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Kalabic, Uros; Chiu, Michael

TR2020-129 September 25, 2020

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Intelligent Transportation Systems Conference

Mitsubishi Electric Research Laboratories, Inc.
201 Broadway, Cambridge, Massachusetts 02139

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Uroš Kalabić Michael Chiu

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I. Introduction

Ridesharing has revolutionized the mobility landscape in the past decade [1], [2]. It has provided a novel way for people to access mobility, but it has also caused problems [3] that have proven difficult to regulate [4], [5].

Different locales have resorted to varying levels of regulation, with some banning the practice of ridesharing outright. However, even detractors must admit that ridesharing does result in possible benefits to society; see, *e.g.*, [6], [7]. The creation of a novel method of transportation has led to new possibilities in regulating mobility [7], [8] and, more importantly, has led to an increased access to the same. This increased accessibility is both a benefit and a drawback, as it leads to the undesirable phenomenon of increased demand for unproductive transportation, *i.e.*, *deadheading* [9]; rideshare vehicles without passengers do not directly serve any useful purpose. Without regulation, it is possible that this phenomenon could be alleviated with the advent of autonomy [10], *i.e.*, for example, autonomy could improve efficiency by causing a decrease in the amount of ridershare drivers competing for a fare. However, the problem of unproductive demand would still not be resolved and this presents an opportunity: Properly regulated, it is possible to gain the advantages of ridesharing and remove its drawbacks.

In this work, we put forth a novel scheme to regulate ridesharing within a city. Our scheme differentiates between productive and unproductive demand for mobility that is caused by ridesharing by comparing the actual cost of ridesharing to the cost of a service that would not result in its negative externalities.

What we propose is a cap-and-trade scheme that credits rideshare service providers, also called transportation network companies (TNCs), a reasonable amount for the allowable social cost of transporting a passenger, and debits the actual social cost of transporting that passenger. We aim to allow the trade of social cost between TNCs so that one in surplus could cover another's deficit. In cases where a single TNC dominates a market, credits and debits may still be traded on the open market amongst non-TNCs.

Cap-and-trade is a market-based mechanism regulating limits and allowances of some market externality. An example of cap-and-trade is the trading of harmful emissions [11], which are widely recognized as a negative externality. The first implementation of cap-and-trade was to the reduction of SO_2 emissions, and it resulted in a 36% reduction in SO_2 emissions between 1990 and 2004, a period in which electricity from coal-fired power plants increased by 25% [12]. In cap-and-trade systems, the regulatory authority imposes a limit, or cap, on the total amount of some negative by-product of an economic activity and issues each participant an allowance, which corresponds to the participant's share of the limit. A participant can acquire additional allowance only through trade, typically done on regulated exchanges. In this way, the mechanism ensures that the limit is kept constant without the need for additional regulation.

Our method is slightly different from a conventional cap-and-trade scheme, in that the allowance is not fixed, but continuously generated according to demand. This is because the intent of our approach is not to limit ridesharing activity, but to ensure that it is performed responsibly. We therefore propose that the amount of credit be the social cost of a rider's selfish route in a world without ridesharing, since it is plausible to expect that riders would exhibit selfish routing behavior if they were to somehow transport themselves from their stated origin to destination without ridesharing. A consequence of our scheme is that it incentivizes increased density in forms of transportation; since negative externalities are caused by vehicles in the form of increased demand on road capacity, these same externalities could be alleviated by increasing the number of passengers per vehicle, *i.e.*, *carpooling*.

The literature has considered cap-and-trade in the context of transportation. For example, [13] shows how such a scheme can lead to a social optimum at equilibrium with many homogenous participants; [14] considers the multi-modal case; [15] extends this work past the consideration

This work was supported by Mitsubishi Electric Research Laboratories.

