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***Abstract*— For control architectures of autonomous and semi-autonomous driving features, we design a vehicle steering controller with limited preview ensuring that the vehicle constraints are satisfied, and that any piecewise clothoidal trajectory, that is possibly generated by a path planner or supervisory algorithm and satisfies constraints on the desired yaw rate and the change of desired yaw rate, is tracked within a preassigned lateral error bound. The design is based on computing a non-maximal, yet polyhedral, robust control invariant (RCI) set for a system subject to bounded disturbances with state-dependent bounds, which also allows to determine the constraints describing the reference trajectories that can be followed. The RCI set is then enforced by model predictive control, where the cost function enforces additional objectives of the vehicle motion.**

I. Introduction

For enabling new driver-assistance and autonomous features in passenger vehicles adequate control algorithms must be developed. Several features, such as lane-keeping, automated lane changing, and eventually autonomous driving, require steering control algorithms capable of tracking time-varying reference profiles, representing the desired vehicle trajectories, with guaranteed performance measures [1]. The control architectures for such features are often arranged as shown in Fig.1: a supervisory unit (SU), such as a path planner or a lane selection algorithm, produces a desired trajectory; a vehicle steering controller (VC), selects the steering control actions to track the desired trajectory; an actuator controller (AC) actuates the steering system.

For proper system operation, each controller must provide some guarantees to the others. For instance, the AC needs to guarantee that it can achieve the commands selected by the VC. The VC must guarantee a tracking error bound along the desired trajectory, so that the SU can plan a robust trajectory accounting for such potential tracking error. In general the tracking error will depend on the reference trajectory, and hence the error bounds will depend on the classes of desired trajectories, according to "agreements" such as: "The VC will ensure performance measure $\mathcal{M}$ as long as the SU generates reference trajectories satisfying property $\mathcal{P}$".

Constraints on the motion of an idealized vehicle following the SU trajectory and on the actual vehicle dynamics can be used for specifying the class of reference trajectories and the performance measure. Then, constrained control can be applied to ensure constraints satisfaction, and hence the enforcement of the agreement. However, since the performance needs to be ensured despite a limited preview of the

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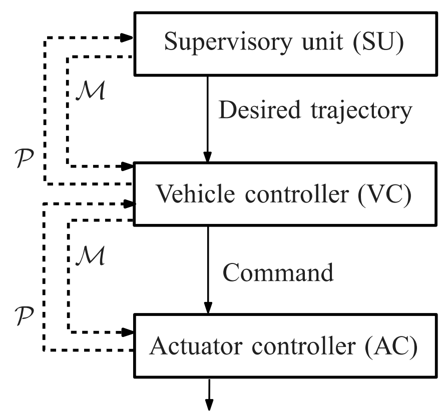


Fig. 1. Example of planning and control stack for driver assistance and automated driving features

trajectory, which can be extended in different ways as long as $\mathcal{P}$ is satisfied, robust constraint satisfaction for all reference trajectories within a certain class must be ensured.

An appealing strategy for achieving this is model predictive control (MPC), which has been shown to be effective in automotive applications [2], [3]. Robust control invariant (RCI) sets can be exploited to achieve guarantees for MPC, as done, for instance, in [4] for vehicle stability control, and in [5] for collision avoidance.

When considering steering control, the shape of the reference trajectory is related to the shape of the road. Road segments are often similar to clothoids [1], i.e., spiral curves where the curvature changes at a constant rate, consistently with a driver changing the steering wheel angle with a constant angular rate. Thus, the overall road is well represented by a piecewise-clothoidal (PWCL) curve, and here we consider PWCL trajectories with bounded curvature *and* curvature rate of change. As a consequence, the trajectories are subject to state-dependent constraints, i.e., the currently allowed range of curvature rate depends on the current curvature. For such trajectories, algorithms for maximal RCI sets [6] results in non-convex sets [7], hard to compute and hard to use for real-time control.

In this paper we propose a design for the VC steering controller based on a recently developed method [8] for constructing convex RCI sets for constrained systems tracking references with state-dependent constraints while ensuring a preassigned bound on the tracking error. The class of references is suitable for modeling PWCL vehicle trajectories, and the error bound provides a guarantee of the VC controller performance measure ($\mathcal{M}$), when the trajectories generated by the SU are in the class of trajectories satisfying the PWCL curvature and curvature rate bounds ($\mathcal{P}$). The method can be used to determine which curvature rate bounds are acceptable, and hence determines at design time the property $\mathcal{P}$ and the control algorithm that enforces the performance measure $\mathcal{M}$ in the "agreement" between the SU and VC. Actuator constraints are accounted for, to avoid negative interactions between VC and AC.

In the rest of this paper, in Section II we describe the model for the vehicle steering control, the physical and performance constraints, and we formalize the problem that we aim at solving. In Section III we compute a polyhedral non-maximal RCI set ensuring that the physical and performance constraints are recursively satisfied, and use it to determine the feasible rate of curvatures. The RCI set is used in Section IV for designing an MPC for lateral vehicle control. Simulation results on different scenarios are reported in Section V, and conclusions in Section VI.

