systems

Constraints are important considerations for aerospace systems such as fighter aircraft and missiles that are open-loop unstable and may become closed-loop unstable if large commands cause the actuator amplitude or rate to saturate. The original work by Kapasouris on continuous-time reference governor [135, 136] used linearized models of fighter jet aircraft to demonstrate the benefits of continuous-time reference governor in handling actuator saturation. Furthermore, maintaining aircraft operation within its flight envelope and preventing aircraft loss of control [233] is fundamentally a constrained control problem. Finally, reference governors can enable structural/maneuver load alleviation in flexible aircraft and helicopters.

The article [194] (see also the thesis [174]) considers several schemes including discrete-time reference governor in the context of pilot command tracking control in aircraft with saturating actuators. The paper [85] (see also a conference paper [167]) investigates a command governor strategy and presents simulation results for fighter aircraft addressing magnitude and rate constraints on actuators (rudder, tailerons, canards), roll rate, side-slip angle and angle of attack. This paper also considers longitudinal constraint enforcement for small civil aircraft with actuator constraints on magnitude of aileron, rudder and elevator deflection and constraint on the angle of attack, and concludes that *"insertion of a command governor as an auxiliary device in the feedback loop involves a significant enhancement of the reliability and performance of both aircrafts."* Simulation results of the hybrid command governor on a P92 commercial aircraft model with constraints imposed on aileron, rudder and elevation magnitude and rates are reported in [90] and the command governor reconfiguration in case of faults is also considered. Another application of hybrid command governor to aircraft flight control is considered in [91]. The paper [59] exploits reference command filtering to ensure that feasible commands are provided to an MPC-based aircraft controller. In [230] reference governors, control allocation and failure mode handling are combined and in [235] reference governors are applied to flight envelope protection in high angle of attack maneuvers and a reference increment model is exploited to automate the computations. In [76], the controller state and command governor is incorporated into a larger scheme which mitigates the aircraft loss of control. In [77], the application of reference and extended command governors are considered to enforce limits on root curvature for a Very Flexible Aircraft model following a commanded flight path angle. In [120], the reference governor is applied to a glider UAV.

Applications of governors to missiles have been studied in [204] and [53]. The former paper develops a continuous-time reference governor (based on the ideas of [137], [136]) as well as the error governor [135] for a bank-to-turn (BTT) missile with saturating actuators. The latter paper considers a command governor application to a bank-to-turn missile steering problem and demonstrates that missile stability can be maintained by avoiding elevator angle saturation. The design relies on output measurements and an observer for estimating the state.

For helicopters, [36] investigates various reference governor designs. Reference [116] experimentally tests reference governors and demonstrates the ability to enforce constraints on main and tail rotor swash plate angles.

Reference governors have been also employed for the obstacle avoidance of a single [165] or of multiple [186] quadrotors and for the constraints management of tethered quadrotors [185] and kites [81]. References [183, 189] exploit explicit reference governors for related applications.

The applications of reference governors to hypersonic vehicles have been studied in [240] and [198]. In the former paper, a reference governor strategy is employed to avoid input saturation and maintain stability with an adaptive controller. In the latter paper, an extended command governor (ECG) is used in the inner (flight control) loop to enforce constraints on the angle-of-attack, pitch rate, elevator deflection, elevator deflection rate, and on elastic deformation constraints. This ECG is integrated with an MPC-based controller in the guidance loop.

Supervisory switching schemes that exploit banks of reference governors are considered in [9,10] for spacecraft relative motion control with force and torque saturation limits and box constraints on positions and orientations. The paper [132] demonstrates that nonlinear quadratic constraints on Line-of-Sight approach cone and thrust magnitude in relative motion problems can be handled by the reference governor. The extended command governor is applied to spacecraft relative motion problems in [199] and its performance is compared with an MPC-based controller. The extended command governor is also a part of the proposed scheme in [80] for landing on an asteroid. The applications of network reference governor to spacecraft constrained relative motion control in presence of communication delays and to spacecraft orientation with flexible appendage is considered in [68–70]. The parameter governor is applied to coordinated control of a spacecraft formation in [93].