U. Kalabić is with Mitsubishi Electric Research Laboratories, Cambdridge, MA 02139, USA kalabic@merl.com

M. Chiu is with the Department of Computer Science, University of Toronto, ON M5S 2E4, Canada chiu@cs.toronto.edu

of travel time to the consideration of harmful emissions; other examples include the review in [16] and more recent work in [17], [18]. The commonality in these approaches is that they pursue quantity management by congestion-pricing, which was originally identified as beneficial by the economist Pigou [19]. They do this to avoid issues around re-distribution of revenue that are caused by congestion-pricing and, more importantly, issues around drivers' insensitivity to marginal pricing. Nevertheless, quantity-based approaches are not without drawbacks; see, *e.g.*, [20]. One issue is the question of who gets to decide correct quantity. Our scheme avoids this problem by allotting the quantity of credits according to stated demand for transportation. Since demand is not directly coupled to route determination, we avoid gamefication that can occur in quantity-based approaches [21].

To our knowledge, the literature has not made important practical considerations of how these credits could be traded. Motivated by the conclusions of [22], which shows that a company's marginal pricing sensitivity is more stable since it is driven by considerations of the balance sheet, and the conclusions of [23], which shows that pricing sensitivity varies greatly amongst drivers, we posit that TNCs, being driven by business considerations, would form a more efficient and stable market as compared to a large group of drivers.

The main contribution of this work is the introduction of a novel mobility regulation scheme. We provide both theoretical analysis and a discussion of practical implementation. Our analysis is performed on a static network, *i.e.*, a traffic network at equilibrium. Although simplified, this model is useful in showing that the cap-and-trade system we propose results in incentivizing denser forms of transportation on roads, such as bus services, reversing a trend caused by ridesharing [24]. This is highly encouraging, since bus services, even though they are a more efficient form of transportation, are often deemed not economically viable, even if this conclusion is often arrived at by considering only a narrow set of parameters [25]. In addition to our analysis, we provide a description of the online algorithm that determines the credits and debits to a TNC. The determination of credits is done in simulation, with a simulator determining the selfish routes of riders moving from origin to destination in a virtual world where ridesharing, along with its positivies and negatives, is absent. We note that the scheme ensures that individual riders' time-preferences are included in the social cost, so that TNCs have a responsibility to provide a guarantee that passengers' time is not being wasted. We also note that, in this work, our focus is on the situation where there are no privately-owned cars; we expect that it would not be too difficult to modify the scheme to include them and provide a discussion on how this could be done.

The paper is structured as follows. Section II motivates the work with examples. Section III presents the scheme in a static, non-atomic scenario and presents some analysis. Section IV presents a method of implementing the scheme in reality. Section V is the conclusion.

Notation: We use conventional mathematical notation. The sets $\mathbb{R}_+$ and $\mathbb{Z}_+$ represent the sets of non-negative real and integer numbers, respectively. For a directed link i, its startpoint is the node $\sigma(i)$ and its endpoint is the node $\tau(i)$.

II. MOTIVATION

We begin by motivating our scheme. We consider the static behavior of traffic, modeled as a static game on a network. Let $\mathcal{G} = \mathcal{V} \times \mathcal{E}$ be a directed graph where $\mathcal{V}$ is the set of nodes or vertexes and $\mathcal{E}$ is the set of directed links or edges. Let $x : \mathcal{E} \to \mathbb{R}_+$ be the vector representing the flows on each link and let $d : \mathcal{E} \to (\mathbb{R}_+ \to \mathbb{R}_+)$ be the vector representing delay functions on each link, where each delay function $d_i : \mathbb{R}_+ \to \mathbb{R}_+$ is piecewise-continuous, positive and nondecreasing, *i.e.*, $d_i(\xi + \eta) \geq d_i(\xi) > 0,\ \forall i \in \mathcal{E},\ \forall \xi, \eta \in \mathbb{R}_+$.