Notation: The sets of real, nonnegative real, positive real, and integer, nonnegative integer, positive integer numbers are denoted by $\mathbb{R}$, $\mathbb{R}_{0+}$, $\mathbb{R}_+$ and $\mathbb{Z}$, $\mathbb{Z}_{0+}$, $\mathbb{Z}_+$, respectively. For $a \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, we denote the stacked vector by $(a, b) = [a' \ b']' \in \mathbb{R}^{n+m}$. Inequalities between vectors are intended componentwise. For a discrete-time signal $x \in \mathbb{R}^n$ with sampling period T_s, $x(t)$ is the value at sampling instant t, i.e., at time $T_s t$, and $x_{k|t}$ denotes the predicted value of x at sample $t+k$, i.e., x_{t+k}, based on data at sample t, where $x_{0|t} = x(t)$.

II. Vehicle Tracking: Modeling and Problem Formulation

We model the reference trajectory as generated by a particle moving at constant speed v_x along a curve with curvature $\kappa = 1/R_T$, R_T being the turn radius. This results in the model for the desired vehicle yaw, $\psi_{\rm des}$, and yaw rate, $\dot\psi_{\rm des}$,

$$\ddot\psi_{\rm des} = v_x \dot\kappa. \tag{1}$$

We sample (1) with sampling period T_s obtaining,

$$\dot\psi_{\rm des}(t+1) = \dot\psi_{\rm des}(t) + \gamma(t), \tag{2}$$

where $\gamma(t) = v_x T_s \dot\kappa(t)$ is an exogenous variable that describes the change of desired yaw rate, or alternatively, the total change of curvature of the reference trajectory during T_s scaled by the velocity v_x. Note that (2) models piecewise clothoidal trajectories where the change of curvature is constant in time periods of length (at least) T_s.

For normal (not at limit-of-performance) driving, the vehicle dynamics based on the single track model with respect to a trajectory with yaw rate $\psi_{\rm des}(t)$, as shown in Fig. 2, is described by [1], [9]

$$\dot x_e(t) = A_e x_e(t) + B_e v_e(t) + D_e d(t), \tag{3}$$

where the state vector is $x_e = [e_y \ \dot e_y \ e_\psi \ \dot e_\psi]'$, e_y, e_ψ are the lateral and yaw rate tracking errors, respectively, the input is

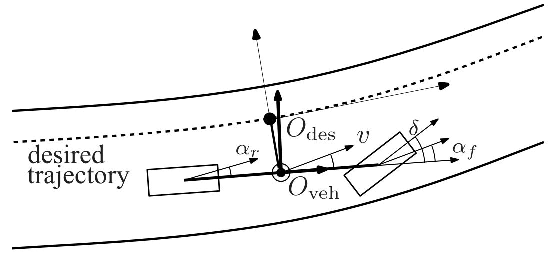


Fig. 2. Schematics of single track vehicle dynamics model, and vehicle ($O_{\rm veh}$) and desired trajectory ($O_{\rm des}$) reference frames.

steering angle $v_e = \delta$, the disturbance is $d = \dot\psi_{\rm des}$, and

$$A_e = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{C_f+C_r}{m v_x} & \frac{C_f+C_r}{m} & -\frac{C_f\ell_f - C_r\ell_r}{m v_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{C_f-C_r}{I_z v_x} & \frac{C_f-C_r}{I_z} & -\frac{C_f\ell_f^2+C_r\ell_r^2}{I_z v_x} \end{bmatrix},$$

$$B_e = \begin{bmatrix} 0 \\ \frac{C_f}{m} \\ 0 \\ \frac{C_f\ell_f}{I_z} \end{bmatrix}, D_e = \begin{bmatrix} 0 \\ -v_x - \frac{C_f\ell_f - C_r\ell_r}{m v_x} \\ 0 \\ -\frac{C_f\ell_f^2+C_r\ell_r^2}{I_z v_x} \end{bmatrix},$$

where m, I_z are the vehicle mass and inertia, C_f and C_r are the front and rear lateral tire stiffnesses, and ℓ_f, ℓ_r are the distances from the center of mass of the front and rear axles.

We formulate (3) in discrete time with sampling period T_s, define as control variable the change of the steering angle $u(t) = \delta(t) - \delta(t-1) = \Delta\delta(t)$, and add (2) resulting in

$$x(t+1) = Ax(t+1) + Bu(t) + D(d(t) + \gamma(t)) \tag{4a}$$
$$d(t+1) = d(t) + \gamma(t), \tag{4b}$$

where $x = [x_e' \ v]'$, $v(t) = \delta(t-1)$.