In [126], a nonlinear reference governor strategy for constrained control on manifolds is introduced and applied to global spacecraft attitude control on $SO(3)$. A reference governor like strategy for spacecraft attitude control that exploits a virtual net of equilibria on $SO(3)$ and a graph search for constraint admissible transitions is developed in [232]. The virtual state governor is considered in [71] to coordinate reaction wheels and thrusters for spacecraft attitude control.

A reference governor approach to guarantee the satisfaction of critical altitude, tether tension, and angle of attack constraints in the presence of realistic setpoint variations and wind disturbance inputs for the Altaeros tethered, lighter-than-air wind energy flying turbine system is demonstrated in [133].

The control systems of aircraft gas turbine engines must handle numerous constraints such as surge avoidance, over-speed and over-temperature limits, combustion lean blowout limit, actuator magnitude and rate limits, etc. The conventional reference and extended command governors have been applied to these problems in [150] and [152]. The application of decentralized reference governors to the distributed gas turbine engine control implementation has been reported in [127], while the application of prioritized and reduced order reference governors has been considered in [226].

5.4 Power Networks

In recent years, the use of reference and command governors has been proposed in several papers to augment the existing load/frequency control and the voltage regulation of multi-area power distribution systems. Typically, these schemes consists of local regulators (usually P and PI controllers) that reject small imbalances around the nominal equilibria. Reference and command governor schemes appear to be a promising solution to enhance these control systems with the capability of managing larger transients (e.g. due to fault occurrence, sudden load changes or changes in the distributed generation), while taking explicitly into account the constraints and the inter-area connections.

For what concerns load frequency regulation, several contribution have been presented for both medium voltage and high voltage grids [39, 42, 42, 44, 218, 219]. These papers emphasize the capability of command of reference offset-governors to change the frequency of the various areas, to manage constraints, and to counteract the effect of possible faults or load/distributed production anomalies. In [92] the developed scheme is also able to take into account possible communication imperfections.

For what concerns voltage regulation, the first paper where a command governor was proposed for the voltage regulation in medium and low voltage distribution networks is probably [41]. Other papers include [40], and [217] that explore the use of a distributed command governor in a power grid with distributed generation.

5.5 Other applications

Several other applications of governors have been presented. A non-exhaustive list includes the power management system of an all-electric ship [208], chemical process control [143], water channel networks [163], wind turbines [28], tokamak reactors for thermonuclear fusion [168], and lab-scale demonstrators such as inverted pendulums [55, 94] and four-tanks systems [38].

6 Connections with other design techniques and future research topics

In this section we discuss the connections between reference/command governors and related control techniques and some potential topics for future research.

6.1 Connections with Model Predictive Control

As predictive control schemes for constrained reference tracking, governors have many common features with Model Predictive Control (MPC) and, in fact, can be designed within the MPC framework [205]. Note also that MPC controllers are often applied to manipulate set-points to lower level controllers which further diminishes the distinction between these schemes. In this sense governors can be viewed as MPC controllers with control horizon of 1 (or longer in extended command governor case), quasi-infinite constraint horizon, special cost function and applied to manipulate references. At the same time, they are special schemes with unique motivation and several unique properties, results, and simplifications (such as finite-time convergence for constant reference commands or design based on reduced constraint set) that are not easily available to more general MPC controllers.

Reference handling in MPC [1, 86, 164], tube MPC [4, 170] and reduced complexity MPC approaches in [158, 159] (arguably) incorporate features similar to reference/command governors. In particular, several control algorithms that integrate MPC with reference governing for setpoint manipulation have been recently proposed with the objective of avoiding constraint violations in transients due to setpoint changes, and enlarging the domain of attraction. Some of these strategies are based on a cascade of a reference governor with an MPC [59, 67, 82], while others, are based on a single algorithm that performs concurrently the setpoint optimization and the control of the plant in a so-called virtual setpoint-augmented MPC [86, 164, 169] that can be seen as the merging of MPC and RG in a single control scheme. Strategies that allow to shape the reference without online state feedback, and then applying MPC for tracking with guarantees that a pre-assigned bound on the tracking error is enforced have been also proposed in [63, 64].

Parameter governors proposed in [157] have similarities with the parameterized nonlinear MPC in [2]. Other techniques for reference tracking in constrained systems include [23–25].