In the static case, traffic achieves a user equilibrium, resulting from the fact that all individuals follow the best route that is available to them. The user equilibrium is given as the solution to the following optimization problem [26],

$$\min_{x} \sum_{i \in \mathcal{E}} \int_0^{x_i} d_i(s) \mathrm{d}s, \tag{1a}$$

$$\text{sub. to } Ax = b, \tag{1b}$$

$$0 \leq x \leq \bar{x}. \tag{1c}$$

Here, the matrix $A : \mathcal{G} \to \{0, \pm 1\}$ represents the graph topology and satisfies,

$$A_{ji} = \mathbb{I}_{j=\sigma(i)} - \mathbb{I}_{j=\tau(i)}, \tag{2}$$

where $\mathbb{I} \in \{0, 1\}$ is an indicator function. The vector $b : \mathcal{V} \to \mathbb{R}_+$ represents the exogenous flows into or out of each node, and is balanced so that $\sum_{j \in \mathcal{V}} b_j = 0$. The vector $\bar{x} : \mathcal{E} \to \mathbb{R}_+$ represents the capacity limit on each link and is strictly positive.

The total delay in the system, termed the social cost, is given by the expression,

$$\sum_{i \in \mathcal{E}} x_i d_i(x_i). \tag{3}$$

In this study, we are concerned with how TNCs can be prevented from worsening the social cost. We begin with the realization that ridesharing can both improve and impair traffic function. Consider that, when a rideshare passenger requests transportation, he creates additional demand for *deadheading*, *i.e.*, transportation from the current location of his rideshare vehicle to the pick-up location. Deadheading creates additional traffic delay without providing productive transportation to satisfy actual demand. Conversely, ridesharing has the potential to improve traffic by allowing *carpooling*, *i.e.*, improving efficiency by allowing multiple riders to share a vehicle. In this way, the flow of traffic can be improved by reducing the number of vehicles servicing a given demand for transportation.

We propose to quantify the positive effect of ridesharing by solving (1) for the expected distribution x of flow at user equilibrium and measuring the social cost (3) thereof in the absence of ridesharing, and relating this to the actual

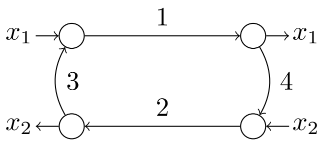

Fig. 1. An example of a traffic network

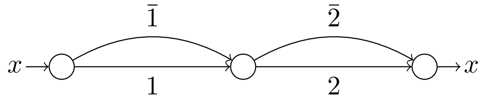

Fig. 2. Another example of a traffic network

cost in the presence of ridesharing. Let $x^* : \mathcal{E} \to \mathbb{R}$ be the minimizer of (1) and let x^R be the actual vector of flows. The actual flow x^R consists of individual vehicles, but individual vehicles could have more or less than one passenger, so we introduce a weighted distribution of vehicles, given in the following definition.

Definition 1. Let $x^{R,n}$, $n = 0, 1, 2, \ldots$, be the distribution of routes of ridesharing vehicle carrying n passengers so that $\sum_{n=0}^{\infty} x^{R,n} = x^R$. A *weighted distribution* of ridesharing vehicles is given by,

$$\tilde{x}^R := \sum_{n=0}^{\infty} n x^{R,n}. \tag{4}$$

To obtain the improvement in social cost, we take the difference between the social cost of the flow x^* at user equlibrium and the social cost of the actual flow x^R,

$$\Delta S := \sum_{i \in \mathcal{E}} \tilde{x}_i^R d_i(x_i^R) - x_i^* d_i(x_i^*). \tag{5}$$

Note that ΔS is the difference in social cost over all passengers, the aggregate of which is represented by the weighted distribution $\tilde{x}^R$, and whose experienced delay is based on the actual distribution of flow x^R. A negative additional cost ΔS implies a net improvement of social cost with the use of ridesharing.