For ensuring that the difference between the SU trajectory and the motion of the vehicle is bounded in a preassigned range, which allows the SU to plan with appropriate safety margins, we enforce the constraints

$$e_{y_{\min}} \le e_y \le e_{y_{\max}}, \tag{5}$$

where usually $e_{y_{\min}} = -e_{y_{\max}}$. When (5) is satisfied, the vehicle position will be in a "tube" of diameter $e_{y_{\max}} - e_{y_{\min}}$ around the reference trajectory. Often, additional constraints have to be imposed on the vehicle system. In particular, the steering angle and angular rate are bounded, for both physical limitations and for safety reasons, as

$$\delta_{\min} \le v \le \delta_{\max}, \tag{6a}$$
$$\Delta\delta_{\min} \le u \le \Delta\delta_{\max}. \tag{6b}$$

Further constraints can be imposed on the state variables of (4), so that together with (5) they result in the state bounds

$$x_{e\min} \le x_e \le x_{e\max}. \tag{7}$$

Thus, (6), (7) describe the performance measure $\mathcal{M}$ that VC has to guarantee in the "agreement" between VC and SU.

Since the input and input rate are bounded, the VC will be able to achieve only a certain range of yaw rates and to react to only a certain range of changes in desired yaw rate, before violating (7), which include (4). Thus, the reference trajectory by the SU generated with dynamics (2) must enforce the constraints,

$$-d_{\max} \le d \le d_{\max}, \qquad -\gamma_{\max} \le \gamma \le \gamma_{\max}, \tag{8}$$

that, for constant v_x, can be related to bounds on the curvature, and curvature step change, i.e., $d_{\max} = v_x \kappa_{\max}$, $\gamma_{\max} = v_x T_s \dot{\kappa}_{\max}$. Note that (2), (8) define the property $\mathcal{P}$ on the reference trajectory for which the VC has to ensure (6), (7), being the performance measure $\mathcal{M}$, according to the agreement between VC and SU. While $d_{\max}$ is related to the achievable steady state yaw rate where (6), (7) are strictly satisfied, and hence to the physical vehicle constraints, $\gamma_{\max}$ determines the transients of the reference trajectories for which (5) must be guaranteed, and hence parametrizes $\mathcal{P}$.

Thus, the objective of the VC is to ensure that (4) satisfies (6), (7) for any reference trajectory satisfying (2), (8). Let $\mathcal{X}$, $\mathcal{U}$, $\mathcal{D}$ be the sets of states x, inputs u, disturbances d satisfying (6), (7) and $d \in [-d_{\max}, d_{\max}]$, respectively.

Problem 1: Given (4), and trajectories generated satisfying (2), (8), determine $\gamma_{\max}$, a (possibly dynamic) controller $h(x, x_c, d, \{\gamma_s\}_{s=0}^{N-1})$, where x_c is the controller internal state, and a set of initial error and reference states $\mathcal{S}_0 \subseteq \mathcal{X} \times \mathcal{D}$ such that (i) if at time $\bar{t}$, $(x(\bar{t}), d(\bar{t})) \in \mathcal{S}_0$, constraints (6), (7) are satisfied for any $t \ge \bar{t}$, (ii) the controller locally guarantees offset-free tracking for reference trajectories with constant yaw rate. □

As suggested in [9], for solving Problem 1 one could solve in receding horizon the finite horizon problem

$$\min_{U(t)} \quad F(x_{N|t}) + \sum_{k=0}^{N-1} L(x_{k|t}, u_{k|t}) \tag{9a}$$

$$\text{s.t.} \quad x_{k+1|t} = f(x_{k|t}, u_{k|t}, d_{k|t}) \tag{9b}$$

$$d_{k+1|t} = d_{k|t} + \gamma_{k|t} \tag{9c}$$

$$(6), (7), \tag{9d}$$

$$x_{0|k} = x(t), \ \gamma_{k|t} = \gamma(t+k), \tag{9e}$$

where $N \in \mathbb{Z}_+$ is the prediction horizon, f is a shorthand for (4), F and L are a terminal and a stage cost, and $\gamma(t+k)$, $k \in \mathbb{Z}_{[0,N-1]}$ denotes the preview on the change of the reference trajectory, $U(t) = [u_{0|t}, \ldots, u_{N-1|t}]$, and $U^*(t)$ denotes the optimal solution of (9). Then, the applied control input is $u(t) = u^*_{0|t}$. Indeed, if (9) admits a solution for any time t and for any reference trajectory $d(\cdot)$ being generated according to (2), (8), the constraints (6), (7) are always satisfied. However, it is well known [10] [5, Remark 6.1] that the recursive feasibility of (9) is not guaranteed, especially with partial preview on $\gamma(t)$.

When the future values along the entire prediction horizon of d are known and there is a feasible solution at time t, a sufficient condition for recursive feasibility is that for $x_{N|t}$, $d_{N|t}$ there exists an input u that ensures the satisfaction of (6), (7), for every $\gamma_{N|t}$ satisfying (2), (8). Next, we design additional constraints for (4) ensuring such property.

III. Invariant set design for piecewise clothoidal trajectory tracking

The property of a set being "maintainable" is formalized in the following definitions, see, e.g., [6], for more details.