The benefits of incorporating reference governing into MPC design to guarantee constraint feasibility and enlarge the constrained domain of attraction has been demonstrated in several applications such as spacecraft control [110, 231], and factory automation [67]. It is finally worth mentioning that a number of “special” reference governor schemes that incorporate some aspects of MPC have been proposed such as extended command governor schemes [105], and schemes mixing the use of cost function based on the reference and on the output error [214]. Continuing research into a synergistic combination of an upper level reference/command governor and lower level MPC-based controller appears to be worthwhile.

6.2 Connections with input shaping

The input shaping techniques have been proposed to minimize residual vibrations in flexible structures, see e.g. [195, 209–211]. Similar to reference/command governors input shapers modify the input to the system, however, they typically are not designed to enforce state and control constraints. The feedforward and reduced order reference governors may be suitable for problems where input shaping has traditionally been used and there are constraints. However, their properties in such applications remain to be further studied.

6.3 Other future research topics

While the subject of reference governors has been researched for over twenty years and, as discussed above, connects naturally with model predictive control and input shaping, a variety of other research directions can be identified.

The nonlinear reference governor results (see e.g., [100, 215] and references therein) extend to the case when the system has uncertain set-bounded constant parameters. At the same time, a non-conservative application of reference governors to systems with uncertain parameters being estimated online remains an area to be further explored. Special assumptions appear to be necessary in this case to guarantee recursive feasibility and other reference governor properties. Combining reference and command governors with direct adaptive controllers also remains to be studied. Resetting adaptive parameters in direct adaptive controllers can yield schemes similar to the controller state and reference governors in [171], however, the main challenge appears to be in a fast and non-conservative prediction of constraint violation for an uncertain system.

From practical standpoint, the infeasibility treatment, i.e., what policy to adopt to recover from situation where the online optimization problem does not admit solution and/or the constraints are violated, deserves further attention. Note that the infeasibility typically occurs as a result of the mismatch between the assumed model and the actual system response. The usual strategy in the scalar or vector reference governor case is to continue applying the previous reference. However, this could lead to governor "hang up", where it stops updating the reference. While this may be a reasonable approach to handle occasional infeasibility, it is more problematic if the "hang up" occurs for prolonged periods of time. The extended command governor has more degrees of freedom to find feasible solutions through optimization. However, where RG is infeasible, ECG may produce discontinuous jumps in the reference that may also not be desirable. Treating constraints as soft [125] avoids infeasibility but may lead to undesirable constraint violations, even when these can be avoided. The use of exact penalty functions [87] may avoid this, at the price of requiring the solution of particular non-smooth optimization problems.

The treatment of the case when constraints are time-varying or reconfigured dynamically is of interest for many applications. While there have been successful treatments for specific examples, see e.g., [147], the theory remains largely to be developed. For initial results in this direction, see [123, 128].

Traditionally, reference and extended command governors modify the set-points to closed-loop systems. While the theory assumes that the set-points are given, in real systems the set-points may be adjusted by a human operator in response to external conditions. This dependence of set-points on external conditions can create a feedback loop encompassing the reference governor and nominal closed-loop system. A closely related situation occurs when the governor augments control signals at the *actuator command* level. This implementation of reference governors is appealing as the design and calibration of the nominal controller can be changed without the need to re-design the governor [124, 132]. The properties of the reference governor in the loop remain to be studied. For initial results in this direction, see [129].

Error governors are schemes related to reference governors, however, they act on the tracking error at the controller input and not on the reference command. They are also primarily intended for handling control input constraints and not output constraints. See [106, 135, 225]. As compared to reference governors, error governors have received relatively little attention; obtaining convergence guarantees for them that are similar to reference/command governor has been elusive. It is interesting that the error governor can be applied with relative ease to direct adaptive controllers [121].

The treatment of uncertainties/disturbances in reference governor design deserves further attention. Currently, robust (set-bounded or ISS Lyapunov function based) treatment of the disturbances appears to be dominant. Stochastic and scenario-based approaches with respect to either reference or disturbance modeling may be pursued to reduce conservatism, similar to related developments in MPC [172].

Finally, further research into the development of real-time optimization algorithms for governors, that take advantage of the specific features of the governor law evaluation, e.g., few variables, large constraints-variables ratio, minimum norm projection form, etc., appears to be worthwhile. The detailed analysis on what algorithms are best for different governors real-time execution deserves further investigation, benchmarking, and formal proofs. This also concerns the preference for the use of QP solvers versus LP solvers in the design procedures [13].

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