Example 1. Consider the network of Fig. 1. Assume $x_1 > x_2 > x_1/2$ and that delays are equal over all links, and consider three cases: no ridesharing, ridesharing without carpooling, and ridesharing with carpooling and two passengers to a car, *i.e.*, $\tilde{x}^R = x^R/2$. In the first case, the flow is at user equilibrium, *i.e.*, $x_1^R = x_1$, $x_2^R = x_2$, $x_3^R = x_4^R = 0$, and $\tilde{x}^R = x^R$. In the second case, the flows along all edges are x_1 because all x_1 vehicles must return to pick up passengers while satsisfying demand and, since $x_1 > x_2$, the vehicles can only satisfy the transportation demand of the flow if $x_2^R = x_1$; the weighted flows are $\tilde{x}_1^R = \tilde{x}_2^R = x_1$ and $\tilde{x}_3^R = \tilde{x}_4^R = 0$. In the third case, the flow along all edges is $x_1/2$ because each vehicle carries two passengers, and the weighted flows are $\tilde{x}_1^R = \tilde{x}_2^R = x_1$ and $\tilde{x}_3^R = \tilde{x}_4^R = 0$. Therefore, in the second case, ridesharing results in an increase in social cost,

$$\sum_{i=1}^{4} \tilde{x}_i^R d_i(x_i^R) - x_i^* d_i(x_i^*) = x_2(d_2(x_1) - d_2(x_2)) > 0.$$

In the third case, ridesharing results in a decrease in cost: $x_1(d_2(x_1/2) - d_2(x_1)) + x_2(d_2(x_1/2) - d_2(x_2)) < 0$. □

Public transportation: We consider the effects that a cap on social costs will have on public transportation. Public transportation can be modeled as a network with effectively unlimited capacity and constant delays. Since public transportation is often slower than ridesharing, all else being equal, selfish behavior exhibits a preference for ridesharing which, due to the sensitivity of roads to density, causes time spent in traffic to increase relatively rapidly with the addition of riders. In the following example, we consider the effect of limiting social costs and how this can incentivize the use of public transportation.

Example 2. Consider the network in Fig. 2. Assume that ΔS is capped at zero, *i.e.*, $\Delta S \leq 0$. Let $d_{\bar{1}} < d_1(0)$ and $d_{\bar{2}} > d_2(x)$. Assume that the flow x is serviced by vehicles with exactly one passenger. At user equilibrium, vehicles utilize link i until $d_i(x_i) > d_{\bar{i}}$. Since link 1 is already oversaturated at $x = 0$, *i.e.*, $d_1(0) > d_{\bar{1}}$, and since link 2 is undersaturated at full flow, *i.e.*, $d_2(x) < d_{\bar{2}}$, the flow at user equilibrium is given by $x_1^* = 0$, $x_{\bar{1}}^* = x$, $x_2^* = x$, and $x_{\bar{2}}^* = 0$. The social cost is equal to $x d_{\bar{1}} + x d_2(x)$.

Let x^R be the actual flow in the network and $\tilde{x}^R$ be the weighted flow. The social cost is equal to $\sum_{i=1,2} \tilde{x}_i^R d(x_i^R) + (x - \tilde{x}_i^R) d_{\bar{1}}$, so that the additional social cost is $\tilde{x}_1^R(d(x_1^R) - d_{\bar{1}}) - \tilde{x}_2^R(d_{\bar{2}} - d(x_2^R)) + x(d_{\bar{2}} - d_2(x))$.

Note that, if $x_2^R = \tilde{x}_2^R = x$ as at user equilibrium, then the constraint $\Delta S \leq 0$ requires that $\tilde{x}_1^R(d(x_1^R) - d_{\bar{1}}) \leq 0$, which implies that $x_1^R = 0$. Therefore, to provide service on link 1, the TNC is forced to make a tradeoff by decreasing in proportional amount the service on link 2 and introducing carpooling because it must satisfy,

$$\tilde{x}_2^R(d_{\bar{2}} - d(x_2^R)) - \tilde{x}_1^R(d(x_1^R) - d_{\bar{1}}) \geq x(d_{\bar{2}} - d_2(x)),$$

and $\tilde{x}_2^R = x_2^R = x$ does not satisfy this requirement for any $\tilde{x}_1^R > 0$.