Definition 1: Given $x(t+1) = f(x(t), u(t), \gamma(t))$ where $x \in \mathcal{X} \subseteq \mathbb{R}^n$, $u \in \mathcal{U} \subseteq \mathbb{R}^m$ and $\gamma \in \mathcal{G} \subseteq \mathbb{R}^d$ are the state, input and disturbance vectors, respectively, $\mathcal{C} \subseteq \mathcal{X}$ is a robust control invariant (RCI) set if

$$x_t \in \mathcal{C} \Rightarrow \exists u_t \in \mathcal{U} : f(x_t, u_t, \gamma_t) \subseteq \mathcal{C}, \ \forall \gamma_t \in \mathcal{G}, \ \forall t \in \mathbb{Z}_{0+}.$$

A set is the *maximal* RCI $\mathcal{C}^*$ in $\mathcal{X}$ if for any RCI $\mathcal{C} \subseteq \mathcal{X}$, $\mathcal{C}^* \supseteq \mathcal{C}$. The robust admissible input (RAI) set for $\mathcal{C}$ is

$$\mathcal{C}_u(x) = \{u \in \mathcal{U} : f(x, u, \gamma) \in \mathcal{C}, \ \forall \gamma \in \mathcal{G}\}.$$

When $\mathcal{G} = \{0\}$, $\mathcal{C}$ and $\mathcal{C}_u$ are called control invariant (CI) and admissible input (AI) set, respectively. □

The maximal RCI is the fixed point of the set sequence $\{\Omega^*_k\}_{k=0}$, where $\Omega^*_0 = \mathcal{X}$, and $\Omega^*_k = \textbf{Pre}(\Omega^*_k, \mathcal{G}) \cap \Omega^*_k$,

$$\textbf{Pre}(\mathcal{S}, \mathcal{G}) \triangleq \{x \in \mathbb{R}^n : \exists u \in \mathcal{U}, f(x, u, \gamma) \subseteq \mathcal{S}, \forall \gamma \in \mathcal{G}\},$$

i.e., **Pre** computes the set of states that can be moved to the $\mathcal{S} \in \mathbb{R}^n$ in one step, robustly to disturbances in $\mathcal{G}$.

For linear systems subject to polyhedral constraints and disturbances bounded in a fixed range, **Pre** is computed by solving linear programs and projections of polyhedra. However, for (4), subject to (6), (7), the disturbance γ must satisfy (8). Thus, the range of disturbances is dependent on a state component of (4), i.e., $\gamma \in \mathcal{G}(d)$. For this case, [7] showed that the maximal RCI set is the union of polyhedra and hence is in general non-convex. To this end, a different algorithm that does not compute $\mathcal{C}^*$ but rather a polyhedral RCI $\mathcal{C} \subseteq \mathcal{C}^*$ is exploited here.

To start, consider a given value for $\gamma_{\max}$. We need to construct a polyhedral RCI set for (4) where the state is (x, d), subject to (6), (7), and where the disturbance γ satisfies (8). Since (8) must always be satisfied, the following holds.

Proposition 1: For every $t \in \mathbb{Z}_{0+}$, $d(t) \in \mathcal{C}^*_d$, $\gamma(t) \in \mathcal{C}^*_\gamma(d)$, where $\mathcal{C}^*_d$ is the maximal CI set for (2) with $\gamma(t)$ as a control input, subject to (8), and $\mathcal{C}^*_\gamma(d)$ is the corresponding AI set. Furthermore, $\mathcal{C}^*_d = \{d : \ d \in [-d_{\max}, d_{\max}]\}$. □

Proposition 1 follows from the trajectories always satisfying (8), and the maximal RCI being the largest set where recursive constraint satisfaction is guaranteed. The last statement is due to (4b), (8) being a constrained integrator.

We modify the approach for tracking constrained reference trajectories in [8] for the case where the system is subject to a time-varying measured disturbance, resulting in Algorithm 1, where $\tilde{\mathcal{G}} = \{\gamma : -\gamma_{\max} \le \gamma \le \gamma_{\max}\}$.

Algorithm 1 (Computation of $\mathcal{C}$):

1: $\bar{\Omega}_0 = \{(x, d) : x \in \mathcal{X}, d \in \mathbb{R}^d\}$, $\Omega_0 = \bar{\Omega}_0 \cap (\mathbb{R}^n \times \mathcal{C}^*_d)$,
2: $k = 0$,
3: `do`
4: $\tilde{\Omega}_{k+1} = \{(x, d) : \exists u \in \mathcal{U}, (Ax + Bu + D(d + \gamma), d + \gamma) \in \bar{\Omega}_k, \forall \gamma \in \tilde{\mathcal{G}}\}$

5: $\bar{\Omega}_{k+1} = \tilde{\Omega}_{k+1} \cap \bar{\Omega}_k$
6: $\Omega_{k+1} = \bar{\Omega}_{k+1} \cap (\mathbb{R}^n \times \mathcal{C}_d^*)$
7: $k \leftarrow k+1$
8: `while` $\Omega_k \neq \Omega_{k-1}$ `and` $\Omega_k \neq \emptyset$
9: `if` $\Omega_k = \emptyset$, `fail`, `else`, $\mathcal{C} = \Omega_k$.

The key differences with the algorithm for $\mathcal{C}^*$ [6] is that in Algorithm 1 robustness is sought with respect to $\gamma \in \tilde{\mathcal{G}}$, rather than to $\gamma \in \mathcal{C}_\gamma^*(d)$, and that two set sequences are built. The sequence $\bar{\Omega}_k$ ignores the constraints on d, both in the initialization and in the iterations. This allows to keep the computation simple, yet alone will cause d to possibly grow to infinity, and the RCI set to be empty. However, the sequence Ω_k is also built, where all values of d outside $\mathcal{C}_d^*$ are removed. This results in enforcing the constraints on d and ignoring the values $d \notin \mathcal{C}_d^*$, which prevents the RCI to become empty due to the relaxation on $\bar{\Omega}_k$, by stopping the iterations if changes occur only outside $\mathcal{C}_d^*$. The set obtained from Algorithm 1 can be proved to be RCI.

Theorem 1: Consider (4) subject to (6), (7), and let $\gamma(t) \in \mathcal{C}^\gamma(d(t))$, such that (8) is always satisfied. Let the set sequence $\{\Omega_k\}_k$ from Algorithm 1 converge in a finite number of steps k^* to a non-empty set. Then, $\Omega_{k^*} = \mathcal{C}$ is a RCI set for (4) subject to (6), (7). Furthermore, $\mathcal{C}$ is a (convex) polyhedron. □

The proof for Theorem 1 are similar to those in [8], and hence omitted due to limited space.

The RCI computed by Algorithm 1 guarantees that if $(x(t), d(t)) \in \mathcal{C}$, for every admissible $d(t+1)$, i.e., for any admissible $\gamma(t)$, there exists $u(t)$ such that $(x(t+1), d(t+1)) \in \mathcal{C} \subseteq \mathcal{X} \times \mathcal{C}_d^*$, i.e., any state trajectory can be extended to satisfy constraints for any admissible reference trajectory. From Theorem 1, $\mathcal{C}$ is polyhedral, and expressed as

$$H_\infty^x [x' \ d']' \leq K_\infty^x. \tag{10}$$

Furthermore, Algorithm 1 can be modified to handle polytopic uncertainties (both multiplicative and additive) in the vehicle model (4) by simply changing the computation of $\tilde{\Omega}_{k+1}$ as shown in [6].

Remark 1: Alternative RCI constructions may be considered, which however appears unrealistic. First, one may ignore the road curvature dynamics. Thus, $d \in [-d_{\max}, d_{\max}]$ is the disturbance in (4a). While $\mathcal{C}^*$ is polyhedral, assuming that the road curvature changes arbitrarily between its minimum and maximum value is extremely conservative, thus imposing limitations on $d_{\max}$ for the RCI to exist, and hence limiting the allowed vehicle maneuvers. Second, one may ignore the constraints $d \in [-d_{\max}, d_{\max}]$. However, in this way d is allowed to grow to arbitrarily large values due to (4b), and since the vehicle steering is limited it will be impossible to track all the admissible references, resulting in an empty RCI set. Ad-hoc solutions ignoring $d \in [-d_{\max}, d_{\max}]$, and using asymptotically stable dynamics in place of (4b), must be tested by trial and error, have no physical motivation, and no guarantee to work in practice. ■

For given (4) (6), (7) and (2), (8), Algorithm 1 may fail and $\mathcal{C} = \emptyset$. This indicates that constraint satisfaction, i.e., the performance metric $\mathcal{M}$, cannot be guaranteed for all the allowed reference trajectories, i.e., the current choice for $\mathcal{P}$. Exploiting this, Algorithm 1 can be used to determine $\mathcal{P}$, by determining the set of acceptable values for $\gamma_{\max}$, i.e., $\Gamma_{\max} = \{\gamma_{\max} : \mathcal{C} \neq \emptyset\}$. Let

$$\begin{aligned} \overline{\gamma}_{\max} = \max_{\overline{\gamma}} \quad & \overline{\gamma} \\ \text{s.t.} \quad & \gamma_{\max} = \overline{\gamma} \implies \mathcal{C} \neq 0, \end{aligned} \tag{11}$$

then $\Gamma_{\max} = [0, \overline{\gamma}_{\max}]$. While apparently complicated, (11) can be solved by bisection. With reference to Fig.1, given the performance metrics $\mathcal{M}$ (by SU) and the vehicle constraints, the RCI set and the trajectory properties $\mathcal{P}$ for which $\mathcal{M}$ can be guaranteed are designed at the same time.