Effectively, the TNC is required to, at some point, suspend service on a segment of the network and perhaps even incentivized to suggest public transportation as an alternative. □

Example 3. Consider the network in Fig. 2 with the same assumptions as in Example 2. With Draconian intention, a designer could violate positivity of the weights $d_{\bar{i}}$ and modify $d_{\bar{1}}$ and $d_{\bar{2}}$ to both be zero, and thus prevent ridesharing on the network altogether. □

III. Cap-and-Trade for Ridesharing

Having presented the benefits of a scheme where additional social cost of ridesharing is limited, we present a mechanism to impose such a limit.

The mechanism relies on a simulation that operates a network in which entities seek to minimize individual travel times. The cost of activity on this network is treated as a maximum allowable social cost, and is equal to the social cost under the assumption of maximum selfishness and non-existence of ridesharing.

TNCs are required to ensure that the aggregate social effect of their activity stays below the aggregate social effect of activity according to the virtual simulation. This means that, although TNCs are allowed to, in determining routes, surpass the social cost of equivalent virtually-simulated routes, at some point they would have to balance the books to ensure that, on average, their effect on traffic is not net-negative.

In the static case, the marginal social cost of a route p^* in the virtual world, credited to a TNC, is given by,

$$\sum_{i \in p^*} d_i(x_i^*) + x_i^* d_i'(x_i^*), \tag{6}$$

where x_i^* is the solution to (1) and d_i' is the right-hand derivative of d_i.

The cost debited from a TNC is the social cost of actual routes. For a single route p, this is given by,

$$\sum_{i \in p} n_i d_i(x_i^R) + \tilde{x}_i^R d_i'(x_i^R), \tag{7}$$

where n_i is the number of passengers in the vehicle on link i.

Proposition 1. *Let x^R be a static distribution of rideshare routes and let $\tilde{x}^R$ be the corresponding, weighted distribution. The cost credited to the rideshare service provider is given by,*

$$C^* := \sum_{i \in \mathcal{E}} x_i^* d_i(\tilde{x}_i^*), \tag{8}$$

where x_i^* is the solution to (1). *The cost debited from the rideshare service provider is given by,*

$$B^* := \sum_{i \in \mathcal{E}} \tilde{x}_i^R d_i(x_i^R). \tag{9}$$

Proof: Integrating the incremental credit (6) over link i and summing over all links, we obtain the required result (8).

Integrating the incremental debit (7) over link i, we obtain,

$$B_i := \sum_{n=0}^{\infty} n \int_{S_i^{n+1}}^{x_i^R} \tilde{x}_i^{R,n} d'(s) \mathrm{d}s + n \int_{S_i^n}^{S_i^{n+1}} \left(d(s) + (s - S_i^n)\, d'(s)\right) \mathrm{d}s,$$

where $S_i^n = \sum_{k=0}^{n-1} \tilde{x}_i^{R,k}$. Solving the integrals, we obtain $B_i = \sum_{n=0}^{\infty} n\tilde{x}_i^{R,n}(d(x_i^R) - d(S_i^{n+1})) + n\tilde{x}_i^{R,n} d(S_i^{n+1}) = \tilde{x}_i^R d_i(x_i^R)$. Summing over all links, we obtain the required result. ☐

The result ensures that a TNC must either improve routing or increase the number of riders to a vehicle. We show this in the following corollary.

Corollary 1. *Assume $B^* \le C^* < \sum_{i\in\mathcal{E}} \tilde{x}_i^R d_i(\tilde{x}_i^R)$. Then $\tilde{x}_i^{R,n} > 0$ for some $n \ge 2$.*

Proof: By assumption, $\sum_{i\in\mathcal{E}} \tilde{x}_i^R d_i(\tilde{x}_i^R) > \sum_{i\in\mathcal{E}} x_i^* d_i(x_i^*) \ge \sum_{i\in\mathcal{E}} \tilde{x}_i^R d_i(x_i^R)$. Therefore there exists $i \in \mathcal{E}$ such that $\tilde{x}_i^R d_i(\tilde{x}_i^R) > \tilde{x}_i^R d_i(x_i^R)$. Therefore, by monotonicity of d_i, $\sum_{n=0}^{\infty} n\tilde{x}_i^{R,n} = \tilde{x}_i^R > x_i^R = \sum_{n=0}^{\infty} \tilde{x}_i^{R,n}$, implying that $\sum_{n=2}^{\infty} n\tilde{x}_i^{R,n} > 0$. ☐

The corollary shows a key effect of our scheme: it incentivizes denser, more efficient forms of transportation.

We next consider the existence of routing that satisfies a debt-credit balance $B^* \le C^*$. Note that a TNC must ensure a balance of all vehicles on the network while satisfying travel demand. Mathematically, we express this requirement with the following constraints,

$$\tilde{x}^R = \sum_{n=0}^{\infty} n\tilde{x}^{R,n},\ x^R = \sum_{n=0}^{\infty} \tilde{x}^{R,n}, \tag{10a}$$

$$A\tilde{x}^R = b,\ Ax^R = 0, \tag{10b}$$

$$0 \le \tilde{x}^{R,n},\ 0 \le x^R \le \bar{x}, \tag{10c}$$

where (10b) represents the demand and balance constraints, respectively. Below, we provide an existence result which ensures that this requirement is satisfied.

Proposition 2. *Assume that there exists a solution x^+ to the optimization problem* (1) *and that there exists a solution x^- to the same problem where the constraint* (1b) *is replaced by $Ax = -b$. Then there exists a finite sequence $\tilde{x}^{R,0}, \tilde{x}^{R,1}, \ldots$ satisfying* (10) *and $B^* \le C^*$.*

Proof: Choose a positive fraction $Q \le 1$ satisfying $x_i^+ d_i(Q(x_i^+ + x_i^-)) \le x_i^+ d_i(x_i^+)$ and $Q(x_i^+ + x_i^-) \le \bar{x}_i$ for all $i \in \mathcal{E}$. Let $x^R = Q(x^+ + x^-)$ so that $Ax^R = 0$ and $0 \le x^R \le \bar{x}$, making x^R feasible.

Let $n_1, n_2, \ldots, n_K \ge 1$ be a sequence of K unique integers and let $a_1, a_2, \ldots, a_K > 0$ be a sequence that sums to one and satisfies $\sum_{k=1}^K a_k/n_k = Q$. Let $\tilde{x}^{R,n_k} = a_k/n_k x^+$ and let $\tilde{x}^{R,n} = 0$ for all $n \ge 1$ that are not members of the integer sequence. Then $\tilde{x}^R = \sum_{n=0}^{\infty} n\tilde{x}^{R,n} = \sum_{k=1}^K n_k a_k/n_k x^+ = x^+$ implying that $\tilde{x}^R$ is feasible.

Let $\tilde{x}^{R,0} = Qx^-$, so that $\sum_{n=0}^{\infty} \tilde{x}^{R,n} = \tilde{x}^{R,0} + \sum_{n=1}^{\infty} \tilde{x}^{R,n} = Qx^- + \sum_{k=1}^K a_k/n_k x^+ = Q(x^+ + x^-) = x^R$ and $\sum_{i\in\mathcal{E}} \tilde{x}_i^R d_i(x_i^R) = \sum_{i\in\mathcal{E}} x_i^+ d_i(Q(x_i^+ + x_i^-)) \le \sum_{i\in\mathcal{E}} x_i^+ d_i(x_i^+)$. Note that x^+ is the user optimal solution so $B^* \le C^*$ is satisfied. ☐

The result implies that, under the non-atomic assumption, we can arbitrarily increase density to ensure that the social cost is bounded, further strengthening our conclusion that the scheme incentivizes denser forms of transportation.