IV. Model predictive control for clothoidal trajectory tracking

Next, the RCI set from Section III is used for guaranteeing recursive feasibility of (9). We achieve integral action, which is practically useful in real implementations for ensuring zero steady state lateral error, by adding to (4)

$$\zeta_{ey}(t+1) = \zeta_{ey}(k) + T_s e_y(t). \tag{12}$$

Then, we define as stage cost

$$L(\xi, u) = z'Qz + u'Ru, \ z = [e_y \ \dot{e}_\psi \ \zeta_{ey}]' = E \cdot \xi, \tag{13}$$

where $Q, R > 0$, $\xi = [x' \ \zeta_{ey}]'$ is the state of (4a) augmented with the integral ζ_{ey} of the lateral error, and we remember that $u = \Delta\delta$. In (13) we do not weigh the yaw error, because it is not possible to drive both lateral error and yaw error to zero, while the vehicle is cornering. In fact, the desired yaw rate (2) is designed for a slip-less dynamics while the vehicle slips. Hence, the vehicle yaw needs to be larger than the desired yaw. However, the lateral error is the key performance measure.

The terminal cost F is designed from the solution P of the Riccati equation for (4a) with $d = 0$, (12), and weight matrices $Q_\xi = E'QE$ and R, resulting in

$$F(\xi) = \xi' P \xi. \tag{14}$$

Thus, from (4), (6), (7), (10), (12), (13), (14),

$$\min_{U(t)} \quad \xi'_{N|t} P \xi_{N|t} + \sum_{k=0}^{N-1} \xi_{k|t} E'QE \xi_{k|t} + u'_{k|t} R u_{k|t} \tag{15a}$$
$$\text{s.t.} \quad \xi_{k+1|t} = A_\xi \xi_{k|t} + B_\xi u_{k|t} + D_\xi (d_{k|t} + \gamma_{k|t}) \tag{15b}$$
$$d_{k+1|t} = d_{k|t} + \gamma_{k|t} \tag{15c}$$
$$x_{e\min} \leq x_{ek|t} \leq x_{e\max}, \ k \in \mathbb{Z}_{[1, N_{\mathrm{pv}}-1]} \tag{15d}$$
$$\delta_{\min} \leq v_{k|t} \leq \delta_{\max}, \ k \in \mathbb{Z}_{[1, N_{\mathrm{pv}}-1]} \tag{15e}$$
$$\Delta\delta_{\min} \leq u_{k|t} \leq \Delta\delta_{\max}, \ k \in \mathbb{Z}_{[0, N-1]} \tag{15f}$$
$$H_\infty [x'_{k|t} \ d_{k|t}]' \leq K_\infty^x, \ k \in \mathbb{Z}_{[N_{\mathrm{pv}}, N]} \tag{15g}$$
$$x_{0|k} = x(t), \ \gamma_{k|t} = \hat{\gamma}_t(t+k), \tag{15h}$$

where (15b) groups (4a), (12), $N_{\mathrm{pv}} \in \mathbb{Z}_{[1,N]}$ is the reliable preview horizon, i.e., the number of steps along which the changes in desired yaw rate $\hat{\gamma}_t(t+k)$ predicted at time t will coincide with the actual future changes in desired yaw rate $\gamma(t+k)$. Since the SU usually provides a long reference trajectory, often $N_{\rm pv} = N$, i.e., the entire preview is reliable, and (15g) becomes a terminal set constraint. However, for the cases where the actual future reference may be different from the predicted one, enforcing the RCI constraints (15g), with $N_{\rm pv}$ being the smallest value of k such that possibly $\hat{\gamma}_t(t+k) \neq \gamma(t+k)$, ensures that feasibility is maintained, as long as (8) always holds.

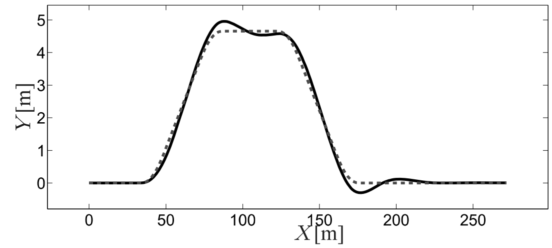
Theorem 2: Given $\gamma_{\max} \in \Gamma_{\max}$ and $N_{\rm pv} \in \mathbb{Z}_{[1,N]}$, consider the control law $u(t) = u^*_{0|t} = h_c(x(t), \zeta_{ey}(t), \{\hat{\gamma}_t(t+k)\}_{k=0}^{N-1})$, where $U^*(t) = [u^*_{0|t} \dots u^*_{N-1|t}]$ is the optimal solution of (15). For every $t \in \mathbb{Z}_+$, let $\gamma(t)$ and $\hat{\gamma}_t(t+k)$ be such that (8) is satisfied and $\hat{\gamma}_t(t+k) = \gamma(t+k)$ for every $k \in \mathbb{Z}_{[0,N_{\rm pv}]}$. Then, for all $(x(0), d(0)) \in \mathcal{C}$, (15) is feasible for every $t \in \mathbb{Z}_{0+}$ for (4) in closed-loop with h_c. □

Theorem 2 follows from enforcing the RCI set in (15) which is invariant for any trajectory satisfying $\mathcal{P}$. The details of the proof are omitted due to limited space.

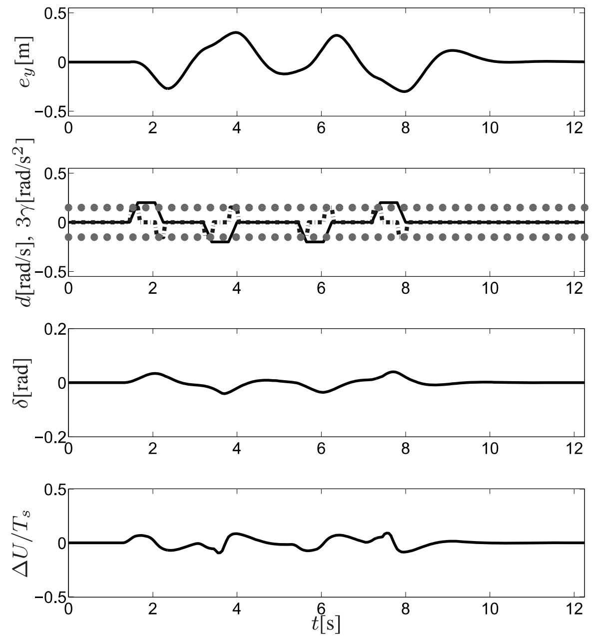
By (14), the MPC is equivalent to LQR when $d = 0$ and the constraints are inactive, which results in guaranteed local stability in an invariant set containing the maximum constraint admissible set [6] of the LQR subject to the MPC constraints. From Theorem 2, the next result follows.

Corollary 1: For any choice $\gamma_{\max} \in \Gamma_{\max}$, the corresponding $\mathcal{C}$, the controller $u(t) = h_c(x(t), \xi(t), \{\hat{\gamma}_t(t+k)\}_{k=0}^{N-1})$ and $\mathcal{S}_0 = \mathcal{C}$ in Theorem 2 solve Problem 1. □

An additional important property of the control law h_c is that (15) results in a quadratic program that can be solved even on platforms with minimal capabilities. Also, the integral action (12) and the cost function (13), (14) do not affect the enforcement of the constraints and hence the performance metrics $\mathcal{M}$, but only which trajectory is selected among those recursively satisfying $\mathcal{M}$. This reduces the tuning time, and prioritizes the "hard" performance objective $\mathcal{M}$ with respect to other "soft" objectives.



(a) Spatial path. Desired (red, dash), and actual (black, solid).



(b) Time trajectories of lateral error, desired yaw rate, steering, input (black, solid), and constraints (red, dash). Second plot from top: yaw rate change (blue, dash-dot) and constraints (magenta, dot) scaled by 3.

Fig. 3. Double lane change with maximum curvature rate of change.

V. SIMULATIONS

We use as simulation vehicle a mid-size SUV on a dry asphalt road. In the simulations the vehicle speed is 80km/h, i.e., 22.22m/s, and the discrete-time model (4a) has sampling with period $T_s = 0.05$s. The bounds in (5), (6) are $\delta_{\max} = -\delta_{\min} = 0.165$[rad], $\Delta\delta_{\max} = -\Delta\delta_{\min} = 0.420$[rad/s], and $e_{y_{\max}} = -e_{y_{\min}} = 0.3$m. In (7) we include the additional constraints $|\dot{e}_1| \leq 2.5$, $|e_2| \leq 0.25$, $|\dot{e}_2| \leq 1$, that, however, are basically always inactive in the reported simulations.

We consider as reference trajectory model (2) subject to (8), where $d_{\max} = 0.5$[rad/s] and we determine $\Gamma_{\max}$ by (11), obtaining $\overline{\gamma}_{\max} = 0.05203$. This is relatively time inexpensive since Algorithm 1 executes in less than 1 minute, as opposed to the one searching for $\mathcal{C}^*$ [7], which did not converge in 12 hours. Here, we choose to proceed with $\gamma_{\max} = 0.05$ and the corresponding $\mathcal{C}$, thus enlarging the sets of feasible trajectories on which the controller optimizes the cost function. A physical interpretation of the values of $\gamma_{\max}$ and $d_{\max}$ can be obtained in terms of corresponding maximum yaw rate, and curvature and turn radius, and corresponding rates of change that are allowed. For the values used in simulations, from $d_{\max}$ we obtain that the minimum turn radius at 80km/h is 44.44m, and from $\gamma_{\max}$ we have that while turning with 100m radius, the admissible radius rate of change is slightly above 20m/s. In all simulations, the vehicle slip angles remain in the linear region, ensuring the validity of model (4). We solved (15) by the method in [11] that in all the simulations presented next, always required less than 3ms.