IV. Implementation

In this section, we consider the real-time implementation of our scheme. Thus far, our analysis has considered a non-atomic model for traffic. Since real traffic is atomic, *i.e.*, cars can be counted and have a finite number, we introduce new notation that replaces flow by count. Let $X : \mathcal{E} \to \mathbb{Z}_+$ be the vector representing the count on each edge and let $\hat{X}$ denote the corresponding count in simulation. Let $D : \mathcal{E} \to (\mathbb{Z}_+ \to \mathbb{Z}_+)$ be the vector representing discrete delay functions on

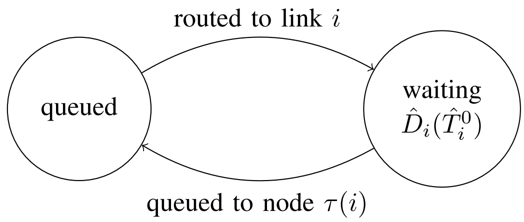

Fig. 3. States of a transportation request

each edge, where each delay function $D_i : \mathbb{Z}_+ \to \mathbb{Z}_+$ is positive and non-decreasing.

A. Credits

The procedure for determining a credit is as follows. When a rider declares an origin and destination, the request is sent to the TNC and the TNC decides whether or not to provide service. After initiating service, the request is put into a simulator queue.

As illustrated in Fig. 3, a transportation request in the simulator has two states: queued and waiting. When queued at node j, the request is routed to a link i satisfying $\sigma(i) = j$ according to the minimum-distance path and transitioned into the waiting state for a delay $\hat{D}_i(\hat{T}_i^0) := D_i(\hat{X}_i(\hat{T}_i^0))$, which is equal to the delay on the link i at the current time $\hat{T}_i^0$. After the duration of the delay, the request is placed back into the queue at node $\tau(i)$ and the process repeats until the request reaches the destination.

Credit for each link is earned after the completion of the delay and it is equal to the sum of the passenger's own delay and the amount of delay caused others,

$$C^{(i)} := \hat{T}_i^1 - \hat{T}_i^0 + \hat{X}_i(\hat{T}_i^1)(D_i(\hat{X}_i(\hat{T}_i^1)) - D_i(\hat{X}_i(\hat{T}_i^1) - 1)), \quad (11)$$

where $\hat{T}_i^1$ is the time at exit and is equal to the time at entry $\hat{T}_i^0$ plus the delay at that time, which is deterministic, and therefore given by $\hat{T}_i^1 = \hat{T}_i^0 + D_i(\hat{X}_i(\hat{T}_i^0))$.

The total credited for a virtual route p^* is $\sum_{i \in p^*} C^{(i)}$.

B. Debits

The procedure for determining debits is similar to determining credits, the main difference being that determining debits is based in reality whereas determining credits is based in simulation.

Debit for each link is incurred after a passenger exits a link and it is equal to sum of the passengers' own delay and the amount of delay caused others,

$$B^{(i)} := n_i(T_i^1 - T_i^0) + X_i(T_i^1)(D_i(X_i(T_i^1)) - D_i(X_i(T_i^1) - 1)), \quad (12)$$

where n_i is the number of passengers in the vehicle on link i.

The total debited for a route p is $\sum_{i \in p} B^{(i)}$.

C. Simulator

To ensure consistency and improve predictability, the simulator algorithm is deterministic. The simulator consists of two components: an integrator and a router. The integrator keeps track of states, implements delays, and performs state transitions. The router determines minimum-path routes for any request. The integrator dynamics have been explained in Section IV-A, where it was shown that the delays are purely determined at the time of routing.

The routing is done by following the minimum-cost path. To do this, the router regularly updates the solution to the all-pairs shortest path problem as new requests for transportation are initiated or completed. The problem can be solved using a dynamic algorithm and a fast procedure to do this is presented in [27].