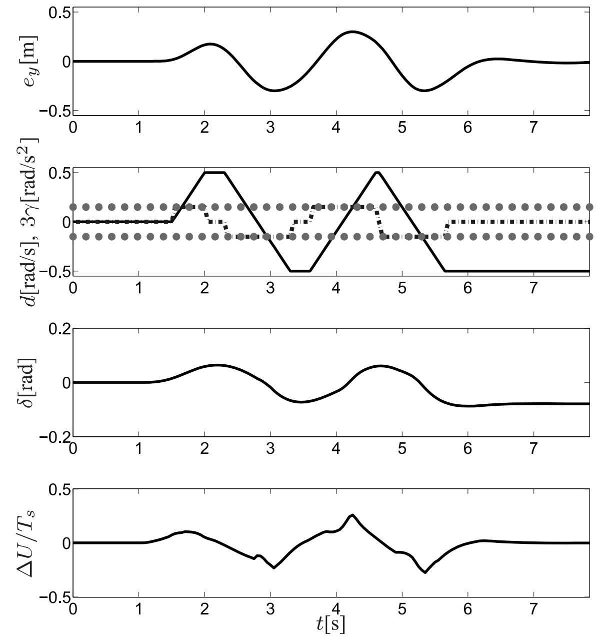
We have also verified that if the entire d is taken as the uncertainty, i.e., without introducing γ, for the given value of $d_{\max}$, $\mathcal{C}^* = \emptyset$. This occurs because d jumping arbitrarily between $d_{\max}$ and $-d_{\max}$ causes the tracking error to violate its bounds before the controller can compensate. Significantly reducing $d_{\max}$ results in a non-empty $\mathcal{C}^*$ but the allowed turn radius range is also reduced, i.e., some maneuvers allowed by the controller developed here and simulated next would no longer be admissible.

The controller h_c uses $N_{\rm prev} = N$. The simulation reported in Fig.3 shows the results for a double lane change with maximum rate of change $\gamma_{\max}$, $-\gamma_{\max}$ and where $N =$

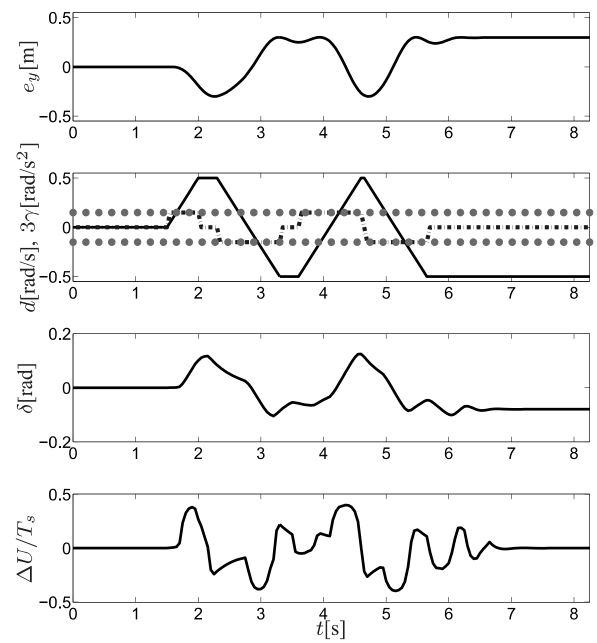
4. Indeed, it is shown in Fig.3 that the tracking constraints, as well as the steering and steering rate constraints are satisfied, despite the short horizon. Fig.4 shows a trajectory with multiple changes in turning direction, each up to the allowed maximum desired yaw rate $d_{\max}$, $-d_{\max}$, achieved with maximum rate of change, $\gamma_{\max}$, $-\gamma_{\max}$. Fig.4(a) shows the behavior for the controller with $N = 10$. The results of Fig.4(b) have been obtained with the controller implemented with $N = 2$, and tracking error weights reduced by 3 orders of magnitude, resulting in a cost function aiming at pointwise minimizing the change of steering and hence conflicting with the objective of tracking the changing reference. While obviously the tracking and actuation performance in Fig.4(b) is affected, all the constraints are still satisfied. Without using the RCI, the maneuver in Fig.4(b) is infeasible unless $N \geq 8$. Even for the initial cost function tuning, i.e., Fig.4(a), without RCI the maneuver is infeasible unless $N \geq 6$.

VI. Conclusions and Future Work

We have proposed a method to design a vehicle steering controller and to determine the parameters of the piecewise-clothoidal the reference trajectories, so that for any trajectory generated with such parameters, the tracking error is within a preassigned bound, and hence the uncertainty in path planning can be bounded accordingly. Future works will investigate thoroughly the cases when the parameters, such as vehicle velocity, are varying and uncertainties affect the vehicle dynamics, and will integrate the steering control with a previously designed path planner method [12] that can generate constrained PWCL reference trajectories according to the parameters determined determined together with the steering control law.



(a) MPC with long horizon and good tuning



(b) Controller with short horizon and bad tuning.

Fig. 4. Simulation of a multiple rapid left-right turns achieving maximum yaw rate and transitioning with maximum rate of change. Time trajectories of lateral error, desired yaw rate, steering, input (black, solid), and constraints (red, dash). Second plot from top: yaw rate change (blue, dash-dot) and constraints (magenta, dot) scaled by 3.

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