D. Cap-and-trade

The issuance of credits and debits can be done by a central service such as a government entity acting through a clearinghouse. Unlike conventional cap-and-trade schemes, however, this needs to be done on demand to match the creation of credits $C^{(i)}$ and debits $B^{(i)}$. For this purpose, transactions need to be handled on an almost continuous basis, as soon as a real vehicle exits a link or a virtual vehicle enters one. Similar to conventional schemes, the cap can be enforced by the imposition of an onerous penalty if it is exceeded. To reduce cyclic effects, the central service could, for example, require the repayment of debts issued at the end of each week or month.

Credits can be traded between TNCs indirectly on the open market. TNCs that are in deficit can purchase credits from the market and sell credits to the market if they are in surplus. The market always exists as a source of supply and demand for credits; when the number of TNCs is low, as is usually the case in ridesharing, TNCs can transact with the market to buy and sell credits.

E. Synthesis

The overall scheme is shown in Fig. 4, which exhibits the advantageous property that there is no feedback from the actions of the TNC to the actions of users. Because there is no feedback from the actual route to the rider or interconnections between virtual routing and actual routing, a TNC cannot directly modify its routing algorithm to increase the limit on social cost since the determination of a rider's virtual route does not depend on actual routing. In reality, the feedback to riders is indirect and is related to long-term usage of the service and satisfaction therewith.

F. Additional considerations

We finish with a discussion of additional practical considerations. Firstly, we note that the integrity of this scheme would depend on an understanding between TNCs and the central service on the structure of delay functions D_i as well as an understanding on how often they can be changed. Updating the rules too frequently may cause service disruptions and, for this reason, we recommend that delay functions

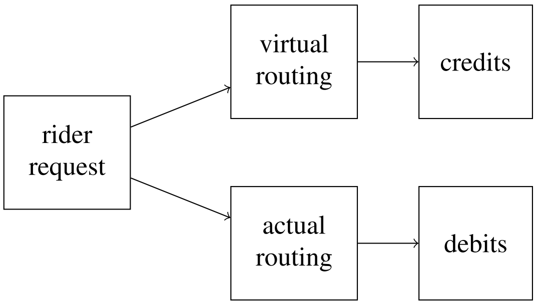

Fig. 4. Schematic of determination of credits and debts

remain on a published and infrequently-updated schedule and mirror the actual delay functions used in transportation planning.

Secondly, we suggest how one could incorporate privately-owned vehicles into the scheme. Again, the recommendation is a schedule that would take into account estimated origin-destination demand over the course of a day and incorporate this into the delay function schedule; this would have the effect of allocating a certain amount of road usage for privately-owned vehicles so that the road may be shared between multiple modes of transportation.

Finally, we recognize that there could be a lack of trust between TNCs and the central service. For example, TNCs may be apprehensive in providing ridership data to the central service in fear of revealing them to competitors. Not only should data be anonymized, it should be encrypted to the full extent possible, perhaps using [28], attempting to hide from competitors the amount of credits and debits issued.

V. Conclusion

In this work, we introduced a cap-and-trade system for the regulation of ridesharing. The scheme uses a virtual simulator to compute social costs of riders' maximal selfishness in the absence of ridesharing, and caps the actual social cost in the presence of ridesharing to be below the virtual social cost. It does this by issuing credits according to virtual routing and debits according to actual routing, and requiring that debts be repaid by credits while allowing both to be traded by transportation network companies (TNCs).

We performed analysis in the case of static networks and showed that the scheme incentivizes denser forms of transportation, like buses. We presented practical implementation steps and discussion and posited that, since there is no interconnection between actual routing and virtual routing, a TNC is not able to use its routing policy to affect the limit on social cost.

Acknowledgment

The authors acknowledge Dr. Matthew Brand of Mitsubishi Electric Research Laboratories for technical discussions.

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